Questioning the world – A dialectic of study and research

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We present and analyse a new statistical inquiry approach based on the theoretical framework of the anthropological theory of the didactic. The analysis is visualised in Q&A diagrams, and the main focus is on the conditions and constraints for shifting from a visiting monument perspective to paradigm of questioning the world. Data are from a design research project in which three teachers and a researcher study, design, implement, reflect and reverse an inquiry project. The results reveal new student and teacher roles and possibilities for a productive dialectic of study and research.

Keywords: Statistics education, the anthropological theory of the didactic, paradigm of questioning the world, inquiry, dialectics of study and research

Paradigm of questioning the world

Key statistical educational ideas suggest the use of real and motivating data, inquiry-based activities and appropriate technological tools (e.g., Bargagliotti et al., 2020). In the present article, we describe, analyse and discuss a case of statistical inquiry in the paradigm of questioning the world (PQW; Chevallard, 2015). The PQW is an inquiry approach that is in contrast to the more established paradigm of 'visiting monuments', in which students study a collection of answers determined by the curriculum. The PQW is driven by an open question, in which the question and the context are taken seriously, not only as an excuse to visit a fixed area of mathematics (Barquero & Bosch, 2015). In the inquiry process, the students are not only expected to answer questions, but also pose smaller derived questions, search for established answers in books or other media and share results. The PQW also includes aspects of 'visiting monuments', for example, when the students study and validate the answers found. An aim of the PQW is to free school mathematics from isolated content activities without a context and instead question the world.

The current paper addresses the following research question: What conditions and constraints can be observed for implementing the PQW in grade-five statistics?

Inquiry in statistics education

The role of inquiry has its origins in science education and, over the last two decades, has migrated into the field of teaching and learning of school mathematics. Inquiry contributes "to a shift in epistemology from seeing knowledge given as faith to knowledge based on thinking, reflection, experimentation and science" (Artigue & Blomhøj, 2013, p. 798). The aim is for students to play an active role in the process of learning. Also, statistics education, for example, the Guidelines for Assessment and Instruction in Statistics Education (Bargagliotti et al., 2020), stresses that students need to participate in statistical inquiry processes and reflect on data. In statistics education, we find different inquiry approaches, such as the cyclic model of plan, problem, data, analysis and conclusion

(Wild & Pfannkuch, 1999), along with the statistical problem-solving process, which includes an interaction between formulating statistical questions, collecting data, analysis and interpretation (Bargagliotti et al., 2020). PQW differ from other inquiry approaches by posing open 'big' questions and include an inquiry process, where students can follow many different paths.

Theoretical framework

We use the theoretical framework known as the anthropological theory of the didactic (ATD; Chevallard, 2015).

S*tudy and research paths* (SRP) are a design tool for teaching within the PQW (Chevallard, 2006). SRP start with a generating question, Q_0, before then following to different paths of student inquiry and study (Bosch & Winsløw, 2015). The students pose derived questions $Q_1 \ldots Q_n$, search for established answers $A^{\Diamond}$, study relevant works $W_1 \ldots W_m$, validate answers $A_1 \ldots A_o$ and develop a final answer, $A^{\heartsuit}$. In contrast to earlier inquiry approaches, the structure of SRP unfolds in a tree-like manner, and the Q&A (tree) diagram refers to the possible paths that students can follow. For more than 15 years, initial experimentations with SRP have focused on designing, realising and analysing SRP at nearly all educational levels (Barquero et al., 2022). In an international context there have been (as far as we know) no experiments with statistical SRP in primary school.

In ATD, knowledge is modelled as *praxeologies* (Chevallard, 1999), which consist of a *practice block* (or know-how) formed by a *type of task* and a *technique* and a *logos block* (or know-why) formed by a *technology*, discourse on the practice block and *theory* that explains and justifies the technology (further elaborated in; Østergaard, 2022). In statistics, knowledge is often categorised as statistical literacy and statistical reasoning (e.g., delMas, 2002). We define statistical literacy as engaging with types of statistical tasks, statistical techniques and technology, and we define statistical reasoning as an activity based on statistical technology and statistical theory (Figure 1) (further elaborated in; Østergaard & Larsen, submitted).

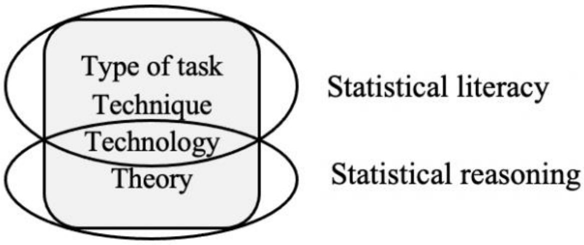

Figure 1. A praxeological illustration of statistical literacy and statistical reasoning

Hence, statistical literacy and statistical reasoning can be seen as a process of developing sequences of statistical praxeologies that gradually become more complex.

Methodology

The methodology of the current study is design research (DR; Bakker, 2018) involving three cycles of 1) *identifying PQW*; 2) *design of the SRP activity*; 3) *implementation*, *observation* and *reflections upon the students' learning*; 4) *reflections* and *revisions* of the SRP activity; and 5) a *posteriori analysis of the design and the values of PQW*. DR allows us to design an inquiry approach based on the theoretical principles of SRP. In our design, we have implemented elements of lesson study because we need to incorporate the teacher's experience in an SRP classroom involving developing and improving lessons from the perspective of the teachers (Miyakawa & Winsløw, 2009). The analysis in this paper mainly focuses on the third implementation and posteriori analysis. The implementation and reflections on students' learning have been video recorded, transcribed and created into Q&A diagrams. In the analysis we exemplify the conditions and constraints of SRP based on student modelling of data.

Context of the study

In the DR process, the first author worked with three fifth-grade mathematics teachers in the processes of planning, implementing, discussing and revising an eight hours long SRP (two times four hours). The three teachers participated in the study of SRP for a total of 50 hours, including interviews, workshops and design, implementation, reflection and revision of the SRP in three different classrooms.

19 fifth-grade students studied the generating question (Q_0) 'Are we physically active?' by using available media and the dynamic data visualisation and modelling tool TinkerPlots. Before the project was implemented, the students had no knowledge about how to work on projects in mathematics and only little knowledge about statistical analysis. To answer Q_0, the students studied answers in different media, for example, newspapers and science papers, and collected data about their own physical activities. The data included 15 questions including daily numbers of steps, time of sleep, time spent on digital and social tools and so forth. The answers found in media and the new raw data led the students to pose smaller questions, develop new answers and compare the new answers with established answers from media. In the SRP the students worked in small groups and discussed their results in whole class settings.

The process of study and research the question 'Are we physically active?'

$A^{\heartsuit}$ was the final answer. The answers $A^{\lozenge}_{2}\ldots A^{\lozenge}_{15}$ could be found in newspapers and science papers and led to the derived questions $Q_{2.1}\ldots Q_{15.1}$. To answer these questions, data were collected, explored and analysed, resulting in the answers $A_{2.1.1}\ldots A_{2.1.x}$. $W_1\ldots W_6$ were those works visited by teacher and students to validate answers and develop more advanced answers. The Q&A process of 'Are we physically active?' is visualised in Figure 2. In the Q&A diagram, it is observable that the students were actively engaged in the process of finding answers, posing questions and visiting works.

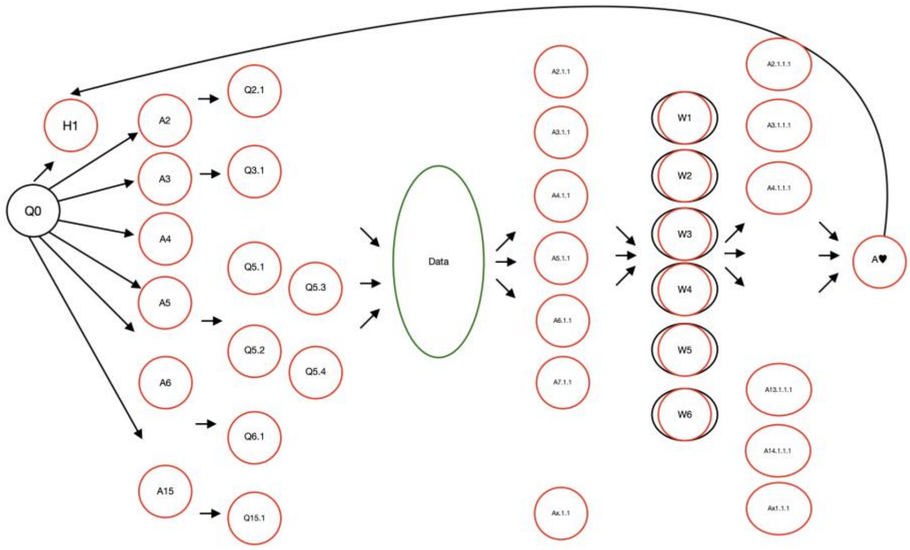

Figure 2. Q&A diagram. Students' questions and answers are in red, teachers in black (Østergaard & Larsen, submitted)

In the SRP, the students and teacher visited six statistical works in total: frequency, relative frequency, minimum and maximum value, mode, mean (W5, Figure 2) and types of diagrams.

Visiting the works of the mean

In the process of analysing the data in TinkerPlots, the students considered different averages like mean, mode or median. In some answers, the students found the arithmetic mean, for example, when the students balanced three sets in making a comparison (Figure 3, left) and when the students used different frequency graphs to conclude (Figure 3, right).

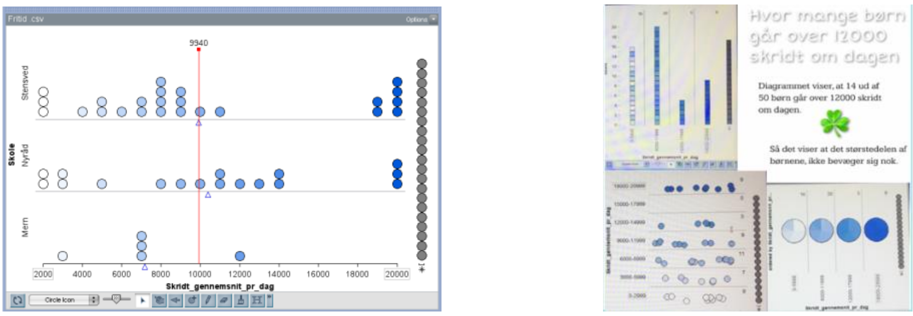

Figure 3. Finding the mean. (left) $Q_{5.3}$: What is the number of steps? $A_{5.3.1}$: Frequency graphs comparing number of steps in three grade five classes, including the mean (red line and triangles). (right) $Q_{3.2}$: How many students walk 12,000 steps? $A_{3.2.1}$: *Reading* frequency graphs and pie charts to conclude whether students are physically active—more or less than 12,000 steps

The techniques of finding the mean are *instrumented* and found in TinkerPlots by separating data, making frequency graphs or bar graphs and by pressing the triangular-icon button (mean). The instrumented process does not require logos directly. Earlier research recommends graphical representation to build students' understanding and as an algorithm for solving statistical problems (e.g., Konold & Harradine, 2014). For students to understand the mean, the teacher and students visit the works of the mean. Figure 4 illustrates the visit of the concept mean (W5).

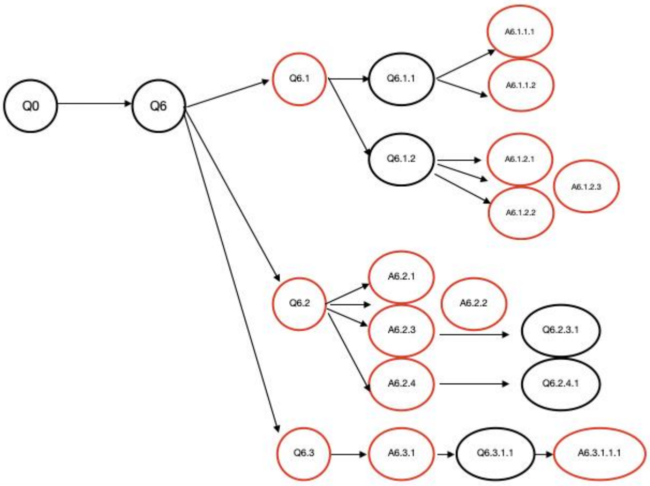

Figure 4. Q&A diagram, visit of the work of the mean

The visit is transcribed below.

Q_0: Are we physically active?

Q_6: What is a good question to pose to answer the overall question?

$Q_{6.1}$: What is the mean of the steps in the three schools?

${}^{w5}Q_{6.1.1.}$: What is the mean? What is that?

${}^{w}A_{6.1.1.1}$: It is just about how many there are.

${}^{w}A_{6.1.1.2}$: It is when you add it all together … in a way you add it, and then, you find how much … like if you have walked 10,000 steps the first day and 12,000 steps the next day. When you add the one to the other … you can give one [thousand] from the 12 [thousand] to the 10 [thousand]; then, the mean is that they have walked 11,000 steps.

${}^{w5}Q_{6.2}$: *A student—Mikkel—and the teacher stand in front of the class.* If I have to find the mean [of height] of me and Mikkel, what do I do then?

${}^{w5}A_{6.2.1}$: Then you have to take some from yourself and give it to Mikkel. Then, you will be at the same height.

${}^{w5}A_{6.2.2}$: It is possible to find the mean in TinkerPlots.

${}^{w5}A_{6.2.3}$: (Showing in TinkerPlots) If I take something from all the tall ones, the ones who are more than the mean and give it to the ones who are short, then the mean will be where the red line is. Then, they (bars) will be an equal height (Figure 4).

${}^{w5}Q_{6.2.4}$: How many steeps do the students in the three schools walk every day?

${}^{w5}A_{6.2.4.1}$: Show the mean of all the steps.

${}^{w5}A_{6.2.4.2}$: Show diagrams where the mean is an integrated part.

${}^{w5}A_{6.2.4.3}$: 13 students walk 12,000 steps or more. The mean is around 10,000 steps a day (Figure 3 and 5).

${}^{w5}Q_{6.2.4.3.1}$: The teacher points out that approximately 10,000 is not a good answer because TinkerPlots can calculate the exact mean.

${}^{w5}A_{6.2.4.4}$: Yes, the students are not physically active enough because they have to walk 12,000 steps a day.

${}^{w5}A_{6.2.4.4.1}$: … And they do not walk 12,000 steps a day that is a good argument.

${}^{w5}Q_{6.3}$: How much time do the students spend in front of a screen (tablet, computer …)?

$^{w5}A_{6.3.1}$: In school B, there are five students who spend more than 400 minutes in front of a screen every day … In school C, one student spends 540 minutes a day in front of the screen, but in total, most students in school B spend time in front of the screen.
$^{w5}Q_{6.3.1.1}$: When you say 'most'—do you then mean students?
$^{w5}A_{6.3.1.1.1}$: We look at the mean.

The type of tasks in visit of the concept mean, W5, is $^{w5}Q_{6.1.}$ 'What is a mean?' Here, $^{w5}Q_{6.1}$ bridges the praxis and logos with a request for the students to deconstruct the algorithm of the arithmetic mean and describe and elaborate the (instrumented) technique. The technology is the students' explanation of process, for example, in $^{w5}A_{6.1.2}$, the students explained the concepts of add and divide (give and take); in $^{w5}A_{6.2.1}$, the student and the teacher physically demonstrated and discussed how to give and take body length to each other; in $^{w5}A_{6.2.3}$, the students explained how a bar graph visualises the mean (Figure 4), and in $^{w5}A_{6.3.1}$, the students then compared the two averages, mode and mean, when describing 'how much'. Theory is not just an explanation of '*how*' to find/calculate the mean. The theory explains why or how results were produced or why a conclusion is justified. The theoretical part in the visit of the mean is not strong, but we see elements when the students in $^{w5}A_{6.2.4.3}$ used the mean in their argumentation of physical activity.

The above analysis is an example of how SRP in the PQW not only allowed the students to inquire but, in a dialectic process, inquire contexts, develop statistical praxeologies and further study these statistical praxeologies.

Inquiry approaches

The study and research of 'Are we physically active?' differed from some inquiry approaches in statistics education according to the dialectic of study and research. In some statistical inquiry approaches, it is somehow expected that students previously have studied statistical works, for example, how to analyse data, how to visualise data and which statistical descriptors to imply, and that the students only need to apply already known statistical work in the process of inquiry. We characterise these inquiry approaches: *First study, then inquire*. In the present SRP, before the projects, the students had little knowledge about how to analyse data. Therefore, the analysis process was experimental, for example, when the students investigated the possibilities in TinkerPlots, looked for patterns and used more informal arguments to answer Q_0. The dialectical process of research and study then became a prerequisite for students to develop sequences of increasingly advanced statistical praxeologies, a process in which students study works to progress the inquiry process.

Teacher's perspective

The two Q&A diagrams (Figures 2 and 4) illustrate how the teachers and students accepted a new didactical contract. The students (Figure 2) were clearly active in the process of finding already established answers, posing derived questions, exploring and analysing data and developing a final answer, while the teacher accepted the role of supervising. In the process of visiting the concept of the mean (Figure 4), the teacher took on a more traditional role, posing leading questions and guiding the students to an informal understanding of the arithmetic mean.

Our hypothesis is that it is only possible for the teacher to implement SRP if they take an inquiry stance, for example, inquire into the paths of how SRP can unfold, investigate new ways to scaffold students' learning and critically reflect on their own practice. In the present study, the teacher was actively engaged in the DR process, developing the SRP using the Q&A diagram as a design tool, observing how the SRP unfolded in the classroom, discussing students' learning opportunities and improving the design. This is in line with earlier research on teacher professional development (Jaworski, 2003).

Discussing conditions and constrains

In the analysis, we have focused on how changing the paradigm, from visiting works to questioning the world, requires new roles of both students and teachers, along with how teachers need to be involved in an inquiry practice to cope with the unpredictability in the different paths that study and research can follow.

Implementing PQW in primary school mathematics in Denmark will require significant modifications to the curriculum. The curriculum will then no longer be merely a collection of mathematical and statistical works to visit, but also big important questions (to the world) that need to be developed. A less pervasive approach for implementing SRP is how SRP can be integrated in the existing Common Mathematical Goals (Danish Ministry of Education, 2019). We have observed several aims already supporting a shift, for example, the two tiers of work in the Danish curriculum. First, the mathematical and statistical work, which has been formulated as very wide proposals like 'Students can do statistical inquiries; they can explore and present their own statistical inquiry; compare sets of data' (Danish Ministry of Education, 2019, p. 7). These aims support implementing SRP as an inquiry approach in mathematics and statistics. Second, mathematical competencies, for example, problem solving and modelling (Niss & Højgaard, 2019), which Danish mathematics teachers are required to incorporate together with the mathematical and statistical work. The content and inquiry approach are two sides of the mathematical discipline, together forming what is going to be taught and learned. In that sense, the implementation of SRP is officially substantiated in the Danish curriculum.

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