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Quantum error correction using multiple nitrogen-vacancy center qubits

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Abstract. Quantum error correction (QEC) is crucial for protecting quantum information from the decoherence caused by the interaction between the system and the environment. Many QEC techniques and algorithms have been proposed and demonstrated in various physical platforms at low temperatures, such as superconducting circuits, Rydberg's atoms, and trapped ions. At room temperature, the QEC realization with nitrogen-vacancy (NV) centers in diamond has become very attractive due to the promising nature of the centers that have a relatively long spin coherence time and can be initialized and read out optically. Here, we investigate the potential realization of a simple repetitive three-qubit QEC scheme in which three NVs are coupled via dipolar coupling. A single NV qubit has been protected using two other coupled NVs which act as ancilla qubits. In this configuration of three NVs, a single NV qubit is protected from bit or phase-flip errors. This work paves the way for realizing five-qubit QEC with NVs at room temperature to preserve a qubit against any arbitrary single-qubit error.

Keywords:
Quantum error correction, NVs, Bit flip error, Phase flip error, Toffoli gate decomposition

1. Introduction

QEC is crucial for quantum computers to perform at their best as it is nearly impossible to completely isolate sensitive quantum systems from the environment which causes decoherence and hence affects the accuracy[1, 2, 3]. Interactions with qubits are essential for performing operations and getting the required output[4]. The most suitable way to achieve efficient information processing is to apply practical QEC techniques[5]. Many QEC techniques have been developed and realized in different materials to address this limitation. Three qubit QEC has been implemented using superconducting qubits, trapped ions[6], neutral atoms[7], and a few other two-level systems. These physical qubits show very long coherence times for the required operations. However, a widespread shortcoming shown by most materials is their operation at very low temperatures, which is challenging.

NV centers have been proven promising for quantum computation at room temperature[8, 9], with a coherence time of up to 2 ms [10, 11]. The electronic structure of NV center is pivotal for their application in quantum technologies, featuring a complex setup of spin states and optical transitions [12]. The ground state of the NV center is characterized as a spin triplet state with three sub-levels: $m_s = 0, +1$ and -1. The zero-field splitting between the $m_s = 0$ and ± 1 is about 2.87 GHz at room temperature, primarily due to spin-spin interactions [8, 13]. In the absence of external magnetic field $m_s = \pm 1$ are degenerate while these lines split into $+1$ and -1 when a non zero magnetic is applied. Using this splitting $m_s = 0$ and $m_s = -1$ are being used as a qubit and are represented as $|0\rangle$ and $|1\rangle$ respectively [15].

Three qubit QEC has been realized in NVs in which two carbon nuclear spins have been used to protect a single electron spin via hyperfine interaction with high fidelity [16, 17]. Nuclear spins have a very long coherence time and high fidelity, compared to electron spins, for quantum operations but they take a relatively longer time for their operation. Haruyama has demonstrated strong dipolar coupling between

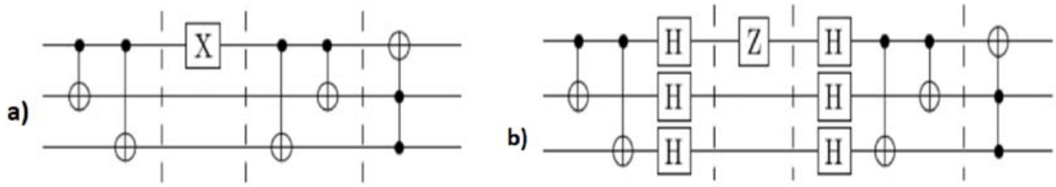

Figure 1. Error correction scheme for[14]. (a) Bit-flip and (b) phase-flip. Hdamarad gates are applied after decoding and before encoding to change the basis therefore the phase-flip error will be converted to the bit-flip error and corrected.

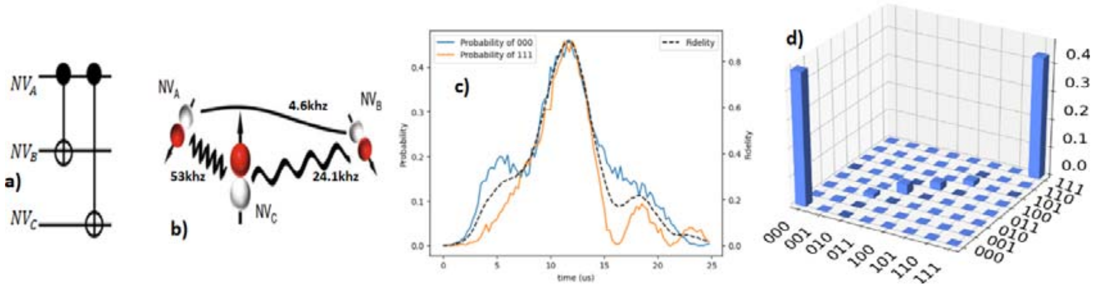

Figure 2. The encoding process is shown. (a) The circuit shows three-qubit encoding which results in an entangled GHZ state. (b) Three NVs are coupled together with different coupling strengths[10]. (c) Probabilities of the prepared state are shown along with the fidelity. This result has been reproduced from [11]. (d) Density matrix showing the encoded state after the measurement.

three NV electron spins at room temperature [10] where the longest coherence time of $428\mu s$ has been observed. Following this work, Mahony has simulated the different configurations of three NVs for different coupling strengths between adjacent NVs[11]. The most realistic configuration contains different coupling strengths for three NVs, resulting in a GHZ state after $11.7\mu s$. In this work, we have proposed a realization of a three-qubit QEC scheme that corrects a single phase-flip or a bit-flip error using entangled electronic spins of three NV centers. The GHZ state proposed by [11] has been considered as the initial step(Encoding) and then the effect of decoherence(Bit-flip, Phase-flip) has been simulated. The errors have been removed by applying a doubly controlled CCX gate.

2. Quantum Error Correction

The simple three-qubit QEC scheme has been used for the realization[14]. This scheme can correct only a single qubit error, meaning that if the error occurs on two qubits at the same time, then the scheme will no longer correct the error. The phase flip error does not affect the probability of state instead it affects the superposition of the coherent state. The extra Hadamarad gates are used for changing the basis after the encoding process which will convert the phase-flip error into the bit-flip error. In the end, both errors can be corrected in the same way. In real physical systems, the bit-flip or phase-flip error can happen with certain probability not necessarily with unit probability. We have simulated the effect of bit flip and phase flip by using a noise modal which shows that probability reduces significantly with the increase in the error probability.

The quantum circuit for the encoding process has been reproduced from the [11]. The encoded circuit is then reversed and simulated to give the decoded state both with and without error. After getting the state with simulated error the correction process has been done. Decomposed Toffoli gate has been implemented for the error correction in this scheme.

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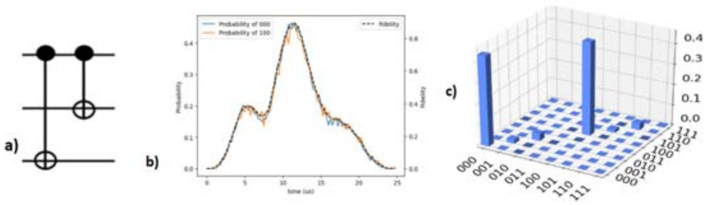

Figure 3. The decoded state without any decoherence. (a) It is the reverse process of encoding which gives the state that was encoded. (b) The probability of $|000\rangle$ and $|100\rangle$ is shown over time. Also, the fidelity of the state is shown. (c) The density matrix for the decoded state.

3. Entanglement generation and coupling of three NV centers

Mahoney [11] has followed the work from Haruyama[10] and simulated the coupling of three NVs namely NV_A, NV_B, and NV_C under dipolar interaction. Many different ideal and realistic configurations of three NVs were considered and analyzed in which the most realistic one is being discussed here. Practically the dipolar coupling depends upon the distance between two NVs which causes the different coupling strengths ($\mathrm{V_{AB}} \neq \mathrm{V_{AC}} \neq \mathrm{V_{BC}}$) between all three NV pairs, where $\mathrm{V_{AB}}$ is the coupling strength between $\mathrm{NV_A}$ and $\mathrm{NV_B}$, $\mathrm{V_{AC}}$ between $\mathrm{NV_A}$ and $\mathrm{NV_C}$ and $\mathrm{V_{BC}}$ between $\mathrm{NV_B}$ and $\mathrm{NV_C}$. In realistic configuration the value for $\mathrm{V_{AB}}$= 4.6 kHz, $\mathrm{V_{AC}}$ = 53 kHz and $\mathrm{V_{BC}}$ = 24.1 kHz resulting in the GHZ state after a free evolution time of 11.7 μs. The coherence time found for the $\mathrm{V_{AB}}$, $\mathrm{V_{AC}}$ and $\mathrm{V_{BC}}$ is 279 μs, 90 μs and 286 μs respectively. This coupled configuration is used as an encoded state for the three-qubit scheme of QEC. This scheme is the same for both errors the only difference is that Hadamarad gates are applied after encoding and before decoding which changes the basis from $|0\rangle$ and $|1\rangle$ to $|+\rangle$ and $|-\rangle$ and the phase-flip error is converted into bit flip error that can be corrected after decoding.

Results and discussion

The decoding process can be done exactly by reversing the operations in the encoding process. Figure(3) shows the result of the decoding process in which the state $1/\sqrt{2}(|000\rangle + |100\rangle)$ has been decoded. This state has been decoded ideally which means that there is no error and this can be seen from the high fidelity (0.89)of this state.

Now we will introduce a phase flip error on the first qubit and change the probability of error to see the effect on the fidelity of the required decoded state. The impact of the phase-flip and bit-flip will be the same after decoding as the scheme will convert the phase-flip error to the bit-flip error.

The error can also be applied to the second and third qubit but it is not significant as long as it is a single qubit error. Here only the first qubit has been protected, the other two qubits are always reinitialized after the complete round because they are used to carry the errors away from the first qubit. Figure (5) shows the behavior of the fidelity of the decoded state with error probability. The probability of error increases which decreases the fidelity and the state loses the information and becomes insignificant.

Figure 6 shows the complete scheme for QEC using three NVs and the partial trace has been shown for the first qubit to see the effectiveness of the error correction. NVa is initialized in $1/\sqrt{2}(\alpha\,|0\rangle + \beta\,|1\rangle)$ while NVb and NVc are in $|0\rangle \otimes |0\rangle$ then they become entangled. The probability of error is increased which causes a decrease in the fidelity of the quantum state. The error correction is done using the Toffoli gate which only changes the output of NVa which is a protected qubit. In Figure 6(b) the partial trace for NVa is shown in which it can be seen that the under first qubit error NVa has undergone error correction and the initial state has been restored. Toffoli gate (CCX) has been used to correct the error from NVa but cannot be implemented directly. A suitable decomposition for the CCX gate, which is composed of single and two-qubit gates, has been discussed by [18].

Implementing the Toffoli gate directly into the configuration of three NVs is quite challenging because

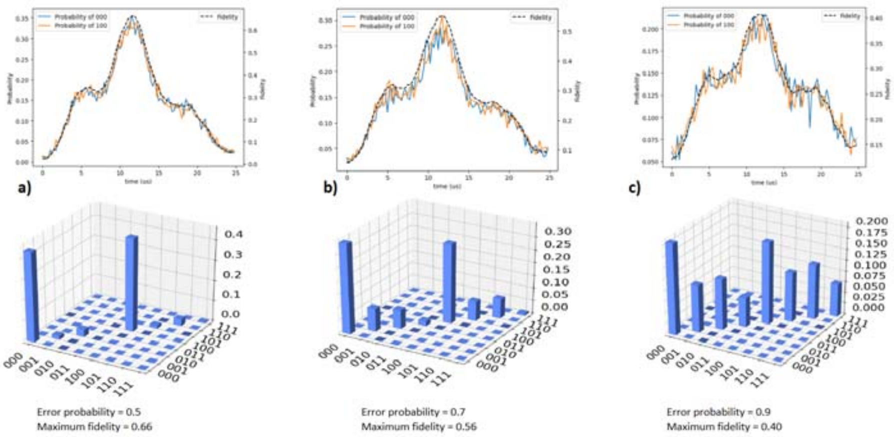

Figure 4. Decoded state having an error on the first qubit with different probabilities. By increasing the probability of error the fidelity of the state decreases.

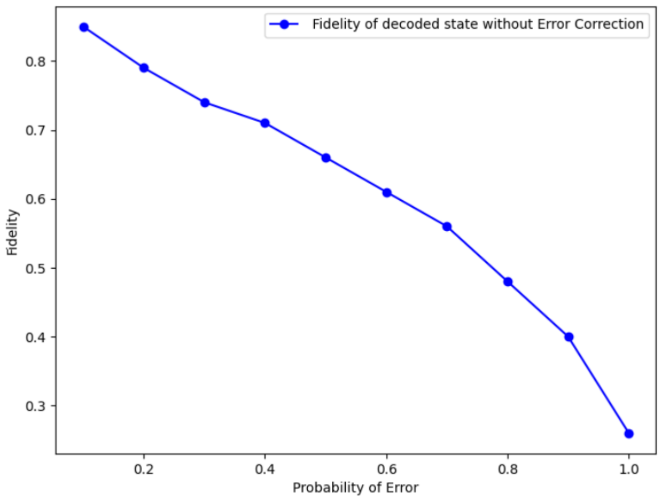

Figure 5. Fidelity of decoded state with increasing probability of error without correction.

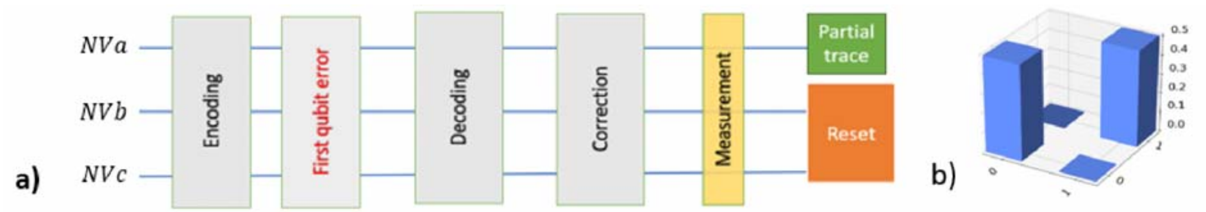

Figure 6. The complete scheme for error correction. (a) shows all the steps in the scheme while (b) shows the partial trace NVa which has been protected.

of individual control and manipulation. Many decompositions have been proposed for the Toffoli gate which are composed of single and two-qubit gates. Here we have used a suitable decomposition, see figure 7, which

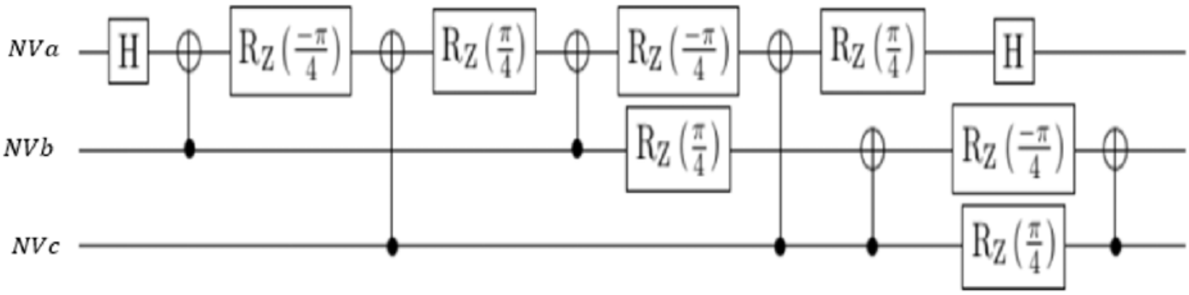

Figure 7. Toffoli gate decomposition into single and two-qubit gates

has less number of gates, and the gates are mostly similar which is easy to realize in NVs.

4. Conclusion

A practical scheme for three-qubit QEC has been proposed. The encoded state has been decoded with different error probabilities and the fidelity behavior has been shown. NVa has been subjected to errors and then corrected which is shown in the partial trace. Decomposed Toffoli gate has been used which is more practical than the actual gate. The exact frequency of the microwave has not been specified for manipulating gates in the decomposed Toffoli gate. This scheme can be more successful by specifying the exact frequency values for all three NVs and the pulse duration to realize the Toffoli gate decomposed in single two-qubit operations.

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Code availability

All codes used to produce simulations in this work are available from authors on reasonable request

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