Bernoulli Trial 2022

1: (2 minutes)

Suppose A is a subset of B and C is a subset of D. If $A \cup C = B \cup D$ and $A \cap C = B \cap D$, then $A = B$ and $C = D$.

2: (2 minutes)

For any positive integer n, let $s(n)$ denote the sum of digits of 2^n. Then there exists a positive integer n such that $s(n) = s(n+1)$.

3: (3 minutes)

The set of all composite odd positive integers less than 121 can be written as a union of 3 arithmetic progressions. (1 is not composite.)

4: (4 minutes)

If $x < y < z$ are positive integers such that $4^x + 4^y + 4^z$ is a square, then $z - 2y + x = -1$.

5: (4 minutes)

For any positive integer n, then

$$\#\{i \in \mathbb{Z} \colon 0 \leq i \leq n, 2 \nmid \binom{n}{i}\}$$

is a power of 2.

6: (3 minutes)

There are infinitely many positive integers N with the following property: if $1 < k \leq N$ and $\gcd(k, N) = 1$, then k is a prime.

7: (3 minutes)

$$\sum_{\substack{m,n=1\\ \gcd(m,n)=1}}^{\infty} \frac{1}{(mn)^2} \notin \mathbb{Q}.$$

8: (3 minutes)

For any positive integer n, there exists a circle in $\mathbb{R}^2$ whose interior contains exactly n points in $\mathbb{Z}^2$.

9: (3 minutes)

Let $P(x)$ be a polynomial of degree m and let $Q(x)$ be a polynomial of degree n such that all the coefficients of P and Q are either 1 or 2022. If $P(x) \mid Q(x)$ as polynomials, then $m+1 \mid n+1$.

10: (4 minutes)

The set $\{1, 2, \ldots, 2022\}$ can be colored with two colors such that any 18-term arithmetic progressions contains both colors.

11: (4 minutes)

For any increasing sequence $\{a_n\}_{n=1}^{\infty}$ of positive integers, there exists a positive integer k such that the sequence $\{k + a_n\}_{n=1}^{\infty}$ contains infinitely many primes.

12: (4 minutes)

$$\int_0^{\pi} \ln\left(\frac{5}{4} - \cos x\right)\,dx > e^{-2022}.$$

13: (4 minutes)

For any integer $n > 1$, the smallest prime divisor of n is less than the smallest prime divisor of $3^n - 2^n$.

14: (4 minutes)

$$\lim_{n\to\infty} \frac{n}{2^n} \sum_{k=1}^{n} \frac{2^k}{k} = 2$$

15: (4 minutes)

Suppose R is a rectangle that can be tiled using rectangles each of which has at least one side of integral length. Then R also has at least one side of integral length.