Experimental Evaluation of Euler Sums

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Abstract

Euler expressed certain sums of the form

$$\sum_{k=1}^{\infty}\Bigl(1+\frac{1}{2^m}+\cdots+\frac{1}{k^m}\Bigr)(k+1)^{-n},$$

where m and n are positive integers, in terms of the Riemann zeta function. In [Borwein et al. 1993], Euler's results were extended to a significantly larger class of sums of this type, including sums with alternating signs.

This research was facilitated by numerical computations using an algorithm that can determine, with high confidence, whether or not a particular numerical value can be expressed as a rational linear combination of several given constants. The present paper presents the numerical techniques used in these computations and lists many of the experimental results that have been obtained.