Proceedings of International Structural Engineering and Construction, 10(1), 2023
Innovative Theory and Practices in Structural Engineering and Construction
Edited by Karatas, A., Iranmanesh, A., Gurgun, A., Yazdani, S., and Singh, A.

ISSN: 2644-108X
www.doi.org/10.14455/ISEC.2023.10(1).STR-85

DYNAMIC ANALYSIS OF THE FOOTBRIDGE EXPOSED TO PEDESTRIAN LOADING

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The submitted paper is dealing with the phenomenon of the human induced vibration caused by walking pedestrians. Pedestrians naturally work as a source of a periodic load, which is typically in the resonance range with some of the natural frequencies of the crossed footbridge. Typical pacing frequency of the normal walk is close to 2 Hz. Unfortunately, the most of footbridges have some of the natural frequencies in the range 1.5 – 3 Hz. These values can be easily reached by pedestrians, which could lead to the uncomfortable level of the structural vibration as could be seen at the Millennium bridge in London. In the submitted paper, we are going to subject the footbridge, acting as the simply supported beam, to the theoretical dynamic analysis. The dynamic analysis will be divided into the modal analysis and the force vibration analysis, which is accomplished by the modal decomposition method. This approach allows us to execute direct integration of the independent equations of motion. Total number of these modal equations is significantly less than number of equations directly assembled by the Finite element method. The pedestrians will be mathematically described by the periodic shifting force with constant velocity of motion.

Keywords: DLF models, Modal analysis, Forced-vibration analysis, Serviceability of footbridges.

1 INTRODUCTION

The human-induced vibration is quite old phenomenon, which must be still considered in the design phase of a footbridge in order to prevent feasible unpleasant consequences and additional costs for damping devices.

In the humankind history, we can find several events, which led to the damage of the bridges under the loading of the crowd of people. The first such accident can be tracked down at the age of ancient Rome, where the stone bridge collapsed under retreated army. Nowadays we are naturally not able to state, what was the precise causation of the collapse. It is not possible to say, if the main reason was in the dynamic effects of running soldiers or their static weight.

2 INVESTIGATED STRUCTURE

The investigated structure, which was chosen for the dynamic analysis presented in this paper, acts as simply supported beam with the span 25.1 m. The cross-section creates reinforced-concrete slab and six steel box-section girders with height of 520 mm. Horizontal distance between individual steel beams is 1100 mm. Height of the reinforced concrete slab is 180 mm. The structure is used almost to the full capacity in a major part of the day since it transfers the pedestrian traffic across the trunk road to the underground station in the urban area of Prague.

There are several reasons for choosing this specific structure. Mainly, it is its simplicity of the static behavior, thereby the additional errors in the phase of modelling, caused by uncertainties in the static acting of complex structures, are omitted (Figure 1).

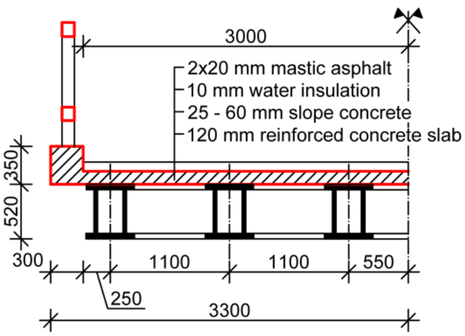


Figure 1. The cross-section (left) and a view the investigated structure (right).

3 MODELLING OF PEDESTRIANS

Currently, the designers and researchers can choose between several options for mathematical modeling of the contact forces induced by pedestrians during walking and running. According to the worldwide and the well-known guideline SÉTRA (2006), the pedestrians should be modeled as direct consideration of the Ground Reaction Forces (GRF), which must be distinguished separately for walking and running. These forces can be used both for vertical and lateral directions. Although this approach seems to be very attractive, the problem is in the complexity of the acting loading, which must be considered as movable along the structure and not every commercial FEM software, used by designers, permits it. On the other hand, the second enlarged approach called Dynamic Loading Factor (DLF) enables both, the movable and stationary usage. This model is based on the theory of Fourier's series, where adequate DLFs and phase shifts are defined for each harmonic. The big advantage of the DLF approach is precisely its stationarity, which allows very simple usage in almost every FE software. But on the other hand, this advantage is simultaneously its big disadvantage, since the stationary force produce greater values of acceleration than the movable one. Several authors provided DLF models for vertical and lateral directions, see e.g., Bachmann and Ammann (1987) and Živanović *et al.* (2005). In this paper, we used DLF model for vertical component of the contact force, which is defined as shown in Eq. (1):

$$F(x,t) = G \cdot \left[1 + \sum_j \alpha_j \sin\left(2\pi f j t - \varphi_j\right)\right] \delta(x - vt) \tag{1}$$

where G is static weight of pedestrian, α_j stands for DLF of actual harmonic, f means pacing frequency, t denotes time, δ is Kronecker delta and v stands for the velocity of motion.

4 COMPUTATIONAL MODEL OF THE FOOTBRIDGE

The two- and three-dimensional theoretical models of the investigated footbridge were created for the theoretic modal analysis. The proportional Rayleigh's relations were used for damping estimation. It means that the damping matrix **C** was assembled as linear combination of the mass and stiffness matrices: $\mathbf{C}=\alpha\mathbf{M}+\beta\mathbf{K}$. The coefficients of linear combination (α and β) were calculated under the assumption that the least amount of damping is received by the first natural frequency. The logarithmic decrement of $\vartheta = 0.09$ was evaluated from the experimentally obtained data.

The first and the simplest model of the footbridge was based on the two-dimensional beam according to the Bernoulli-Euler's bending theory, see Figure 2. This model was primarily used for verifying the programmed procedures and its validation. The stiffness **K** and mass **M** matrices were derived by the FEM ($EI_y = 5.1 \times 10^6$ kNm^2, $\mu = 5.18 \times 10^3$ kg/m).

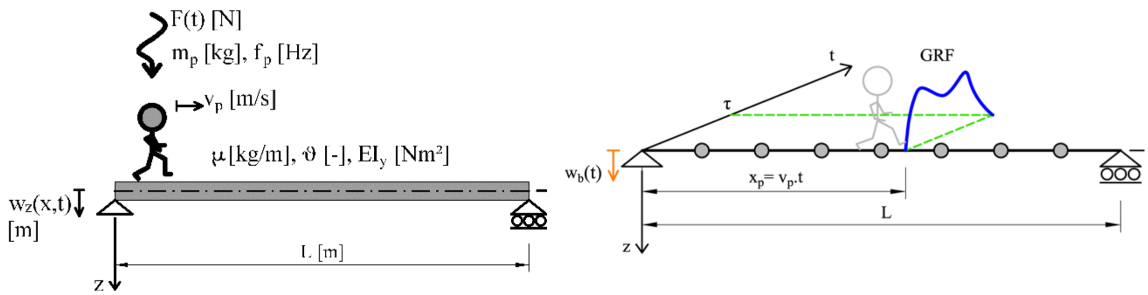


Figure 2. Bernoulli-Euler's 2D beam model (left) and its discretization (right).

The self-programmed inverse iteration method with Gramm-Schmidt orthogonalization was used to determine the natural frequencies and corresponding mode shapes of the computational model, which were normalized with respect to the mass matrix.

The three-dimensional theoretical model was formed in software Dlubal-RFEM 5-31-01. The steel girders were simplified by elastic 1-D beam elements and the concrete slab by 2-D elements with constant thickness. Connection between the girders and the concrete slab was considered as perfectly rigid. The modal analysis was treated in the GUI of the software using the Subspace iteration method. The resultant mode shapes were normalized with respect to the mass matrix of the structure.

5 RESPONSE CALCULATION

The models of the footbridge were loaded by the DLF force model, which was in motion along the footbridge deck with constant velocity of motion. In a general time moment t_j, the force acts somewhere between the nodes due to the motion of the contact point, and therefore has to be recalculated into the equivalent nodal loading. This procedure was done by the two-dimensional linear basis functions, which will degenerate into simple one-dimensional linear basis functions in the case that the load is moving along the edge (e.g., A-B Figure 3) of the square element (beam model). One of the four basis functions as well as the recalculation of the fictive force of 1 kN into the nodes A-D is depicted in Figure 3. When we had finally the vector of nodal forces, we transformed the equations of motion (EoM) to the modal space by the modal decomposition method, see e.g., Baťa *et. al.* (1987) as shown in Eq. (2).

$$\ddot{q}_j(t) + 2\xi_j\omega_j\dot{q}_j(t) + \omega_j^2 q_j(t) = \phi_j^T \mathbf{f}(t) \tag{2}$$

where $\ddot{q}_j$(t) means modal acceleration, ξ_j is critical damping ratio, ω_j denotes j^{th} natural frequency, $\dot{q}_j$(t) stands for modal velocity, q_j(t) expresses modal deflection, ϕ_j^T is the j^{th} mode shape and *f*(t) means loading vector. After assembling the EoM in modal domain, the resultant responses were calculated by the Newmark's implicit integration scheme, see e.g., Kabe and Sako (2020), with step Δt=0.03s. Lastly, the calculated responses were inversely transformed back to the original domain.

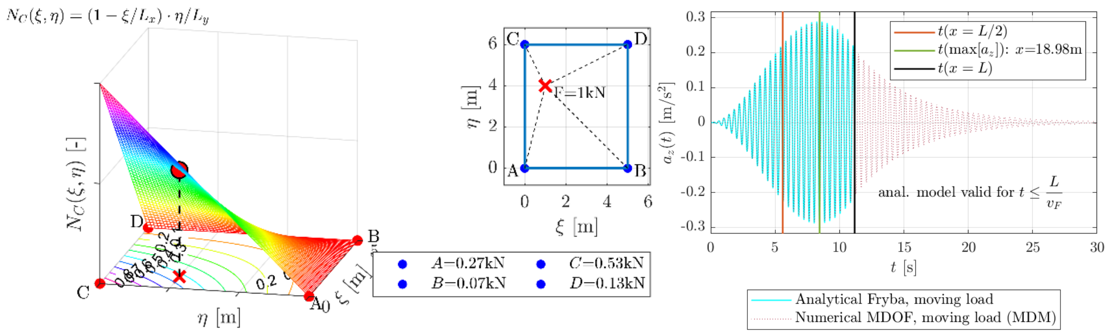


Figure 3. Recalculation process of the fictive force into nodes A-D.

The pacing frequency f (1) of the periodic force corresponds to the jogging, whereas Bachmann's DLF model is defined for running with $f > 3.2$ Hz. This is the reason why we have decided to interpolate in order to obtain the equivalent DLF model between marginal values for running and walking.

6 EXPERIMENT

Firstly, the experiment was aimed only at evaluation of the main natural frequencies, which yielded primarily the first natural bending frequency $f(1)$=2.72 Hz. The excitation of the footbridge during this phase was realized by the force impulses of jumping people. The more detailed experimental modal analysis was carried out using the electrodynamic exciter TIRAVIB 5140. The response was measured by piezoelectric accelerometers in the previously defined network of nodes. Second part of the experiment was focused on the forced vibration. The nodes in quarters and in the middle of the span were chosen and pedestrians excited the structure in the frequencies $f(1)$ and $f(1)/2$.

7 RESULTS AND CONCLUSION

Figure 4 and Table 1 summarize the obtained results of the theoretical and experimental modal analysis and their comparison. The agreement between frequencies was measured by the difference $\Delta(j) = [(f_{\text{th.}}(j) - f_{\text{ex.}}(j))/f_{\text{th.}}(j)]\cdot 100$.

The modal assurance criterion (MAC) was used, for prediction of the accordance level between theoretically and experimentally obtained mode shapes defined in ČSN 73 6209 (Czech Standardization Institute 2019).

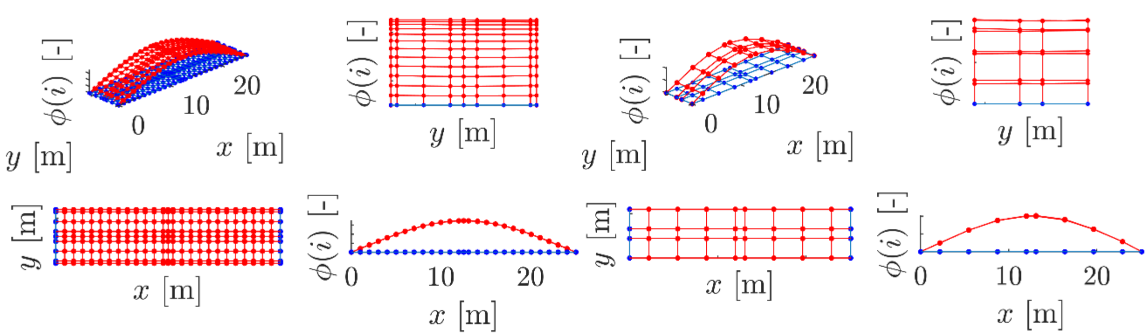
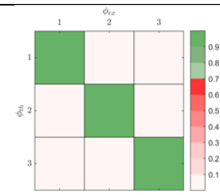


Figure 4. The first computed mode shape (left) and the first measured mode shape (right).

Table 1. Measured and computed natural frequencies and graphical representation of the MAC matrix.

Frequency mode	**Theor. [Hz]**	**Exp.[1] [Hz]**	**Exp.[2] [Hz]**	**Δ [%]**	**MAC matrix**
1 – vert. bend.	2.71	2.72	2.70	-0.4	
2 – torsional	4.59	5.06	-	-10.2	
3 – vert. bend.	8.81	9.31	8.88	-5.7	
4 – torsional	12.12	12.28	-	-1.3	
5 – vert. bend.	20.45	-	19.75	-	

As can be seen in Table 1 and Figure 4, the theoretical and experimental mode shapes are in excellent agreement. Following Figure 5 delineates the results of the theoretical dynamic analysis with pacing frequency $f = 2.71$ Hz. The acceleration at the midspan can be noticed in Figure 5 (left), while the right side indicates the deflection of the nodes at time $t = 5.04$ s loaded by the couple of synchronized pedestrians.

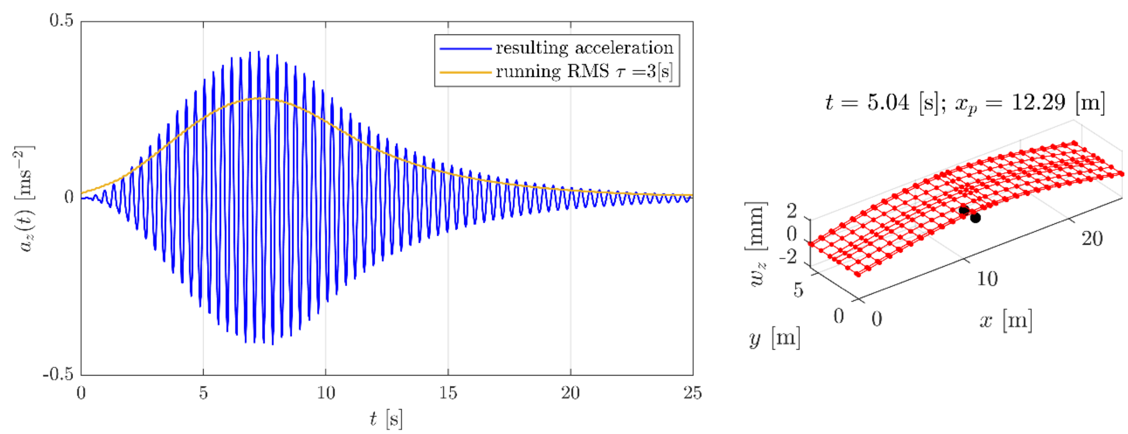


Figure 5. Resulting acceleration at the midspan (left) and shape of excited deflection at time $t = 5.04$ s.

Table 2 summarizes the maximal and minimal values of acceleration and compared them with the in-situ experiment. The theoretical accelerations were calculated at the 2-D and 3-D for DLF models according to Bachmann (#1), Schultze (#2) and Kreuzinger (#3).

Table 2. Measured and computed maximal and minimal values of acceleration.

#	Frequency	Max. [ms^{-2}]	Min. [ms^{-2}]	Max. [ms^{-2}]		Min. [ms^{-2}]	
				2-D	3-D	2-D	3-D
		Experiment		Theory			
1	2.71 Hz	0.41	-0.49	0.48	**0.42**	-0.48	**-0.41**
	1.36 Hz			0.05	0.18	-0.05	-0.18
2	1.36 Hz	0.09	-0.09	0.05	0.06	-0.06	-0.05
3	1.36 Hz			0.11	**0.10**	-0.10	**-0.10**

It is obvious, that the best accordance between experiment and calculation was achieved for Kreuzinger model in case of walking. Bachmann's model yielded very good agreement with experimental data. The programmed procedures for obtaining equivalent nodal loading were verified and compared with the analytical solution of the simply supported beam provided by Frýba (1989) with great conformity, see Figure 3 (right). The footbridge structure did not exceed the criterion limit for pedestrian comfort of 0.7 ms^{-2}, therefore we can state that the footbridge is sufficiently usable. The review of individual approaches in the national guidelines may be found in (Banas and Jankowski 2020, Banas 2020).

In this study we presented the results of the theoretical and experimental modal and forced-vibration analysis. Structure was loaded by moving periodic forces, which represent perfectly synchronized pedestrians. The greater number of synchronized pedestrians should be modeled as an action of one pedestrian multiplied by square root of the number of all pedestrians, see e.g., Zoltowski *et. al.* (2022).

Acknowledgements

The submitted paper has been created thanks to the project No. SGS22/089/OHK1/2T/11, which is hereby gratefully acknowledged.

References

Bachmann, H., and Ammann, W., *Vibrations in Structures Induced by Man and Machine*, Structureal Engineering Documents, 3e, International Association for Bridge and Structural Engineering, Zürich, 1987.

Banas, A., and Jankowski, R., *Experimental and Numerical Study on Dynamics of Two Footbridges with Different Shapes of Girders*, Applied Sciences, MDPI, 10(13), 4505, July, 2020.

Banas, A., *Assessment of Pedestrian Comfort on Footbridges: Case Study*, Builder, PWB Media Zdziebłowski Sp. J., 277(8), 26-31, July, 2020.

Baťa, M., Plachý, V., and Trávníček, F., *Dynamics of Building Structures*, SNTL/ALFA, Prague, 1987.

Czech Standardization Institute, *ČSN 73 6209 (508313): Loading Tests on Bridges*, Czech Office for Standards, Metrology and Testing, January, 2019.

Frýba, L., *Vibration of Solids and Structures under Moving Load*, 2nd Edition, Academia, Prague, 1989 (in Czech).

Kabe, A. M., and Sako, B. H., *Structural Dynamics: Fundamentals and Advanced Applications*, Academic Press, London, June, 2020.

SÉTRA, *Footbridges – Assessment of Vibrational Behaviour of Footbridges under Pedestrian Loading*, Technical Guide, AFGC, Paris, October, 2006.

Živanović, S., Pavic, A., and Reynolds, P., *Vibration Serviceability of Footbridges under Human-induced Excitation: A Literature Review*, Journal of Sound and Vibration, Elsevier, 279(1-2), 1-74, January, 2005.

Zoltowski, K., Banas, A., Binczyk, M., and Kalitowski, P., *Control of the Bridge Span Vibration with High Coefficient Passive Damper: Theoretical Consideration and Application*, Engineering Structures, Elsevier, 254, 113781, March, 2022.