Structural Integrity of CD-ROMs in High Speed Drives

Report prepared for RM plc

by

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6th August 2001

Summary

This report investigates the mechanics of the use of CDs in high speed disk drives. A stress analysis of a rotating disk is undertaken and it is shown that stresses in the disk scale with the square of rotational speed. It is concluded that failure of an uncracked disk in a 52 speed drive by material yield is unlikely. Fracture mechanics is then employed to investigate the stability of cracks in CDs. A critical crack length of 12.5mm is predicted for a disk operating in a 52 speed drive. This compares with experimental observations of failures from 6 mm cracks. Reasons for the discrepancy are discussed. It is shown that the critical crack length scales inversely with the fourth power of the rotational speed. This means that cracks in drives operating at 40X or slower are likely to cross the index track of the CD before they reach the critical size for fast fracture. In contrast, fast fracture appears to be possible for cracked disks in higher speed drives. Some consideration is given to disk containment issues

Contents

1. Introduction

Over the last five years CD-ROMs have become the industry standard media for distribution of PC software. With the advent of writeable CD-R disks, they are increasingly becoming used for data storage and backup. Hence, almost all PCs now have a CD or DVD drive. Pressure to reduce access times has led to progressive increases in the rotational speed of the drives. Original single speed drives rotated at approximately 200 rpm, whereas recent 52 speed drives rotate at approximately 10500 rpm. There are obvious implications for the structural integrity of the CD disk, since stresses in the disk caused by centrepetal acceleration as the disk rotates will increase with speed. Moreover, should a disk failure occur, the energy released will also be much higher with the higher speed drives, so that there may be a potential hazard to the PC user. RM have recently experienced a small number of customer reports of disk failures in 52 speed drives, where the disk fragments have not been completely contained within the disk drive. This report has therefore been commissioned in order to understand the structural integrity issues associated with the use of such high speed drives.

The geometry of a typical CD is shown in Fig. 1. The disk has an inner radius, r_i of 7.5mm and an outer radius, r_0, of 60mm. The thickness of the disk may vary slightly, but a typical disk was measured and found to have a thickness of 1.0mm. Disk material is normally a Polycarbonate thermoplastic, such as the Dow Calibre 1080 DVD resin [1]. Relevant material properties are [1]:

Density, ρ (kg/m^3)	1200
Young's Modulus (MPa)	2300
Tensile Yield Strength (MPa)	60
Izod Impact Strength (notched) (J/m)	270

Clearly, there will be some variation in these properties according to the precise formulation and moulding process, but these figures may be taken as typical for a good quality resin. A more precise figure of density was obtained by weighing a CD to determine its mass (16g) and dividing by the volume to give ρ = 1437 kg/m^3.

2. Nomenclature

The following nomenclature will be used in this report

D	Amplitude of disk vibration
E	Kinetic energy of rotating disk
F	Out of balance force
I	Moment of inertia about axis of rotation
K	Stress Intensity Factor
K_{IC}	Critical Stress Intensity Factor, or Fracture Toughness
m	Mass of disk
P	Contact force between spindle and disk
p	Contact pressure between spindle and disk
r	Radial co-ordinate
r_I	Inner radius of CD
r_o	Outer radius of CD
t	Time

w	Disk thickness
x	Displacement of disk
α	Half-angle of disk/spindle contact
ν	Poisson's Ratio for disk material
θ	Angular co-ordinate
ρ	Density of disk material
ω	Angular velocity of disk

3. Stress Analysis

An appropriate initial approach to the problem will be to determine the stresses in the rotating CD and compare these to the yield stress of the material, given above. This places a practical upper limit on the use of the disk, since if the material is loaded beyond this point, permanent deformation will occur. Our initial assumption will be that the disk remains elastic, i.e. that the stresses remain below the yield stress and there is no permanent deformation. In this regime, strains and displacements will be proportional to stress and the well-known equations of elasticity theory may be employed. Once the analysis has been completed we will be able to check whether this initial assumption was, indeed, correct. We shall assume that the disk is operating at a constant speed and that stresses induced by the drive mechanism (which provides the small torque necessary to overcome air resistance) are small compared to the stresses induced by centrepetal acceleration. In these circumstances, the disk stresses are independent of angular position and depend only on radius, r. The three principal stress directions will be:

(i) A radial stress, σ_{rr}
(ii) A tangential, or hoop, stress, $\sigma_{\theta\theta}$
(iii) A stress normal to the plane of the disk, σ_{zz}

The CD is very thin compared to its other dimensions and this allows us to make the assumption of 'plane stress' [2] so that the σ_{zz} stress component can be taken as zero. We are therefore only concerned with stresses in the radial and hoop directions. For the axisymmetric case that we have here (no angular variation), the usual partial differential equations of elasticity can be reduced to a single ordinary differential equation, which has the well-known result for the case of a rotating disk [2]:

$$\sigma_{rr} = A - \frac{B}{r^2} - \left(\frac{3+\nu}{8}\right)\rho\omega^2 r^2 \qquad (1)$$

$$\sigma_{\theta\theta} = A + \frac{B}{r^2} - \left(\frac{1+3\nu}{8}\right)\rho\omega^2 r^2 \qquad (2)$$

where ω is the angular velocity of the disk and ν is Poisson's ratio for the material. A and B are arbitrary constants in the solution, whose values must be determined from the boundary conditions. It is convenient first to normalise the equations by dividing by ω^2 to give

$$\frac{\sigma_{rr}}{\omega^2} = \frac{A}{\omega^2} - \frac{B}{\omega^2 r^2} - \left(\frac{3+\nu}{8}\right)^2 \rho r^2 \quad (3)$$

$$\frac{\sigma_{\theta\theta}}{\omega^2} = \frac{A}{\omega^2} + \frac{B}{\omega^2 r^2} - \left(\frac{1+3\nu}{8}\right)\rho r^2 \quad (4)$$

A precise value for ν was not available, but may be taken to be 0.3 for a wide range of engineering materials and this value was therefore assumed.

The relevant boundary conditions are that the disk has free (i.e. unloaded) boundaries at the inner and outer surfaces. For a free boundary, the stress component normal to the boundary must be zero, giving in our case:

$$\sigma_{rr} = 0 \qquad r = r_i \quad (5)$$

$$\sigma_{rr} = 0 \qquad r = r_o \quad (6)$$

Substitution of these two boundary conditions into equations (3) and (4) is sufficient to determine the two arbitrary constants, A and B. For $r_i = 7.5 \times 10^{-3}$ m, $r_o = 60 \times 10^{-3}$, $\rho = 1437$ kg/m^3 and $\nu = 0.3$, this gives $A/\omega^2 = 2.13$ and $B/\omega^2 = 1.16 \times 10^{-4}$. Hence

$$\frac{\sigma_{rr}}{\omega^2} = 2.13 - \frac{1.16 \times 10^{-4}}{r^2} - 593 r^2 \quad (7)$$

and

$$\frac{\sigma_{\theta\theta}}{\omega^2} = 2.13 + \frac{1.16 \times 10^{-4}}{r^2} - 341 r^2 \quad (8)$$

where r is in metres, ω in rad/s and σ in N/m^2. It will readily be appreciated that the stresses at any radial position, r, vary in proportion to ω^2, so that a CD in a 52X drive will be subjected to stresses 2704 times higher than those if it were in a single speed drive.

The variation of normalised stress with radial position. It will be seen that the hoop stress is everywhere higher than the radial stress and that the maximum value of stress is the hoop stress component at the inner radius. This is consistent with the experimentally observed location of cracks, which appear to start at the inner radius and propagate radially (i.e. normal to the hoop stress direction – cracks normally propagate in a direction normal to the largest principal stress). Inserting the specific rotational speed in a 52X drive (10500 rpm = 1099 rad/s) allows us to calculate the value of the maximum stress, which is 5.04 MPa. This may be compared to the quoted yield strength for the material of 60 MPa. It is clear that the material is still a long way from reaching its yield strength, so that our initial assumption that the disk remains elastic is correct. Further, there would appear to be no danger of an uncracked disk failing in the drive by yield.

4. Fracture Mechanics

The procedure described in the previous section represents a typical stress analysis of an engineering component for design against material yield. In many situations, however, this is not adequate to ensure the integrity of the component since we must recognise that the component may contain flaws or cracks which might propagate and cause component failure during its lifetime. It is therefore usual with safety-critical components to investigate the component's damage-tolerance, i.e. to determine the size of the largest crack which might be tolerated before the component fails. In this way we will be able to assess how frequently to inspect the component for cracks and what size of crack might be considered hazardous. It is perhaps appropriate to start by explaining that a component subject to cracking generally experiences three phases during its life:

(i) *Crack Initiation* – where microscopic flaws in the material develop and coalesce to form an observable crack

(ii) *Stable Crack Propagation* – where cracks propagate in a stable and controlled manner under load. Often this process takes place under cyclic (time-varying) load and is known as fatigue crack propagation [3]

(iii) *Unstable Crack Propagation* – Once the crack reaches a certain length it may become unstable and propagate extremely fast, at velocities approaching the speed of sound in the material. Usually, of course, this leads to component failure.

In the context of the CD-ROM problem we can see that cracks might initiate by the development of small flaws in the area of maximum stress at the inner radius. These might then propagate by fatigue as the disk is repeatedly spun up in the drive and then stopped. Eventually a critical crack size will be reached and the disk will break by unstable crack propagation.

In order to understand why cracks reach a critical size it is perhaps most helpful to consider the energy-approach developed by Griffith [4]. As a crack propagates it allows the surrounding material to relax and stored elastic strain energy is released. The larger the crack, the greater the affected volume of material and more energy is available for release. As the crack propagates, however, energy is consumed in the creation of new crack surfaces (which have a higher energy-state than when bonded to adjacent atoms). Thus, Griffith postulated that the onset of instability might be predicted by comparing the elastic energy released with the energy of the new surfaces created. The onset of instability would correspond to the crack length where these two quantities are equal.

Whilst Griffith's approach provides a helpful physical explanation of why a crack's propensity to propagate depends on its length as well as the loading to which it is subjected, a more useful and modern approach is provided by the concept fracture mechanics [5]. This approach has been developed in the past 50 years as a response to the requirement to analyse flawed structures. Particular impetus was given to its development by failures such as the Comet crashes of the mid 1950s. In the fracture mechanics approach, we focus on the stress field around the tip of a crack in an elastic material. The elastic stress solution predicts infinite stresses at a sharp crack tip, which is not physically reasonable, but the stresses in the neighbourhood of the crack tip can be characterised by a stress intensity factor K.

K is defined as

$$K = C\sigma\sqrt{\pi a} \qquad (9)$$

where σ is the stress at some reference location remote from the crack, *a* is the crack length[1], and C is a numerical constant which depends on the geometric configuration and the location chosen for the reference stress. For most configurations C may be taken to be close to 1.0. For example, for an edge crack in a semi-infinite plate (Fig. 3.) the value of C is 1.1215 [5]. The normal units quoted for K are MPa∉m. It will be seen from (9) that the stress intensity factor for a crack varies in direct proportion to the applied stress, but as the square root of the crack length. Experimentally it is found that the onset of unstable crack growth occurs when the stress intensity factor for the crack reaches a critical value, i.e.

$$K = K_{IC} \qquad (10)$$

where K_{IC} is known as the 'critical stress intensity factor' or 'fracture toughness'[2]. This may be regarded as a material property[3] and may be measured experimentally using, for example, compact tension test pieces [5].

To apply the foregoing to the CD, we need a value of K_{IC} for Polycarbonate. Polycarbonate is generally regarded as a 'tough' polymer (i.e. it is resistant to cracking and impact). Values for K_{IC} are not easy to find in the literature, but Shackelford [6] quotes a range of values between 1.0 and 2.6 MPa∉m for Polycarbonate. Taking the lower of these values, assuming C =1.0, and using the maximum stress in the disk (σ = 5.04 MPa) we can obtain an estimate of critical crack length by substitution into (9) and (10), i.e.

$$a = \frac{1}{\pi}\left(\frac{K_{IC}}{C\sigma}\right)^2 \qquad (11)$$

This yields a value for *a* of 12.5 mm. This compares with RM experience [7] which suggests that cracks of approximately 6mm long may be critical. The discrepancy may be explained by a number of factors, which will be discussed in the next section. Of particular interest, however is that equation (11) reveals that the critical crack length is proportional to $(1/\sigma^2)$. Since σ is proportional to ω^2 (8), we can conclude that the critical crack length is proportional to $(1/\omega^4)$. This may be used to predict critical crack lengths at other speeds, either using the materials data employed here, or by extrapolation from RM's experimental results. The results are shown in Table 1.

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1 For a buried crack with two crack tips, the crack length is normally taken to be 2*a*. For the usual case of a surface breaking crack with a single crack tip, *a* is the crack length

2 A related material property, called the 'critical strain energy release rate' is sometimes given the name 'toughness'. Care must be taken not to confuse 'toughness' and 'fracture toughness' when looking up material properties. Fortunately, the units are different.

3 It is found that there is some variation of the measured property with the thickness of the specimen, but this is a complication which need not be considered for the present purposes.

Speed of drive	Predicted critical crack size (mm)	
	Using material constants, as outlined above	By extrapolation from RM experience at 52X
16X	1395[4]	669[4]
32X	87.1[4]	41.8
40X	35.7[4]	17.1
48X	17.2	8.3
52X	12.5	6.0

Table 1 – Predicted critical crack sizes for various drive speeds

It is, of course, apparent that the critical crack size decreases rapidly as the drive speed increases, so that the risk of disk failure by fast fracture is greatly increased. A number of engineering components are designed to encourage a safe mode of failure. An example of this would be in pressure vessel design, where use of a material with a high fracture toughness and low yield stress leads to a 'leak before break' condition, so that the vessel yields and allows fluid to escape instead of exploding catastrophically. The above figures suggest an analogous situation for CDs. The index track of a CD is the innermost data track and is located at a radius of approximately 22mm. Thus, a crack longer than about 15mm will cross the index track and render the disk unreadable in the drive before a critical failure will take place. Thus, the figures above suggest that a 40X drive is unlikely to encounter problems with disk cracking: cracked disks will become unusable well before there is a danger of unstable propagation. On the other hand, with speeds of 48X and higher there would appear to be a possibility of cracks reaching a critical size before the index track is breached. This appears to be in accordance with RM experience, where failures are not generally observed on 40X drives and slower [8].

5. Other factors

As described above, there is a difference between the predicted critical crack size of 12.5 mm and the experimentally observed value of about 6 mm. This suggests that we have either over-estimated the material's fracture toughness, or under-estimated the stresses. A number of possible explanations can be advanced:

Poor material

Whilst we have taken the lowest value of the range of fracture toughnesses quoted in the literature it is possible that this is not representative of CD-ROM material. There has been downward pressure on the price of CDs in recent years and it is possible that this has resulted in manufacturers using lower quality resin or inferior manufacturing routes, leading to lower fracture toughness. It would be instructive to carry out tests on compact tension specimens manufactured from CDs to establish a more accurate value for the fracture toughness of the material, together with its variability from disk to disk.

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4 For lower speeds the predicted critical crack length exceeds the maximum physical crack length ($r_o - r_i$). In part this is due to the simplifying assumption used that C = constant (1.0), whereas in practice it will vary with the ratio $a/(r_o - r_i)$, particularly as this ratio approaches unity. Physically, however, it is likely that these disks will fail by yield of the remaining ligament of material rather than by fast fracture.

Dynamic forces

The stress solution employed in Section 2 assumes that the disk is rotating at a constant speed. Of course, the disk will have to be accelerated to its maximum speed and decelerated again. This will cause dynamic stresses within the disk (since a force is required to accelerate a mass). In general, the magnitude of these stresses will increase as the acceleration time is reduced, since faster accelerations will be called for. However, the precise values will depend on the specified acceleration profile. This may be one reason why disks fail in some designs of drive but not in others operating at the same speed. A full finite element (i.e. numerical) stress analysis would be required to determine what these stresses are during any given velocity-time profile.

A further instance of dynamic force occurs due to disk imbalance. Our analysis has assumed that the disk is perfectly balanced (i.e. its centre of mass is coincident with the axis of rotation). This will never be precisely true and the degree of imbalance may be represented by the product of the mass, m, and the distance between the centre of mass and axis of rotation, e. The resulting out of balance force F, will be then be found by multiplying by ω^2 so that

$$F = me\omega^2 \qquad (12)$$

This force will cause the disk to be forced sideways on the central spindle as it rotates, causing additional stresses. Further, since the out of balance force rotates with the disk, it will cause vibration in the spindle mounting system leading to movement of the axis of rotation. This will set up further dynamic stresses in the disk. Whilst these effects are difficult to quantify precisely, it is clear that in general they will increase with ω^2 so that they become much more important with high speed drives. Also, they provide a further explanation why some drives may perform better than others, since differences in the disk clamping arrangement and spindle suspension system will affect the degree of out of balance and the vibrational response.

It will also be seen that the out of balance force depends on the (*me)* value for the disk. Whilst limits are specified in the international standard [9], it is possible that problems have increased as media costs have fallen [10].

It is much more difficult to calculate the stresses caused by out of balance than in the case of balanced disk. However, it is possible to estimate an order of magnitude. The main effect will be to introduce a contact force, P between the spindle and the disk which will be equal to the dynamic out of balance force (Fig. 4). Thus,

$$P = me\omega^2 \qquad (13)$$

Let us assume error in the position of the centre of the disk with respect to the spindle axis of 0.3 mm [7], (i.e. e = 0.3 mm). Thus, with m = 16 g we obtain a contact force of P = 5.8 N. In practice, we will not see a point force, since the disk and spindle will deform, resulting in a pressure distribution (Fig. 5). The precise form of this distribution will depend on the elastic properties of the spindle and disk and on their initial mismatch in radii. We must therefore assume a contact angle, α. If the disk and spindle are reasonably close in radii, a total contact angle of $2\alpha = 90^o$ might be a reasonable assumption. In this case, the pressure distribution will be approximately Hertzian [11], so that

$$p(\theta) = p_0\sqrt{1-(\theta/\alpha)^2} \tag{14}$$

The value of the peak contact pressure, p_0, may be found by ensuring equilibrium with the total contact force, i.e.

$$P = \int_{-\alpha}^{\alpha} p_0 w r_i \sqrt{1-(\theta/\alpha)^2}\, d\theta \tag{15}$$

where w is the disk thickness. For P = 5.8 N this gives a peak contact pressure of 0.62 MPa. The pressure distribution leads directly to a compressive radial stress in the contact region. Tensile tangential stresses are also developed, however, [12,13], and these reach a maximum at the edge of the contact region. For a contact of this size the value of this tangential stress will be approximately equal to p_0, so that an additional tangential stress of about 0.6 MPa might be estimated. The estimate will depend on the contact angle assumed, so that smaller contacts will lead to higher stresses.

The total stress required to cause a critical crack length of 6mm with $K_{IC} = 1$ MPa√m is 7.1 MPa (11). Thus, an additional stress of 2.1 MPa would be required and our estimate of the stresses caused by imbalance does not appear to explain all the discrepancy, unless the imbalance is much higher than estimated or the contact angle much less. Note, however, that our estimate of stresses due to imbalance has assumed that the spindle does not vibrate. In practice, spindle vibration will result, but it is very difficult to predict the resulting amplitude. The frequency of vibration will almost certainly contain a significant component at the rotational speed of the disk, so that the spindle position in any particular direction as a function of time can be described by

$$x(t) = D\sin\omega t \tag{16}$$

The acceleration of the disk/spindle assembly will therefore be

$$\ddot{x}(t) = -\omega^2 D\sin\omega t \tag{17}$$

and the amplitude of the contact force required to accelerate the disk will be

$$\Delta P = m\omega^2 D \tag{18}$$

Thus, for example, a vibration amplitude of 0.5 mm would be required to produce a force amplitude of 10 N. This seems unlikely it practice.

Residual stress

Most engineering analyses of the sort we have undertaken tend to assume that the component is stress-free when unloaded. This is often far from the case, since the manufacturing process may have induced stresses in the material. These need to be self-equilibrating, i.e. for our case

$$\int_{r_i}^{r_o} \sigma_{rr} dr = 0 \quad (16)$$

Thus, a region of tensile residual stresses must be counteracted by balancing compression elsewhere. Nevertheless it is possible that the manufacturing process may cause residual tensile stresses towards the inside of the disk, exacerbating cracking in this region. The critical crack length will be correspondingly reduced. Once the crack has started to propagate in an unstable manner it is unlikely that compressive stresses further out will cause it to stop. It is quite common in engineering materials to experience residual stresses of a similar order of magnitude to the yield stress of the material. Thus it is entirely possible that residual stresses alone could account for the discrepancy between the predicted and experimental results

Discussion

A number of possible reasons for the discrepancy between the predicted and experimentally observed critical crack length have been examined. Approximate calculations suggest that stresses caused by out of balance and the resulting vibration are unlikely to explain all of the discrepancy. The existence of residual stresses in the disk might account for the discrepancy without any additional effects.

6. Containment

Since there is a possibility of CDs cracking in high speed drives it is important to consider whether the resulting debris can be contained. This is a common consideration for rotating machinery, for instance aircraft engine manufacturers are required to demonstrate (by test!) that all debris can be contained in the event of the failure of a fan blade at the front of the engine. An appropriate place to start is to calculate the amount of kinetic energy present in the rotating disk. The energy, E, in a rotating body is given by

$$E = \frac{1}{2} I \omega^2 \quad (17)$$

where I is the moment of inertia about the axis of rotation. For a hollow disk, I is given by [14]

$$I = \frac{1}{2} m (r_o^2 - r_i^2) \quad (18)$$

Hence, the total energy contained in a CD rotating in a 52X drive is in the region of 17 J. In engineering terms, this is not a particularly large amount of energy, so it should be relatively straightforward to contain. Further, the peripheral speed of the disk is only 66 m/s. The speed of any emerging fragments is unlikely to exceed this value, so the velocities are not particularly high, either[5]. However, it should be borne in mind that the front panel of the drive tray is usually made from a polymer which will have a low fracture

[5] Whilst not high in engineering terms, fragments of CD travelling at this speed may well present a hazard to the user of the PC.

toughness itself and which will be rather flexible. Similar comments can be made regarding the drive tray and its retaining latch. Thus, although the energies and velocities concerned are quite low, certain elements of the surround are poorly designed from a containment point of view. In general the best means of absorbing energy is by plastic (i.e. permanent) deformation. This is the reason that cars are designed with 'crumple zones' to protect the occupants in case of impact. Materials such as mild steel will absorb a large amount of energy in this way, whereas polymers will not. Thus, consideration should be given to introducing greater energy absorption capability into the front of the drive tray and the latches if the incidence of non-containment is to be reduced. One possibility is to incorporate metal in these components, although there are other options (e.g. energy absorbing foam).

7. Conclusions

An analysis of the stress state within a CD revolving at constant angular velocity has shown that the magnitude of the stress increases with the square of the angular velocity. At a speed of 10500 rpm (corresponding to a 52X drive) stresses are well below the yield stress of CD material, so that failure of an uncracked disk by yield is unlikely. A fracture mechanics analysis of defects emanating from the inner bore of the CD has revealed that critical crack size for failure by fast fracture varies inversely with the fourth power of rotational speed. The predicted critical crack length in a 52X drive is 12.5mm. This compares with experimentally observed failures from cracks of about half this length. Extrapolation from these experiments suggests that failures will be unlikely in drives operating at 40X or slower, since a crack will cross the index track before it reaches its critical length.

Reasons for the discrepancy between predicted and observed critical crack sizes have been discussed and include dynamic loading and residual stresses. Some general consideration of the containment problem has been undertaken.

8. References

1. Datasheet on Calibre 1080 DVD resin, Dow Chemical Company. Obtained from www.dow.com.

2. Benham, P.P., Crawford, R.J., and Armstrong, C.G., 'Mechanics of Engineering Materials', 2nd edition, Longman, 1996.

3. Suresh, S., 'Fatigue of Materials', 2nd edition, Cambridge University Press, 1998.

4. Griffith, A.A., 'The phenomenon of rupture and flow in solids', Phil. Trans. Roy. Soc., London, A221, 163-197, 1921.

5. Ewalds, H.L., and Wanhill, R.J.H., 'Fracture Mechanics', Arnold, 1984

6. Shackleford, J.F., 'Introduction to Materials Science for Engineers', 3rd edition, Macmillan, 1992.

7. Sweeney, T., and Davidson, D., RM plc, private communication.

8. CD Disc Crush – Egress Event Data Pack, RM plc, July 2001.

9. ISO 10149, ISO, Geneva, 1995.

10. www.mediascience.com

11. Johnson, K.L., ‘Contact Mechanics’, Cambridge University Press, 1985.

12. Persson, A., ‘On the stress distribution of cylindrical elastic bodies in contact’, PhD thesis, Chalmers Technical University, Göteborg, 1964

13. Hou, J.P., and Hills, D.A., ‘Review of progress on plane analysis of cracks emanating from fastener holes’, Solid Mechanics Group report No. 43, University of Oxford, Department of Engineering Science, 2001.

14. Howatson, A.M., Lund, P.G., and Todd., J.D., ‘Engineering Tables and Data’, Chapman and Hall, 1972.

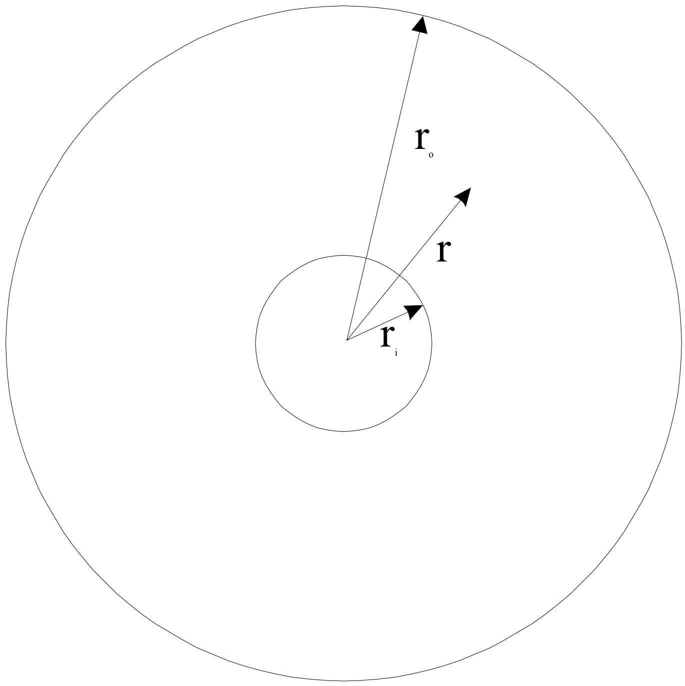

Figure 1. – Geometry of CD

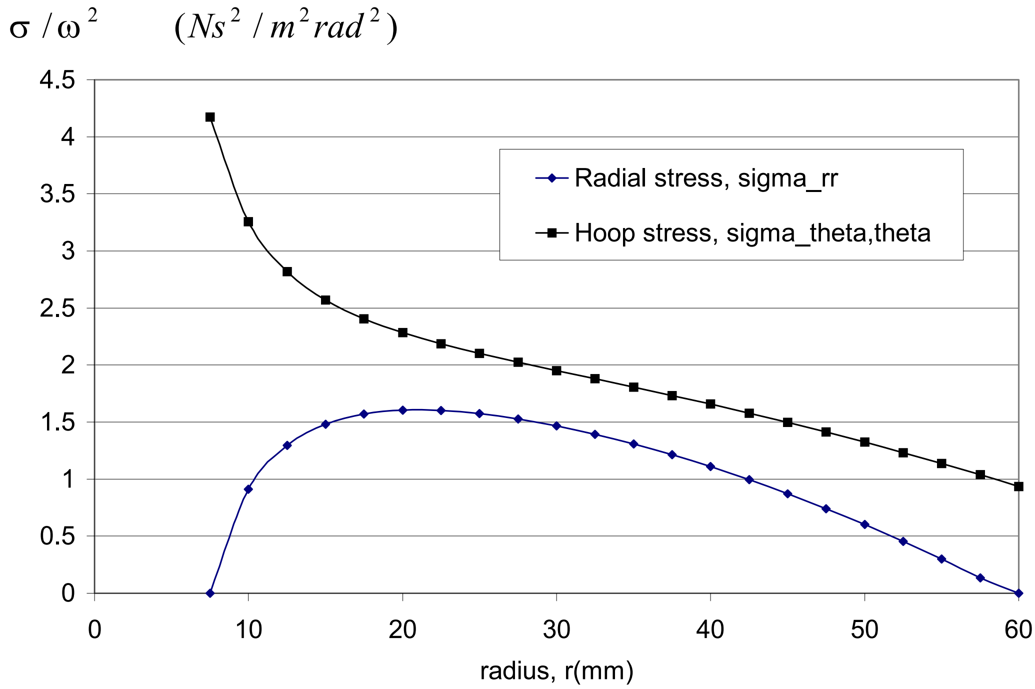

Figure 2 – Variation of stress with radius in an uncracked CD

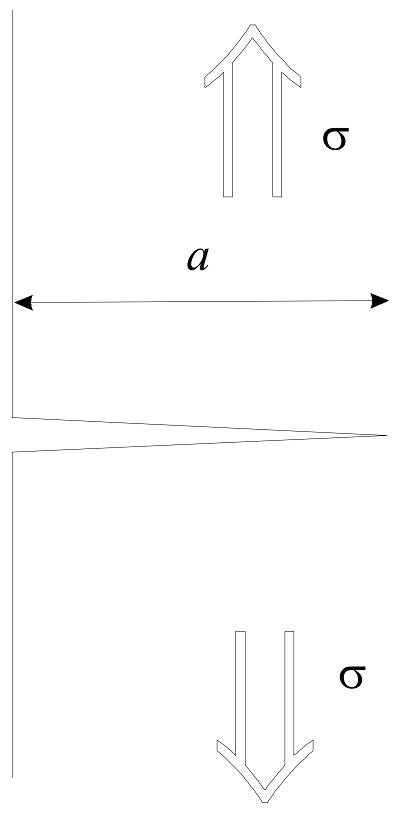

Figure 3 – Edge crack in a semi-infinite plate, loaded by uniform remote tension.

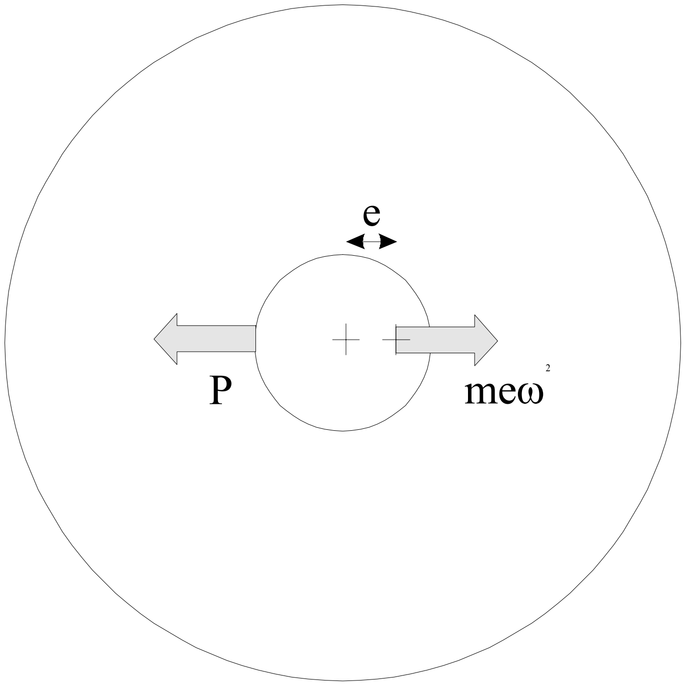

Figure 4 – Out of balance force resulting from eccentricity, e, between spindle axis and centre of mass of the disk, together with corresponding contact force, P.

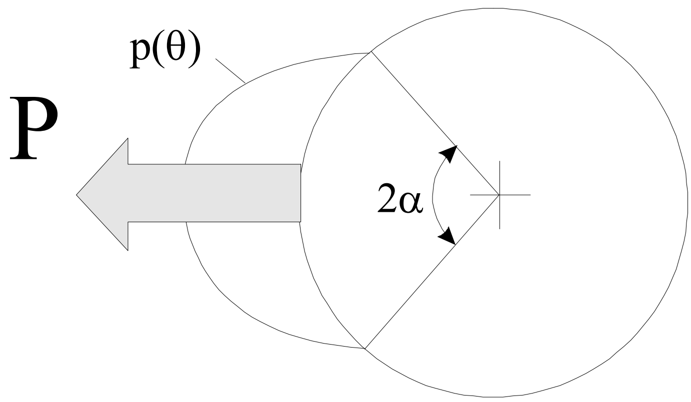

Figure 5 – Realisation of contact force P as distribution of pressure over a contact angle of 2α

APPENDIX – Biographical note

David Nowell took a first class degree in engineering from Clare College, University of Cambridge in 1982. He then undertook the Advanced Course in Production Methods and Management before joining British Rail in 1983 as an Engineering Management Trainee. Between 1986 and 1988 he studied for a doctorate in engineering on fretting fatigue at the University of Oxford and was awarded the degree of D.Phil. in 1989. Since 1988 he has been a university lecturer in engineering science at the University of Oxford and is an Official Student (i.e. tutorial fellow) of Christ Church. In 1999 he was awarded the title of Reader in Engineering Science by the University. He is a Chartered Engineer and a Fellow of the Institution of Mechanical Engineers.

David has undertaken research in structural integrity since 1986 and is the author or co-author of over 100 journal papers and conference publications, together with two books. Since 1990 he has been involved with the Rolls-Royce University Technology Centre in Solid Mechanics at the University of Oxford. He has been Director of the Centre since 1999. He has undertaken consultancy work for a number of industrial companies, including Rolls-Royce plc, DERA, Alcan International, OCS, and HCL Fasteners.