REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

12[2.05].—J. H. Ahlberg, E. N. Nilson & J. L. Walsh, *The Theory of Splines and Their Applications,* Academic Press, New York, 1967, xi + 284 pp., 24 cm. Price $13.50.

The theory of spline functions has been developed rather recently. However, since Schoenberg's first paper appeared in 1946, the number of research articles devoted to or connected with the theory of splines has grown very fast. The authors have tried to systematize and organize this material and in this reviewer's opinion succeeded rather well.

The simplest spline approximation can be formulated in the following way: Given a function $f(x)$ in the interval $0 \leqq x \leqq 1$. Let $0 = x_1 < x_2 < \cdots < x_n = 1$ be n points. A function $\phi_n(x)$ is a spline approximation if: (1) $\phi_n(x_i) = f(x_i)$, $i = 1, 2, \cdots n$, (2) $\phi_n(x)$ is a polynomial of order three in every subinterval $x_i \leqq x \leqq x_{i+1}$ and (3) $\phi_n(x)$ is as smooth as possible.

Then the following questions arise: Does $\phi_n(x)$ exist for all n ? Does $\phi_n \rightarrow f$ converge for $n \rightarrow \infty$? How fast does ϕ_n converge to f ? All these questions are answered in the first part of the book and other remarkable properties of $\phi_n(x)$ are derived. The rest of the book is devoted to more general spline approximations in one and two space dimensions.

H. O. K.

13[2.05].—E. W. Cheney, *Introduction to Approximation Theory,* McGraw-Hill Book Co., New York, 1966, xii + 259 pp., 24 cm. Price $10.95.

This eminently readable book is intended to be used as a text for a first course in approximation theory. Uniform approximation of functions is emphasized and the discussion is not only theoretical, but provides usable algorithms as well.

An introductory chapter presents some of the major theoretical tools, and assigns an important role to convexity considerations. There follow chapters on the Chebyshev solution of inconsistent linear equations and Chebyshev approximation by polynomials and other linear families. The next chapter treats least-squares approximation and related topics. The scene then shifts back to uniform approximation by rational functions. The final chapter offers a miscellany of topics too interesting to be omitted from the book. Throughout the book the author provides interesting proofs and occasionally new approaches of his own. The approximately 430 problems are an extremely valuable supplement to the text, as is an impressive set of notes to each chapter, which provide the historical context of much of the material and suggestions for further reading. It is not surprising, in view of the great scope of these notes, that there are a few minor misstatements in this part of the book. For example, there is no proof of V. Markoff's theorem in Rogosinski's paper, hence certainly not "the simplest proof". (There is a simple proof of a simpler theorem.) The reader sent to Dickinson's paper for more information about Chebyshev polynomials will not be helped much. These, however, are quibbles in the face of the author's achievement of having written a useful book which is also pleasurable to read.

THEODORE RIVLIN

IBM Research Center
Yorktown Heights, New York 10598

14[2.05].—GÜNTER MEINARDUS, *Approximation of Functions: Theory and Numerical Methods*, translated by Larry L. Schumaker, Springer-Verlag, New York Inc., 1967, viii + 198 pp., 24 cm. Price $13.50.

This is a translation of the German edition which appeared in 1964. It differs in detail from that edition by inclusion of new work on comparison theorems for regular Haar systems and on segment approximation.

H. O. K.

15[2.05].—LEOPOLDO NACHBIN, *Elements of Approximation Theory*, D. Van Nostrand Co., Inc., Princeton, N. J., 1967, xii + 119 pp., 20 cm. Price $2.75.

It is appropriate to begin by pointing out that the subject matter of this book is not *best* approximation; the author is rather concerned with the problem of *arbitrarily good* approximation.

More precisely, the author works within the framework of a given function algebra $C(E)$, consisting of all continuous scalar (real or complex, depending on the circumstances) functions on a completely regular topological space E. Such algebras are given the compact-open topology and the general problem is then to characterize the closure of various subsets S of $C(E)$.

The results given include the following cases: S is a lattice (Kakutani-Stone theorem), an ideal, a subalgebra of $C(E)$ (Stone-Weierstrass), or a convex sublattice (Choquet-Deny). In particular, these results imply criteria for the density of S in $C(E)$.

In addition to these well-known theorems, there is a careful presentation of a general *weighted* approximation problem. This problem is a generalization to the $C(E)$ context of the classical Bernstein problem on R^1 or R^N, and is largely based on recent work by the author. The problem is reduced back to the one-dimensional Bernstein problem and various criteria for its solution are then established, making use of analytic or quasi-analytic functions on R^1.

The book will be accessible to readers with a modest background in analysis (Taylor and Fourier series, Stirling's formula) and general topology (partition of unity, Urysohn's lemma). The necessary functional analysis of locally convex spaces is developed in the early chapters. The Denjoy-Carleman theorem on quasi-analytic functions is the only other major result needed and references for its proof are provided. There is an extensive bibliography, but no index or exercises.

It is clear that numerical analysts will find material on approximation more relevant to their profession in, for example, the books of Cheney or Rice. On the other hand, Nachbin's book provides an interesting blend of hard and soft analysis, and more importantly, it collects together for the first time the main closure theorems in function algebras. For these reasons the book represents an important contribution to the mathematical literature. But it also merits additional kudos: the author is noted for (among other things) the clarity of his mathematical exposition and the present book continues in this trend. It is a pleasure to read!

R. B. HOLMES

Purdue University
Lafayette, Indiana 47907

16[2.05, 7, 12].—JOHN F. HART, E. W. CHENEY, CHARLES L. LAWSON, HANS J. MAEHLY, CHARLES K. MESZTENYI, JOHN R. RICE, HENRY G. THACHER, JR. & CHRISTOPH WITZGALL, *Computer Approximations*, John Wiley & Sons, Inc., New York, 1968, x + 343 pp., 23 cm. Price $17.50.

This encyclopedic work represents the culmination of the combined efforts of the authors and other contributors to provide an extensive set of useful approximations for computer subroutines, along the lines of the pioneer work of Hastings [1]. The stated aim of the book is "to acquaint the user with methods for designing function subroutines (also called mathematical function routines), and in the case of the most commonly needed functions, to provide him with the necessary tables to do so efficiently."

The book divides into two parts: the first four chapters deal with the problems of computation and approximation of functions in general; the last two chapters (following one devoted to a description and use of the tables) consist, respectively, of summaries of the relevant mathematical and computational properties of the elementary and selected higher transcendental functions and tables of coefficients for least maximum error approximations of appropriate forms for accuracies extending to 25 significant figures.

The wealth of material may be more readily inferred from the following summarization of the contents of the individual chapters and appendices.

The introduction to Chapter 1 (The Design of a Function Subroutine) emphasizes that the efficient computation of a function no longer is simply mathematical in the classical sense, but requires a thorough knowledge of the manner of operation and potentialities of the computing equipment involved. The principal matters considered in this chapter are certain general considerations in preparing a function subroutine, the main types of function subroutine, special programming techniques, subroutine errors, and final steps in preparing a subroutine.

Chapter 2 (General Methods of Computing Functions) summarizes the various techniques for evaluating a function; these include infinite expansions (series, continued fractions, infinite products), recurrence and difference relations, iterative techniques, integral representations, differential equations, polynomial and rational approximations, and transformations for the acceleration of convergence.

Chapter 3 (Least Maximum Approximations) contains a discussion of the characteristic properties of least maximum approximations, together with a description of the second algorithm of Remez for the determination of the optimum polynomial approximation of given degree to a specified function in the sense of Chebyshev. Also considered are nearly least maximum approximations resulting from the truncation of Chebyshev expansions, several algorithms for determining best rational approximations, and segmented approximation.

Chapter 4 (The Choice and Application of Approximations) deals with various ways in which to increase the efficiency of procedures for approximating a function. These include the reduction of the range over which the approximation is performed, utilizing such properties as periodicity, addition formulas, symmetry, and recurrence relations. Illustrative comparisons of approximations of comparable accuracy to the same function are made with respect to the relative number of arithmetic and logical operations required in their computer evaluation. The significant problem of improving the conditioning of approximations is discussed with the aid of two examples. Various ways of evaluating a polynomial, such as Horner's (nested) form, the product form, the orthogonal polynomial form, Newton's form, and streamlined forms (Pan, Motzkin-Belaga), are described and considered with respect to ill-conditionedness. The evaluation of rational functions by means of associated continued fractions and J-fractions is also considered in some detail. This chapter closes with a brief presentation of methods and algorithms for transforming one polynomial or rational form into an equivalent form.

Chapter 5 (Description and Use of the Tables) serves as an introduction to the concluding chapters with their appendices. A description is given of the tabular material, its preparation, checking, and use.

The specific functions discussed in Chapter 6 (Function Notes) include the square root, cube root, exponential and hyperbolic functions, the logarithm function, direct and inverse trigonometric functions, the gamma function and its logarithm, the error function, the Bessel functions $J_n(x)$ and $Y_n(x)$, and the complete elliptic integrals of the first and second kinds. For each function the notes include definition and analytic behavior, fundamental formulas, error propagation, design of subroutines, checking, relevant constants, a priori computation, and index tables (for guidance in using the tables of coefficients).

Three appendices, following Chapter 6, are entitled, respectively, Conversion Algorithms, Bibliography of Approximations, and Decimal and Octal Constants. The four algorithms listed (in FORTRAN language) are concerned with the conversion of rational functions to and from equivalent continued fraction forms. The bibliography of approximations includes an approximation list arranged in the categories set forth in the FMRC *Index* [2] and a list of 62 sources of the approximations, which includes the Russian handbook of Lyusternik et al. [3]. The list of constants gives approximations both to 35D and 40 octal places of 74 basic numbers required in the function subroutines under consideration.

These appendices are followed by a list of 115 references to the literature on the approximation of functions. It seems appropriate here to mention the recent supplementary bibliography compiled by Lawson [4].

Chapter 7 (Tables of Coefficients), which concludes the book, gives in a space of 151 pages the coefficients, up to 25S, of least maximum error approximations of appropriate forms to the functions discussed in Chapter 6.

A work of this magnitude almost invariably has a number of typographical and other kinds of errors. Those found by this reviewer, which are listed elsewhere in this issue, do not seriously detract from the value of this excellent book.

This definitive collection of computer approximations, which represents the result of a systematic approach to new methods and new approximations, should be an indispensable reference work for all those engaged in scientific computation.

J. W. W.

1. Cecil Hastings, Jr., Jeanne T. Hayward & James P. Wong, Jr., *Approximations for Digital Computers*, Princeton Univ. Press, Princeton, N. J., 1955. (See *MTAC*, v. 9, 1955, pp. 121–123, RMT **56**.)

2. A. Fletcher, J. C. P. Miller, L. Rosenhead & L. J. Comrie, *An Index of Mathematical Tables*, second edition, Addison-Wesley Publishing Co., Inc., Reading, Mass., 1962. (See *Math. Comp.*, v. 17, 1963, pp. 302–303, RMT **33**.)

3. L. A. Lyusternik, O. A. Chervonenkis & A. R. Yanpol'skii, *Handbook for Computing Elementary Functions*, Pergamon Press, New York, 1965. (See *Math. Comp.*, v. 20, 1966, pp. 452–453, RMT **64**.)

4. Charles L. Lawson, *Bibliography of Recent Publications in Approximation Theory with Emphasis on Computer Applications*, Technical Memorandum No. 201, Jet Propulsion Laboratory, Pasadena, Calif., 16 August 1968.

17[2.10].—Arne P. Olson, "Gaussian quadratures for $\int_1^\infty \exp(-x)f(x)dx/x^m$ and $\int_1^\infty g(x)dx/x^m$," tables appearing in the microfiche section of this issue.

Abscissas and weights of N-point Gaussian quadrature formulas for the integrals in the title are given in Table II to 16 significant figures for $N = 2(1)10$, $M = 0(1)10$. Table I contains 24S values of the moments $a_{n,l} = \int_1^\infty \exp(-t)dt/t^{n-l}$ for $n - l = 10(-1)-19$. As a possible application of Table II, the author mentions the evaluation of the series

$$\sum_{k=0}^{\infty} E_m(t + kh) = \int_1^\infty \frac{\exp(-xt)dx}{[1 - \exp(-hx)]x^m} \quad (E_m \text{ the exponential integral}) ,$$

which occurs in the calculation of neutron collision rates in infinite slab geometry.

W. G.

18[2.45, 12].—Donald E. Knuth, *The Art of Programming*, Vol. I: *Fundamental Algorithms*, Addison-Wesley Publishing Co., Reading, Mass., 1968, xxi + 634 pp., 25 cm. Price $19.50.

Most people think of mathematics as being a very complicated subject. On the contrary, if we define the complexity of a problem as the amount of information required to describe the problem and its solution, it is apparent that mathematics can deal only with simple problems. Further, if we consider the universe of all problems, almost all of them are too complex for solution by mathematical methods, which rely heavily on abstraction and conceptualization, which themselves reflect the limitations of the brain. Thus, while it is reasonable to expect a mathematical solution to the four-color problem, it is not reasonable to expect a mathematical solution to the problem of language translation, which can be stated only in terms of massive amounts of information from grammars and dictionaries, and whose solution will probably require an algorithm of comparable size. The importance of the computer is that it permits the consideration of such irreducibly complex problems.

It is not surprising therefore, that what looks like the most authoritative work on computer programming is entitled "The Art of Computer Programming". Programming, which is concerned with algorithms for solving complex problems, is itself a complex problem, and we would be mistaken if we expected it to have a neat and tidy theory. There is simply too much detailed information associated with computer programs for them to be analyzed completely and axiomatically, particularly by human intelligence. Certain aspects of certain algorithms are amenable to theoretical treatment, but the great bulk of programming decisions are made by various combinations of experience, taste, habit, intuition, philosophy and guesswork.

This book, by one of the best-known workers in the field, is the first volume of a seven-volume set. The complete contents of the set are:

The scope of this set is so large that we must regard it as an attempt at 'the' reference work on programming. The provisional schedule is for one volume to be published each year.

Chapter 1 starts by introducing the notion of an algorithm (pp. 1–9), and then continues with the basic mathematics necessary for comparing the efficiencies of algorithms (pp. 10–119). This is concerned mainly with finite and infinite series.

The next section (pp. 120–181) is devoted to a hypothetical computer called MIX which is described in detail, and which the remainder of the book uses to illustrate programming examples. The author's decision to use machine language for most of his examples was not made lightly, and the author defends it persuasively.

It is clear that a knowledge of machine language is necessary in order to describe assemblers or compilers, so a specific language must be introduced. However, it is not clear that there is any advantage in using machine language to specify or discuss algorithms, apart from the ability to determine their efficiency, and even that is getting more difficult with the increased parallelism in today's hardware. The most dangerous aspect of the use of machine language is that it perpetuates the view that 'real' programming is done in machine language, and that those who use Fortran or Snobol do not really know what is going on. This misapprehension is the one that is giving the computing world the most trouble—many otherwise competent programmers believe that their talent for writing clever machine-language code is a virtue rather than a vice. The real truth is that the Fortran or Snobol programmer wishes to have a more powerful grasp of his problem, and his decision represents a desire to ignore details, rather than an inability to learn machine language. It seems difficult to believe that the casual user wanting to find out about data-structures will take the trouble to learn MIX, so some of the expertise contained in this book will remain hidden. It seems unlikely that many instructors will implement a MIX interpreter for use by their classes, but will continue teaching the machine language of whatever machine is accessible to them, and using higher-level languages for discussion of specific algorithms.

The next section (pp. 182–227) introduces some fundamental programming techniques, including subroutines, coroutines, a MIX interpreter written in MIX, and input/output. Of these, the section on coroutines is perhaps the most interesting, concerning itself with routines each of which thinks it is calling the others as subroutines. This is an unfamiliar notion to many programmers and is given clear treatment. This section provides a quite sophisticated introduction to machine-language programming.

Chapter 2 (pp. 228–464), is entitled 'Information Structures' and gives an excellent survey of the sort of data-structures found in programming. Section 2.2 on linear lists (pp. 234–304) describes stacks, queues, dequeues, (double-ended queues!), sequential allocation, linked allocation, circular lists, doubly linked lists, arrays and orthogonal lists. The treatment is well organized and comprehensive; a surprising omission is the gimmick of using the exclusive-or of the right and left pointers to save space in a double-linked list.

Section 2.3 (pp. 305–422) on trees is perhaps the most important section in the book. There are subsections on traversal, binary trees, other representations of trees, mathematical properties of trees, lists and garbage collection. The orientation towards trees rather than lists, the more usual data-structure, is presumably made to allow the treatment to be linked naturally with the theory of graphs and trees. This is probably the first adequate treatment of this area in the literature. The author points out, quite rightly, that such techniques can be used to advantage in many problems, and are not restricted to explicit 'list-processing' languages.

The chapter ends with sections on multi-linked structures (pp. 423–434), dynamic storage allocation (pp. 435–455) and a history and bibliography (pp. 456–464). As the author points out, there is little known about storage allocation, and one is left with a strong feeling that this treatment is far from complete.

It has in the past often been impossible to find books appropriate for coursework in Computer Science beyond the first undergraduate semester or two, except in the more mathematical areas. Nearly all graduate courses have had to be based on papers in journals and the instructor's own work. This book could provide for some time an excellent basis for a course on data-structures, and may serve as additional reading in other courses. The many excellent exercises should provide valuable practice for the student (and the teacher). A large part of the book (pp. 465–606) is devoted to answers to the exercises. These range in difficulty from trivial to unsolved problems of significant research interest.

This is surely a 'must' for those who aim to be experts in the field of programming. The mind boggles at the fact that in a book written with such style and accuracy there can be a reference to section 12.3.4, which, if the length of this volume is any indication, must be fully 3000 pages away.

MALCOLM C. HARRISON

Courant Institute of Mathematical Sciences
New York University
New York, New York 10012

19[3].—JOAN R. WESTLAKE, *A Handbook of Numerical Matrix Inversion and Solution of Linear Equations*, John Wiley & Sons, Inc., New York, 1968, viii + 171 pp., 23 cm. Price $10.95.

This is, as the title indicates, a handbook. A two-page introduction is followed by brief descriptions of the standard direct and iterative methods for a total of 85 pages, with no theory and (at this stage) no evaluation. This is followed by chapters on measures of condition (four pages), measures of error (five pages), scaling (two pages), operational counts (four pages), comments and comparisons (14 pages), including some test results. In the appendix are a glossary (nine pages), a collection of basic theorems (ten pages), and a set of test matrices, and finally a list of references (126 items), a table of symbols, and an index. No programs are given. Each method is briefly but clearly described and the selection is quite reasonable. It should be a useful and convenient reference for the purpose intended.

A. S. H.

20[3].—ANDRÉ KORGANOFF & MONICA PAVEL-PARVU, *Éléments de la Théorie des Matrices Carrées et Rectangles en Analyse Numérique*, Dunod, Paris, 1967, xx + 441 pp., 25 cm. Price 98 F.

This is the second volume in a series entitled "Méthodes de calcul numérique," of which the first, *Algèbre non linéaire*, appeared in 1961 as a collection of papers on the subject, edited by the senior author of this volume. The first volume provides a fairly elementary but rigorous development of methods available at that time for solving nonlinear equations and systems of equations, the most sophisticated chapter being the first on error analysis.

The present volume, by contrast, treats only a limited aspect of the subject, but treats it in considerable depth and at a rather high level of sophistication. Primarily it is concerned with the Moore-Penrose generalized inverse, a subject which, along with still more general "generalized inverses," has recently led to a rapidly expanding literature. Presumably the eigenvalue problem will be the subject of a later volume.

The book is divided into three "parts." Of the three chapters in this part, the first provides a survey of certain notions from functional analysis, and the remaining two continue in a similar vein with the theory of norms as the guiding principle, these having been introduced already in the first chapter.

In Part 2, the first chapter starts out quite generally to discuss, for a given matrix a, the most general solutions of the equations $e_g\, a = a$ and $a\, e_d = a$, thence the most general solutions of $a'_g\, a = e_d$, and $a\, a'_d = e_g$, and proceeds to impose further conditions to arrive at the principal object of study, the Moore-Penrose generalized inverse. This is applied to the problem $a \times b = c$ in the next chapter. The third chapter introduces a new approach (though already foreshadowed) in terms of minimizing norms of x and $a\,x - c$. However, the development is meager except for the Euclidean norm, which leads back again to Moore and Penrose. Finally, the last chapter describes a number of numerical methods for computing the generalized inverse.

There are no exercises, but there are numerical examples. A two-page general bibliography of items on matrices and functional analysis is given at the outset, and a rather extensive special bibliography is given at the end of each part. There is no index.

This is by no means an elementary text. But the numerical analyst with some degree of mathematical maturity can find here a great deal of interesting material. Although the literature on generalized inverses has expanded considerably since the book went to press, it gives a quite complete and systematic coverage of the theory up to that time, and the diverse points of approach suggest aspects of the subject that are by no means yet fully explored.

A. S. H.

21[3].—E. H. CUTHILL, *Tables of Inverses and Determinants of Finite Segments of the Hilbert Matrix,* Applied Mathematics Laboratory, Naval Ship Research & Development Center, Washington, D. C., ms. of 9 typewritten pp. + 346 computer sheets deposited in the UMT file.

The main table (Appendix A) gives on 326 computer sheets the exact (integer) values of the elements of the inverses of the first 37 segments of the Hilbert matrix. The symmetry of the inverse matrices is exploited through the printing of only those elements situated on or below the main diagonal. Included are the exact values of the determinants of these segments.

The underlying calculations were performed in variable-precision rational arithmetic on an IBM 360/50 system at the IBM Boston Programming Center, using programs written in PL/1-FORMAC. The program used in calculating the results in Appendix A is listed in Appendix B.

The exact values of the determinants of the segments of the Hilbert matrix were calculated independently from the well-known formula

$$\det H_n = \prod_{r=1}^{n-1} (r!)^4 / \prod_{r=1}^{2n-1} r!\,,$$

corresponding to $n = 2(1)62$. (Beyond this point the range permitted by PL/1-FORMAC was exceeded.) These 61 numbers (which are reciprocals of integers) are separately tabulated in Appendix C, and the underlying program is listed in Appendix D.

Relevant mathematical formulas, as well as details of the computer calculations, are given in the introductory text, to which is appended a useful list of six references.

These extensive manuscript tables greatly exceed the range of similar earlier tables, such as those of Savage & Lukacs [1] and R. B. Smith [2], to which the present author refers.

J. W. W.

1. Richard Savage & Eugene Lukacs, "Tables of inverses of finite segments of the Hilbert matrix," in *Contributions to the Solution of Systems of Linear Equations and the Determination of Eigenvalues*, NBS Applied Mathematics Series No. 39, U. S. Government Printing Office, Washington, D. C., 1954, pp. 105–108.

2. Richard B. Smith, *Table of Inverses of Two Ill-Conditioned Matrices*, Westinghouse Electric Corporation, Bettis Atomic Power Division, Pittsburgh, Pa., 1957. (See *MTAC*, v. 11, 1957, p. 216, RMT **95.**)

22[3, 4].—Joel N. Franklin, *Matrix Theory*, Prentiss-Hall, Inc., Englewood Cliffs, N. J., 1968, xii + 292 pp., 23 cm. Price $10.95.

The author states in his preface that this book, developed from a course given over the past ten years, intended originally to be a preparation for courses in numerical analysis, but in fact attended by juniors, seniors, and graduate students majoring in mathematics, economics, science, or engineering. Thus, Chapter 3 (optional) is entitled "Matrix analysis of differential equations," and here and there are to be found more concrete applications. The book is probably unique in that, while presupposing almost nothing at the outset, it very quickly but easily arrives at the main theoretical portion dealing with normal forms and perturbation theory, and concludes with a long chapter of nearly 100 pages on numerical methods for inversion and the evaluation of eigenvalues and eigenvectors.

The first two chapters develop the theory of determinants, and that of linear bases (56 pages). Chapter 6, entitled "Variational principles and perturbation theory," includes the minimax and separation theorems for Hermitian matrices, Weyl's inequalities, the Gershgorin theorem, norms and condition numbers, and ends with a continuity theorem. For solving systems and inverting matrices only triangular factorization is included, but with special attention to band matrices; and among iterative methods chief attention is given to Gauss-Seidel, with mention of overrelaxation. For eigenvalues the power method with deflation (but not the inverse power method) is given; reduction to Hessenberg form for a general matrix, and unitary tridiagonalization of a Hermitian matrix with the Givens application of the Sturm sequence; and, finally, the QR method.

A set of exercises of reasonable difficulty follows each section, and there is a three-page index. Unfortunately there is no bibliography, and only very few references (a half dozen or so).

A. S. H.

23[7].—D. S. Mitrinovic, *Kompleksna Analiza* (*Complex Analysis*), Gradjevinska Knjiga, Belgrade, Yugoslavia, 1967, xii + 312 pp., 24 cm.

This volume in the series *Matematicki Metodi u Fizici i Tehnici* consists mainly of text and numerous examples on complex numbers and functions of a complex variable, in the Serbian language. Its connection with computation arises mainly from the appended *Mali Atlas Konformnog Preslikavanja* (*Small Atlas of Conformal Representation*), by D. V. Slavi´. This atlas contains 30 finely drawn diagrams showing level curves u = constant and v = constant in the z-plane when $w = u + iv$ and $z = x + iy$ are connected by functional relationships. The relationships considered are as follows, where the reviewer has grouped pages together, somewhat arbitrarily, for the sake of conciseness.

282–285: $z = w^2, z^2 = w, z = 1/w^2, z^2 = 1/w$.
286–288: $z = (1 + w^2)/w, z = 2w/(1 + w^2), z = 1/w$.
289–290: $(1 + z^2)/z = w, z^2 = 1 + w^2$.
291–293: $z^2 = 1/(1 + w^2), 2z/(1 + z^2) = w, 1/(1 + z^2) = w$.
294–295: $z = e^w, z + \log z = w$.
296–298: $z = \sin w, z = \operatorname{cosec} w, z = \tan w$.
299–301: $z^2 = e^w - 1, e^z = w, \exp (1/z) = w$.
302–304: $2z = w + e^w, 2z = w^2 + \log (w^2), e^z = 1 + w^2$.
305–308: $e^z = \sin w, e^z = \tan w, \sin z = e^w, e^z + e^w = 1$.
309–311: $\sin z = w, \tan z = w, \operatorname{cosec} z = w$.

It will be noticed that the transformations include several well known in applied mathematics; for example, flow due to two-dimensional point source superposed on uniform stream (p. 295) and edge effect for a parallel plate condenser (p. 302).

At first sight one tends to regret the lack of numerical scales along definite axes, as in some similar diagrams in various editions of Jahnke and Emde, but this reaction appears on consideration to be hardly justified. In many cases the same diagram may be used to illustrate several slightly different functional relationships. Thus the diagram on page 296 may be used, as stated on the page, for

$$z = \sin w\,, \quad z = \cos w\,, \quad z = \sinh w\,, \quad z = \cosh w\,.$$

For the sake of conciseness, the reviewer has listed only one equation (chosen somewhat arbitrarily) for each diagram, but in fact the author's 30 diagrams relate to 92 stated equations.

Where a diagram relates to more than one equation, the user may visualize the axes in the manner appropriate to whichever equation he chooses. In most cases, consideration of the positions of singularities and other special points is sufficient to determine the positions of the axes and the scale, and to enable a few of the level curves to be quickly identified. It then remains only to note that, as explained on page 312, the intervals in u and v are normally $\frac{1}{4}$; but if one of u and v is an angle, both are taken at interval $\pi/18$ ($= 10°$), and small meshes continue to appear approximately square.

A. Fletcher

Department of Applied Mathematics
University of Liverpool
Liverpool 3, England

24[7].—Albert D. Wheelon, *Tables of Summable Series and Integrals Involving Bessel Functions*, Holden-Day, San Francisco, Calif., 1968, 125 pp., 24 cm. Price $8.50.

The present volume is divided into two parts as is clearly suggested by the title. Part I, by A. D. Wheelon, comprises 14 chapters and is a short glossary of sums of series. The introductory chapter notes several techniques for finding sums of series. Further, each chapter gives historical comments on the series and illustrates how the sums might possibly be evaluated in closed form. The material on methods of summing series is not extensive. In a glossary, perhaps, this is to be expected. On the other hand, certain well-known procedures are not even mentioned; for instance, use of the ψ-function, the logarithmic derivative of the gamma function, to sum series of the form $\sum f(n)/g(n)$ where $f(n)$ and $g(n)$ are co-prime polynomials in n, such that the degree of g exceeds that of f by at least 2. Connections with hypergeometric series are not noted.

If a sum does not depend on a parameter and is known in terms of a named mathematical constant (e.g., π, e), this is given. In any event, 8D values are presented. If a sum depends on a parameter and can be expressed in terms of standard special functions, this is usually given. Here, also, the sum of the series is given for 1 and sometimes 2 values of the parameters to 8D. The first 6 chapters deal with $\sum h_n f(n)/g(n)$, $h_n = 1$ or $(-)^n$ and $f(n)$ and $g(n)$ as described above where the degree of $g(n)$ does not exceed 6. Chapters 7, 8 and 9 give series containing factorial, exponential and logarithmic functions, respectively. Chapters 10 and 11 give series containing trigonometric and inverse trigonometric functions, respectively. Series containing Bessel functions and Legendre polynomials are found in Chapters 12 and 13, respectively. Some double sums are delineated in Chapter 14 which concludes Part I. The glossary no doubt will be useful to some readers, but as it comprises but 57 pages, it is clear that the glossary in no way can be considered extensive.

Part II is a short glossary of integrals involving Bessel functions by A. D. Wheelon and J. T. Robacker. It is a slightly revised version of a previous report published in 1954 which was reviewed in these annals; see *Math Comp.* v. 9, 1955, p. 223. In connection with the previous reviewer's remarks, the condition of validity for formula 11.306 has been added and formula 1.207 has been omitted in the present edition. The previous reviewer's note concerning formula 11.201 on p. 46 is confusing as formula 11.201 is on p. 36 while formula 112.01 is on p. 46. On pp. 105, 106, the formulas for 3.701 and 3.702 are incorrect. Correct forms can be deduced from formula 3.401 on p. 100. But for no choice of the parameters does this reduce to the stated results. Also, the value of the integral in 4.501 on p. 112 should read $\frac{1}{2}\pi[\ln (a/2)-\gamma]$. The denominator parameters for the ${}_3F_2$ in 64.302 on p. 124 are $3/2$ and $\mu + 3/2$.

As noted by the previous reviewer, the work on developing the 1964 report was halted when the authors learned of the similar but more extensive work which was then under preparation and available since 1954 as *Tables of Integral Transforms*, by A. Erdélyi et al., McGraw-Hill Book Co., Inc., New York, 1954 (see *Math Comp.*, v. 10, 1956, pp. 252–254). Most of the integrals in the present edition can be found in the above reference or in *Integrals of Bessel Functions* by Y. L. Luke, McGraw-Hill Book Co., Inc., New York, 1962 (see *Math. Comp.* v. 17, 1963, pp. 318–320). A virtue of the compilations by Wheelon and Robacker is their indication of sources for the formulas. In this connection, the authors of the present edition note on p. 64 that the references by Erdélyi et al. and Luke appeared after the collection was completed and consequently no explicit references to formulas in these sources are given. The statement seems incongruous as explicit references to formulas in the well-known *Handbook of Mathematical Functions* edited by M. Abramowitz and I. Stegun which first appeared in 1964 (see *Math. Comp.*, v. 19, 1965, pp. 147–149) are given.

Y. L. L.

25[7].—WILLARD MILLER, JR., *Lie Theory and Special Functions*, Academic Press, Inc., New York, 1968, xv + 338 pp., 23 cm. Price $16.50.

An excellent description of this volume, its purpose as well as its scope, is given in the preface:

"This monograph is the result of an attempt to understand the role played by special function theory in the formalism of mathematical physics. It demonstrates explicitly that special functions which arise in the study of mathematical models of physical phenomena and the identities which these functions obey are in many cases dictated by symmetry groups admitted by the models. In particular it will be shown that the factorization method, a powerful tool for computing eigenvalues and recurrence relations for solutions of second order ordinary differential equations (Infeld and Hull [1]), is equivalent to the representation theory of four local Lie groups. A detailed study of these four groups and their Lie algebras leads to a unified treatment of a significant proportion of special function theory, especially that part of the theory which is most useful in mathematical physics.

"Most of the identities for special functions derived in this book are known in one form or another. Our principal aim is not to derive new results but rather to provide insight into the structure of special function theory. Thus, all of the identities obtained here will be given an explicit group-theoretic interpretation instead of being considered merely as the result of some formal manipulation of infinite series.

"The primary tools needed to deduce our results are multiplier representations of local Lie groups and representations of Lie algebras by generalized Lie derivatives. These concepts are introduced in Chapter 1 along with a brief survey of classical Lie theory. In Chapter 2 we state our main theme: Special functions occur as matrix elements and basis vectors corresponding to multiplier representations of local Lie groups. Chapters 3–6 are devoted to the explicit analysis of the special function theory of the complex local Lie groups with four-dimensional Lie algebras $\mathcal{G}(0, 0)$, $\mathcal{G}(0, 1)$, $\mathcal{G}(1, 0)$ and six-dimensional Lie algebra $\mathfrak{T}_6$. Many fundamental properties of hypergeometric, confluent hypergeometric, and Bessel functions are obtained from this analysis. Furthermore, in Chapter 7 it is shown that the representation theory of these four Lie groups is completely equivalent to the factorization method of Infeld and Hull.

"In Chapter 8 we determine the scope of our analysis by constructing the rudiments of a classification theory of generalized Lie derivatives. This classification theory enables us to decide in what sense the results of Chapters 3–6 are complete. Chapter 8 is more difficult than the rest of the book since it presupposes a knowledge of some rather deep results in local Lie theory and an acquaintance with the cohomology theory of Lie algebras. Hence, it may be omitted on a first reading.

"Finally, in Chapter 9 we apply some of the results of the classification theory to obtain identities for special functions which are not related to the factorization method.

"It should be noted that we will be primarily concerned with the representation theory of local Lie groups, a subject which was developed in the nineteenth century. The more recent and sophisticated theory of global Lie groups is by itself too narrow to obtain many fundamental identities for special functions. Also, unitary representations of Lie groups will occur only as special cases of our results; they will not be our primary concern.

"The scope of this book is modest: we study no Lie algebras with dimension greater than 6. Furthermore, in the six-dimensional case, $\mathfrak{J}_6$, we do not give complete results. (The so-called addition theorems of Gegenbauer type for Bessel functions would be obtained from such an analysis.) However, it should be clear to the reader that our methods can be generalized to higher dimensional Lie algebras.

"We will almost exclusively be concerned with the derivation of recursion relations and addition theorems. The manifold applications of group theory in the derivation of orthogonality relations and integral transforms of special functions will rarely be considered. For these applications see the encyclopedic work of Vilenkin. The overlap in subject matter between that book and this one is relatively small, except in the study of unitary representations of real Lie groups."

WILHELM MAGNUS

Courant Institute of Mathematical Sciences
New York University
New York, New York 10012

26[7].—RUDOLF ONDREJKA, *Exact Values of* 2^n, $n = 1(1)4000$, ms. of 519 computer sheets deposited in the UMT file.

This impressive table of the exact values of the first 4000 powers of 2, which was computed in 1961 on an IBM 709 system, constitutes the first volume of an extensive unpublished series of such tables.

In private correspondence with the editors the author has revealed that in October and November 1966 and in the period from May through August 1967 he extended this initial table by computing the next 29,219 powers of 2 on an IBM 7090 system. These additional powers occupy a total of 27,023 computer sheets, arranged in 74 volumes, which are in the possession of the author.

A further statistic supplied by the author is that the total number of digits in all 75 volumes is 166,115,268. This digit count is also given for each volume; thus, the volume under review, for example, contains 2,410,843 digits. A useful index has been supplied for the entire set of tables.

The selection of 33,219 as the total number of entries in this immense tabulation was based on the author's plan to include all powers of 2 whose individual lengths do not exceed 10,000 decimal digits.

The tabular entries are clearly printed in groups of five digits, with 19 such pentads on each line. The appropriate exponent is printed beside each entry, and successive powers are conveniently separated by a double space.

In a brief abstract the author has added the information that tabular values were spot checked by comparison with the published results of others, particularly those relating to the known Mersenne primes.

J. W. W.

27[7].—H. R. AGGARWAL & VAHE SAGHERIAN, *An Extension of the Tables of the Quotient Functions of the Third Kind*, Stanford Research Institute, Menlo Park, Calif., ms. of 18 typewritten pages deposited in the UMT file.

The quotient functions of the third kind refer herein to the ratios

$$h_\nu^{(1)}(z) = zH_{\nu-1}^{(1)}(z)/H_\nu^{(1)}(z) \quad \text{and} \quad h_\nu^{(2)}(z) = zH_{\nu-1}^{(2)}(z)/H_\nu^{(2)}(z)\ ,$$

where $H_\nu^{(1)}(z)$ and $H_\nu^{(2)}(z)$ are the Hankel functions of the first and second kinds, respectively, of order ν.

The tables consist of 5D values of $[2/(2n - 1)]\mathcal{R}[h_n^{(1)}(x)]$ and $\mathcal{I}[h_n^{(1)}(x)]$ for $n = 0(1)10$, $x = 0(0.2)15$. Graphs of the tabulated quantities are included. The ratio $h_\nu^{(2)}(x)$ is not tabulated, since it is merely the complex conjugate of $h_\nu^{(1)}(x)$.

Appropriate reference is made to the related tables of Onoe [1], [2].

The authors include a discussion of the asymptotic forms of the functions $h_\nu^{(1)}(z)$ and $h_\nu^{(2)}(z)$, and they state that such functions arise in the study of the propagation of sinusoidal sound waves in loud-speaker horns and in the diffraction of steady plane elastic waves by circular cylindrical discontinuties embedded in an infinite medium.

J. W. W.

1. M. Onoe, "Formulae and tables, the modified quotients of cylinder functions," Report of the Institute of Industrial Science, University of Tokyo, v. 4, 1955, pp. 1–22. (See *MTAC*, v. 10, 1956, p. 53, RMT **29.**)
2. M. Onoe, *Tables of Modified Quotients of Bessel Functions of the First Kind for Real and Imaginary Arguments*, Columbia Univ. Press, New York, 1958. (See *MTAC*, v. 13, 1959, p. 131, RMT **22.**)

28[7, 13.05].—Shigetoshi Katsura & Katsuhiro Nishihara, *Tables of Integrals of Products of Bessel Functions*. II, Department of Applied Physics, Tôhoku University, Sendai, Japan, undated, ms. of 21 computer sheets deposited in the UMT file.

These manuscript tables, which constitute a sequel to earlier manuscript tables by Kilpatrick, Katsura & Inoue [1], gives 16S decimal values (in floating-point form) to the coefficients of p^l in the polynomial expression for the integral

$$\int_0^\infty J_{1/2}(pt)J_{3/2+m}(bt)J_{3/2+n}(ct)t^{-3/2}dt$$

for $l = -\frac{1}{2}\ (1)\ m + n + 7/2$; $m, n = 0(1)6$, $m + n$ even; $b, c = 1$ and 2. These data were calculated on an IBM 7090, using double-precision arithmetic. A spot check revealed that several entries are accurate to only 13S.

Details of the evaluation of the integral by the calculus of residues are set forth in the introductory text of seven pages, to which is appended a list of eight references.

These tables have been used by the authors in the calculation of the molecular distribution function for a square-well potential gas.

J. W. W.

1. J. E. Kilpatrick, Shigetoshi Katsura & Yuji Inoue, *Tables of Integrals of Products of Bessel Functions*, Rice University, Houston, Texas and Tôhoku University, Sendai, Japan, 1966. (See *Math. Comp.*, v. 21, 1967, p. 267, UMT **27.**)

29[9].—Edward T. Ordman, *Tables of the Class Number for Negative Prime Discriminants*, National Bureau of Standards, 1968, 18 Xeroxed computer sheets deposited in the UMT file.

There are deposited here two tables of class numbers $h(-p)$: (1) those for the first 2455 primes of the form $8n + 3$, lying in the range $3 \leqq p \leqq 102059$, (2) those for the first 2445 primes of the form $8n + 7$, lying in the range $7 \leqq p \leqq 102103$. Each table required only a few minutes computer time on a Univac 1108 in August, 1968. The program computed and counted the reduced forms for these (negative prime) discriminants.

A number of checks were made and no error was discovered. The first table was computed originally for the specific purpose [1] of determining those p with $h(-p) = 25$ and $p < 163 \cdot 25^2$. This accounts for the upper limit on p indicated above. Of course, the tables will have many other uses.

From theory, each of these class numbers is odd, and we list the first and last examples for each class number $h = 1(2)25$.

h	$8n + 3$		$8n + 7$	
1	3	163*	7	7*
3	59	907	23	31*
5	131	2683	47	127*
7	251	5923	71	487*
9	419	10627	199	1423*
11	659	15667	167	1303*
13	1019	20563	191	2143*
15	971	34483	239	2647
17	1571	37123	383	4447
19	2099	38707	311	5527
21	1931	61483	431	5647
23	1811	90787	647	6703
25	3851	93307	479	5503

It is highly probable that the "last examples" in these tables are the largest that exist, but that has been proven only for those cases marked with an asterisk * [1].

Aside from this printed version, the tables are also kept on punched cards, as are the earlier tables of $h(p)$, $p \equiv 1 (\mathrm{mod}\ 4)$, that were deposited in the UMT file and reviewed previously [2].

D. S.

1. Daniel Shanks, "On Gauss's class number problems," *Math. Comp.*, v. 23, 1969, pp. 151–163.
2. K. E. Kloss et al., *Class Number of Primes of the Form* $4n + 1$, RMT **10**, *Math. Comp.*, v. 23, 1969, pp. 213–214.

30[9].—J. H. Jordan & J. R. Rabung, *A Table of Primes of* $Z[(-2)^{1/2}]$, Washington State University, Pullman, Washington, July 1968, twenty computer sheets deposited in the UMT file.

Each (rational) prime p of the form $8m + 1$ or $8m + 3$ has a unique decomposition

$$p = a^2 + 2b^2 = (a + b(-2)^{1/2})(a - b(-2)^{1/2}).$$

This table gives a and b for the 4793 such primes $p < 10^5$, from

$$3 = 1^2 + 1^2 \cdot 2 \quad \text{to} \quad 99971 = 207^2 + 169^2 \cdot 2.$$

It is printed on 19+ sheets of computer paper, 250 primes per page, and was computed on an IBM 360, Mod 67. No details are given concerning the method or computer time.

The same table is contained (with much else) in Cunningham's valuable book [1] that is long out of print.

The present table is similar in character and scope to this Computing Center's earlier table of Gaussian primes. Our detailed and lengthy commentary [2] on that earlier table could have analogous remarks here, but we largely leave examination of such analogies to any interested reader. The largest complex prime in $Z[(-2)^{1/2}]$ presently known [3] to the undersigned is the quite modest:

$$179991 + (-2)^{1/2}.$$

Clearly, larger complex primes here would not be difficult to find.

It may be of interest to add that while the primes $p = a^2 + 2b^2$ constitute asymptotically one-half of the primes, and this does not differ from the primes

$$p = a^2 + nb^2$$

with $n = -2, 1, 3, 4, 7$, the number of composites $c = a^2 + 2b^2$ is exceptionally large [4].

D. S.

1. A. J. C. Cunningham, *Quadratic Partitions*, Hodgson, London, 1904.
2. L. G. Diehl & J. H. Jordan, *A Table of Gaussian Primes*, UMT **19**, *Math. Comp.*, v. 21, 1967, pp. 260–262.
3. Daniel Shanks, "On the conjecture of Hardy and Littlewood concerning the number of primes of the form $n^2 + a$," *Math. Comp.*, v. 14, 1960, pp. 321–332. (We check their count by $\pi(10^5) - \bar{\pi}_2(10^5) - 1 = 4793$ from our Table 3.)
4. Daniel Shanks & Larry P. Schmid, "Variations on a theorem of Landau. Part I," *Math. Comp.*, v. 20, 1966, Sect. 6, pp. 560–561.

31[9].—Beth H. Hannon & William L. Morris, *Tables of Arithmetical Functions Related to the Fibonacci Numbers*, Report ORNL-4261, Oak Ridge National Laboratory, Oak Ridge, Tenn., June 1968, iii + 57 pp., 28 cm.

Five arithmetical functions related to the Fibonacci numbers, u_n, are herein tabulated for all positive integer arguments m to 15600, inclusive.

The first of these, designated by $\pi(m)$, is called the *Pisano period* of m; it is the least integer k such that $u_k \equiv 0 \pmod{m}$ and $u_{k+1} \equiv 1 \pmod{m}$. Closely related to this function is the *restricted period* of m, here denoted by $\alpha(m)$, which is the least integer n such that $u_n \equiv 0 \pmod{m}$. This is generally called the "rank of apparition" or "entry point," and has been previously tabulated [1], [2] for all primes less than 10^5. The quotient $\beta(m) = \pi(m)/\alpha(m)$ is also tabulated in this report.

J. D. Fulton and the second of the present authors [3] have established the existence of two new arithmetical functions of the Fibonacci numbers, by virtue of a fixed-point theorem; namely, "$\pi(m) = m$ if and only if $m = (24)5^{\lambda-1}$ for some integer $\lambda > 1$"; and an iteration theorem: "There exists a unique smallest positive integer ω such that $\pi^{\omega+1}(m) = \pi^{\omega}(m)$, where $\pi^{n+1}(m) = \pi^n\{\pi(m)\}$ for $n \geqq 1$." The authors call these functions $\omega(m)$ and $\lambda(m)$ such that $\pi^{\omega}(m) = (24)5^{\lambda-1}$, the *Fibonacci frequency* of m and the *Leonardo logarithm* of m, respectively.

The tables are arranged so that the five functional values for each of 300 consecutive arguments appear on each page. Equivalent Latin letters are used in the headings because of the resulting convenience in printing directly from the computer output tapes. The computation of the table was performed on an IBM 360/75 system in one hour.

These attractively printed tables supplement earlier tables, which have been restricted to tabular arguments that are primes. This restriction, however, is not serious with respect to the functions $\alpha(m)$ and $\pi(m)$, since their values for composite m equal the least common multiples of the values corresponding to the constituent prime powers.

Important references not listed by the authors include a paper by Wall [4] and a book by Jarden [5], which has a very extensive bibliography, arranged chronologically.

J. W. W.

1. Marvin Wunderlich, *Tables of Fibonacci Entry Points*, The Fibonacci Association, San Jose State College, San Jose, Calif., January 1965. (For a joint review of this and the following reference, see *Math. Comp.*, v. 20, 1966, pp. 618–619, RMT **87** and **88**.)

2. Douglas Lind, Robert A. Morris & Leonard D. Shapiro, *Tables of Fibonacci Entry Points, Part Two*, The Fibonacci Association, San Jose State College, San Jose, Calif., September 1965.

3. John D. Fulton & William L. Morris, "On arithmetical functions related to the Fibonacci numbers," *Acta Arithmetica*. (To appear.)

4. D. D. Wall, "Fibonacci series modulo *m*," *Amer. Math. Monthly*, v. 67, 1960, pp. 525–532.

5. Dov Jarden, *Recurring Sequences*, second edition, Riveon Lematematika, Jerusalem, 1966. (See *Math. Comp.*, v. 23, 1969, pp. 212–213, RMT **9**.)

32[9, 10, 11, 12].—R. F. Churchhouse & J. C. Herz, Editors, *Computers in Mathematical Research*, North-Holland Publishing Co., Amsterdam, 1968, xi + 185 pp., 23 cm. Price $9.00.

This book consists of fifteen papers and an extensive bibliography about the application of computers to mathematical research.

The papers are as follows:

"Machines and pure mathematics," by D. H. Lehmer discusses some of the opportunities for involving, not replacing, the pure mathematician with the computer. The author also submits a case for the construction of special purpose hardware for application to mathematical research.

"Congruences for modular forms," by A. O. L. Atkin describes in general terms the author's attempts to extend and generalize congruence properties of modular forms with the aid of a computer.

"Covering sets and systems of congruences," by R. F. Churchhouse describes the application of a computer to the problem of determining the number of distinct solutions of a system of congruences, and, in particular, to determine covering sets of congruences.

"A tabulation of some information concerning finite fields," by J. H. Conway gives a brief account of some extensive tables of information concerning finite fields computed by the author and M. J. T. Guy.

"On a specific similarity of finite semigroups," by P. Deussen describes and characterizes a refinement of the notion of similarity suggested by the theory of finite automata.

"Calculs algébriques dans l'anneau des vecteurs de Witt," by J. J. Duby is concerned with the application of a computer to the algebraic manipulation of Witt vectors.

"An algorithm that investigates the planarity of a network," by A. J. W. Duijvestijn reports on the author's investigations of planarity of networks using a computer.

"On some number-theoretical problems treated with a computer," by C. E. Fröberg briefly discusses the application of computers to Wilson and Fermat remainders, inverses of twin primes and Möbius power series.

"L'usage heuristique des ordinateurs en mathématiques pures," by G. Glaeser discusses some experiences in using a computer in analytical mathematical research.

"Sur la cyclabilité des graphes," by J. C. Herz discusses the cyclability of graphs and describes an investigation of hypo-hamiltonian graphs using a computer.

"A problem in stable homotopy theory and the digital computer," by A. Liulevicius describes an algorithm and its machine implementation for the construction of families of vector spaces which start spectral sequences approximating the p-primary components of the homotopy groups of spheres.

"Obtention automatique des équations de Runge et Kutta," by J. Martinet and Y. Siret discusses the automatic derivation of Runge-Kutta formulae. A systematic method is given for writing Runge-Kutta formulae of arbitrary rank and order.

"A method for computing the simple character table of a finite group," by J. K. S. McKay describes a method for determining the character table of a finite group given in terms of a generator and relation presentation.

"Periodic forests of stunted trees: the identification of distinct forests," by J. C. P. Miller describes in detail the identification of distinct individual forests, and some results obtained using computer examination of the various forests and tesselations are given.

"Computations on certain binary branching processes," by S. M. Ulam gives a brief account of some computations performed on a combinatorial problem suggested by a certain schematized model of evolution.

The bibliography contains some 300 references. It is intended to cover all published papers involving the use of computers as an aid to mathematical research except for papers published after 1966, papers of restricted circulation, and papers on boolean function minimization, error-correcting codes, experimental testing of numerical methods, construction of function tables and studies in "automatic proving."

HAROLD BROWN

Ohio State University
Columbus, Ohio 43210

33[10, 11].—C. J. Bouwkamp, *Catalogue of Solutions of the Rectangular $3 \times 4 \times 5$ Solid Pentomino Problem*, Technological University Eindhoven and Philips Research Laboratories, N. V. Philips' Gloeilampenfabrieken, Eindhoven, Netherlands, July 1967, 310 pp., 30 cm. One copy deposited in the UMT file.

In this remarkable document, which resembles a small town telephone directory both in size and composition, Professor Bouwkamp, who is more widely known for his contributions to electromagnetic theory, presents a catalog of the 3940 solutions which he and an assortment of computers succeeded in finding, over a period of several years, to the $3 \times 4 \times 5$ solid pentomino problem, described by this reviewer [1].

Although the programming of two-dimensional problems, pioneered in 1958 by Dana Scott, has now become commonplace, with 2339 solutions to the 6×10 rectangle of (plane) pentominoes having been found independently by numerous investigators, this catalog appears to be the first publication to document the exhaustive computer search for the solutions of a three-dimensional problem.

In addition to the obvious contribution to polyominology, these methods should also be applicable to crystallography, organic chemistry, and other disciplines in which it is of interest to enumerate the possible ways in which a given set of three-dimensional building blocks can be fitted together.

Solomon W. Golomb

University of Southern California
University Park
Los Angeles, California 90007

1. S. W. Golomb, *Polyominoes*, Charles Scribner's Sons, New York, 1965, pp. 116–118.

34[12].—John M. Blatt, *Introduction to Fortran* IV *Programming*, Goodyear Publishing Co., Pacific Palisades, Calif., 1968, xi + 313 pp. Price $5.25.

By now there are various versions of Fortran IV—different dialects as it were. This particular text by John Blatt is geared specifically to users of computers using the IBM G-level Fortran IV, a language which is implemented in the student compiler WATFOR for the IBM 360.

Certain features will look unfamiliar to the seasoned programmer, but this textbook is not intended for him; it is aimed at the novice whose task is to learn the WATFOR version of Fortran IV in minimum time. From this point of view, the author succeeds admirably. In clear terms he covers each of the topics thoroughly and with pedagogic finesse. His text is abundant with good examples, well-documented and to the point, and each chapter contains a series of pertinent questions, the answers to which are also supplied.

Most textbooks on the subject would stop at this point, but John Blatt goes further. With customary thoroughness, the author expounds his philosophy on computers and languages and pinpoints the frequent inconsistencies in this field. Thus, in addition to supplying the beginning student with all the tools necessary for learning efficiently, he also provides him with an up-to-date background on the state of the art so that, indeed, there would be little, if any, need to seek other supplementary material.

Henry Mullish

Courant Institute of Mathematical Sciences
New York University
New York, New York 10012

35[12].—Mario V. Farina, *Programming in BASIC*, Prentiss-Hall, Inc., Englewood Cliffs, N. J., 1968, ix + 164 pp., 28 cm.

BASIC is the name of a programming language used on teletype consoles by students at Dartmouth College in time-sharing communication with the GE-265 and GE-645 computers. It does not have the flexibility of Fortran IV which it resembles in many ways, but what it lacks in repertoire it more than makes up for in terms of usefulness.

This text is an attempt to illustrate the BASIC language and it is to the credit of the author that he accomplishes his task magnificently. The book assumes no previous knowledge of either time-sharing features or programming know-how and, without realizing it, the reader is introduced to the language with ease and clear understanding.

On a recent visit to Dartmouth, I was told that it takes a student about two hours to learn BASIC. This may be somewhat of an exaggeration or may apply only to those specially gifted students we are used to seeing around computers nowadays. However, it takes only about two hours to get through Mr. Farina's book and it has the added advantage that if one has to learn BASIC one can enjoy it at the same time.

Henry Mullish

Courant Institute of Mathematical Sciences
New York University
New York, New York 10012

36[12].—Mario V. Farina, *COBOL Simplified*, Prentiss-Hall, Inc., Englewood Cliffs, N. J., 1968, xii + 528 pp., 28 cm.

Once again Mario Farina has shown that all it takes to write a clear, meaningful and substantial textbook on a computer language, is a thorough understanding of the subject matter, a penchant for short, precise statements, a sensitive pedagogic technique plus a desire to patiently present the subject matter in a logical order and in a manner designed to inform the reader in the most palatable way possible.

COBOL *Simplified*, despite its 528 easy-to-read pages, is an excellent contribution to the literature. It will not appeal to the COBOL expert or to one who is searching for all manner of novel or exotic techniques. Instead, it is ideally suited for the serious beginning student and very little by way of prerequisites is expected of him.

The structure of the COBOL language is carefully explained in the course of the 60 lessons each of which is accompanied by clear, appropriate sketches and program segments followed by review questions to which full answers are found in the rear of the book.

This text might not be a literary masterpiece but it most certainly accomplishes its stated goal, i.e. to teach those who need to learn the language quickly. I think this would make an excellent choice for any student of business. It is probably suitable for a semester's course in college although it may be used to great advantage as a self-instructional text.

HENRY MULLISH

Courant Institute of Mathematical Sciences
New York University
New York, New York 10012

37[12].—BRYAN HIGMAN, *A Comparative Study of Programming Languages*, American Elsevier Publishing Co., Inc., New York, 1967, iii + 164 pp., 22 cm. Price $8.50.

There is an essential need for comparative studies of programming languages. Unfortunately, this book does not contribute much to filling the void. It is a little book which gives an unbalanced and much too superficial view of programming languages. It is not suitable as a textbook but might be useful to someone with a knowledge of, say, Fortran who wants a feel for other programming languages with a decided "European" view. About a third of the book attempts to lay the formal groundwork for the comparative study of languages; this is however later ignored and instead a philosophical view is presented. The introduction to the basics of programming linguistics is good.

MAX GOLDSTEIN

Courant Institute of Mathematical Sciences
New York University
New York, New York 10012

38[12].—WILLIAM F. SHARP, *Basic*, The Free Press, New York, 1967, xi + 137 pp., 26 cm. Price $6.75 Hardbound, $3.95 Paperbound.

As an introduction to Computer Programming, using the BASIC language should prove to be most successful with students of both scientific or business orientation. It is clear, precise, amply illustrated and written in a style which is both interesting and engaging.

The first seventy-two pages are devoted to the various instructions and procedures, questions being asked along the way and their answers supplied in full. Once the fundamentals have been learned the student is told (page 71)—"you now know something about computers. You know they are *not* giant electronic brains, but they can be programmed to do rather clever things; and you have a fair notion of the manner in which this is done."

Who could ask for more?

The appendices will probably prove to be of great use to particular students. The first describes the Dartmouth /GE system while the second describes the UWBIC (University of Washington) system. The rest of the book lists some useful programs written in BASIC.

All in all this is a fine book carefully thought out and prepared and deserves a good measure of success.

HENRY MULLISH

Courant Institute of Mathematical Sciences
New York University
New York, New York 10012

39[12].—KEITH NICOL, *Elementary Programming and ALGOL*, McGraw-Hill Book Co., New York, 1965, viii + 147 pp., 24 cm. Price $6.50.

This hard cover book by Keith Nicol of the Edinburgh School of Computer Services is an honest attempt to present the fundamental principles of computers. These basic concepts are incorporated in the first four chapters but are written in a needlessly detailed fashion. To the uninitiated this could be most discouraging, and even for the initiated it makes for difficult reading.

The next three chapters deal with various hints on programming (a questionable practice since the reader still doesn't know what a program is all about), computer hardware and applications of computers. These chapters could have been omitted in large part since they do not add very much to the understanding of programming per se.

It is not until we arrive at Chapter 8 that we encounter the introduction to ALGOL programming itself which, according to the cover, is the principle purpose of the book—'a teach-yourself-programming book which will have a general appeal.'

The introduction to Algol is, indeed, clear and well planned but it lacks a sufficiently developed sequence of problems to satisfy most students. For this reason the book is not suitable for classroom use but rather for individual reading of a somewhat superficial nature.

HENRY MULLISH

Courant Institute of Mathematical Sciences
New York University
New York, New York 10012

40[12].—JOHN H. FASAL, *Nomography*, Frederick Ungar Publishing Co., New York, 1968, xviii + 382 pp., 26 cm. Price $14.50.

With the enormous expansion in the use of large-scale digital computers, the impression is generally held that there is less need for nomography. A nomogram, however, is not really a computing device. Rather, it is a method of presenting the voluminous results of computing with two or more independent variables in a more useful and compact form. Therefore, the increase in the volume of available data reinforces the need for nomography.

In this century, the demands of nomography have inspired many ingenious mathematical techniques. Most of these are described in this book. They include the basic three-scale nomograms designated by III, V, and N (or Z) which represent three straight line scales that are parallel, concurrent, or N-shaped, respectively. Also included are the nomograms designed by ICC, IIC, and CCC, where I represents a straight scale and C represents a curved scale. Combinations of these are used when there are more than two independent variables. An important process, described here, is the transformation of a given function into "partial functions" which conform to the relations inherent in the basic nomograms.

Frequently, when two nomograms are combined, the functional scale at the end of one alignment does not have the desired shape or graduation distribution to begin the second alignment. A technique, developed by the author, called Tangent Line Alignment (TLA), makes use of an auxiliary curve which is the envelope of straight lines joining corresponding numerical values of the two scales. Hence, one can proceed from one scale to the next by drawing a tangent line to this curve. This speeds the process and reduces the probability of error associated with reading one scale and then entering the second scale by interpolation.

It is well known that not all functions can be precisely nomographed. However, since all technical data have limited accuracy, one can usually devise a nomogram with the desired accuracy. Similarly, it is shown that empirical data, which have no known precise mathematical expression, can be satisfactorily approximated.

A large portion of the book is devoted to the anamorphic transformations of the nomograms to produce scales that are more uniform, more readable, and which permit greater accuracy. This process, which has not received as much attention in other books, frequently distinguishes the skill of the expert from the labor of an amateur.

The thoroughness of the exposition of the detailed construction of nomograms makes this work the most practical book in English. To this same end, the more esoteric mathematical techniques have been omitted in favor of the simpler techniques, even though the latter sometimes may require more steps.

Michael Goldberg

5823 Potomac Avenue, N.W.
Washington, D. C. 20016