A new class of Fermat pseudoprimes and few remarks about Cipolla pseudoprimes

Marius Coman
Bucuresti, Romania
email: mariuscoman13@gmail.com

Abstract. I wrote an article entitled “A formula for generating primes and a possible infinite series of Poulet numbers”; the sequence I was talking about not only that is, indeed, infinite, but is also already known as the sequence of Cipolla pseudoprimes to base 2. Starting from comparing Cipolla pseudoprimes and some of my notes I discovered a new class of pseudoprimes.

Introduction

The article I was talking about in Abstract was my second encounter with Cipolla pseudoprimes. I first submitted a sequence to OEIS (A217853) to define a subset of Fermat pseudoprimes to base 3, *i.e.* numbers of the form (3^(4*k + 2) - 1)/8. I just later saw the note of Mr. Bruno Berselli on this sequence, that for p prime, p = 2*k + 1, is obtained the generating formula for Cipolla pseudoprimes to base 3, namely (9^p – 1)/8, and I made the connection with my further article, in which I was talking about the numbers of the form (4^p – 1)/3, namely Cipolla pseudoprimes to base 2.

The formula (3^(4*k + 2) - 1)/8 generates Fermat pseudoprimes to base 3 not only for k = (p – 1)/2, where p prime (which gives the formula for Cipolla pseudoprimes to base 3), but for other values of k too.

The first few Cipolla pseudoprimes to base 3 are 91, 7381, 597871, 3922632451, 317733228541, 2084647712458321, 168856464709124011 (for more of them, see the sequence A210454 in OEIS).

The first few terms generated by the formula above are 91, 7381, 597871, 48427561, 3922632451, 317733228541, 25736391511831, 2084647712458321, 168856464709124011 (for more of them, see the sequence A217853 in OEIS).

It can be seen that the formula generates until the number 168856464709124011 three more Fermat pseudoprimes to base 3: 48427561, 3922632451 and 25736391511831.

It seemed logic to try to generalize the formula (3^(4*k + 2) - 1)/8, hoping that it can be obtained a class of pseudoprimes that would contain the set of Cipolla pseudoprimes, but instead of this I obtained something even more interesting, an entirely different class of Fermat pseudoprimes (containing pseudoprimes which are not in the Cipolla sequence and, *vice versa*, not containing pseudoprimes that are in Cipolla sequence).

A formula that generates Fermat pseudoprimes

Conjecture: The formula (n^(n*k + k + n - 1) – 1)/(n^2 – 1) generates an infinity of Fermat pseudoprimes to base n for any integer n, n > 1.

Verifying the conjecture

For n = 2 the formula becomes (2^(3*k + 1) - 1)/3 and generates the following Fermat pseudoprimes to base 2, for k = 3, 7, 11: 341, 1398101, 5726623061.

For n = 3 the formula becomes (3^(4*k + 2) - 1)/8 and generates Fermat pseudoprimes to base 3 for 14 values of k from 1 to 20.

For n = 4 the formula becomes (4^(5*k + 3) - 1)/15 and generates the following Fermat pseudoprime to base 4, for k = 1 : 4369.

For n = 5 the formula becomes (5^(6*k + 4) - 1)/24 and generates the following Fermat pseudoprime to base 5, for k = 1 : 406901.

Unfortunatelly the first term of the sequence (corresponding to k = 1) for n = 7 is larger than 10^10 and I do not have the possibility to extend the verifying, but seems there is enough data to justify the conjecture.

Conclusion

It can easily be seen that, for n = 2, the sequence of Cipolla pseudoprimes to base 2 contains until the pseudoprime 5726623061 two more pseudoprimes than the pseudoprimes I defined above (5461 and 22369621 – for the sequence of Cipolla pseudoprimes to base 2 see the sequence A210454 in OEIS) and I have shown above that Cipolla pseudoprimes to base 3 contains until the pseudoprime 168856464709124011 three less pseudoprimes than the pseudoprimes I defined above so it’s no need for a further proof that neither one of the two classes is not a subset of the other.

Reference

Cipolla Pseudoprimes, Y. Hamahata and Y. Kokobun