A set of formulas for primes

by
Simon Plouffe
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Abstract

In 1947, W. H. Mills published a paper describing a formula that gives primes : if A = 1.3063778838630806904686144926... then $\lfloor A^{3^n} \rfloor$ is always prime, here $\lfloor x \rfloor$ is the integral part of x. Later in 1951, E. M. Wright published another formula, if $g_0 = \alpha = 1.9287800\ldots$ and $g_{n+1} = 2^{g_n}$ then

$$\lfloor g_n \rfloor = \left\lfloor 2^{\cdot^{\cdot^{2^{2^{\alpha}}}}} \right\rfloor \text{ is always prime.}$$

The primes are uniquely determined by α , the prime sequence is 3, 13, 16381, …
The growth rate of these functions is very high since the fourth term of Wright formula is a 4932 digit prime and the 8'th prime of Mills formula is a 762 digit prime.
A new set of formulas is presented here, giving an arbitrary number of primes minimizing the growth rate. The first one is : if $a_0 = 43.8046877158\ldots$ and $a_{n+1} = a_n^{\frac{5}{4}}$, then if $S(n)$ is the rounded values of a_n, $S(n) = 113$, 367, 1607, 10177, 102217, 1827697, 67201679, 6084503671, …. Other exponents can also give primes like 11/10, or 101/100. If a_0 is well chosen then it is conjectured that any exponent > 1 can also give an arbitrary series of primes. When the exponent is 3/2 it is conjectured that all the primes are within a series of trees. The method for obtaining the formulas is explained. Five formulas are presented and all results are empirical.

Résumé

En 1947, W. H. Mills publiait un article montrant une formule qui peut donner un nombre arbitraire de nombres premiers. Si A = 1.3063778838630806904686144926... alors $\lfloor A^{3^n} \rfloor$ donne une suite arbitraire de nombres tous premiers. , ici $\lfloor x \rfloor$ est le plancher de x. Plus tard en 1951, E. M. Wright en proposait une autre, si $g_0 = \alpha = 1.9287800\ldots$ et $g_{n+1} = 2^{g_n}$ alors

$$\lfloor g_n \rfloor = \left\lfloor 2^{\cdot^{\cdot^{2^{2^{\alpha}}}}} \right\rfloor \text{ est toujours premier.}$$

Les premiers consécutifs sont uniquement représentés par α. La suite de premiers est 3, 13, 16381,…
Le taux de croissance de ces 2 fonctions est assez élevé puisque le 4ème terme de la suite de Wright a 4932 chiffres décimaux. La croissance de celle de Mills est moins élevée, le 8ème terme a quand même une taille de 762 chiffres. Une série de formules est présentée ici qui minimise le taux de croissance et qui possède les mêmes propriétés de fournir une suite de premiers de longueur arbitraire. Si $a_0 = 43.8046877158\ldots$ et $a_{n+1} = a_n^{\frac{5}{4}}$ alors la suite $S(n) = \{ a_n \}$: l'arrondi de a_n, est une suite de premiers de longueur arbitraire. Ici l'exposant $^5/_4$ peut être abaissé à $^{11}/_{10}$, ou même $^{101}/_{100}$. Si a_0 est bien choisi il est conjecturé que l'exposant peut être aussi près de 1 que l'on veut. Un autre modèle est basé sur $\{c^n\}$, où { } est l'arrondi avec c = 31622.77671… et n > 1 permet d'obtenir 387 nombres premiers d'affilée. Cinq formules sont présentées et tous les résultats sont empiriques.

Introduction

The first type of prime formula to consider is for example, given a_0 a real constant > 0 and $a_{n+1} = a(n)10$, if $a_0 = 7.3327334517988679...$ then the sequence $73, 733, 7333, 73327, 733273, ...$ is a sequence of primes but fails for obvious reasons after a few terms. If the base is changed to any other fixed size base, taking into account that the average gap between primes is increasing then eventually the process fails to give any more primes.

If we choose a function that grows faster like n^n, we get better results. The best start constant found is $c = 0.265588372943143390897129453665466129438 9...$ giving 19 primes.
But fails at 23 (beginning at n = 3). Here $a_n = \lfloor cn^n \rfloor$.

7
67
829
12391
218723
4455833
102894377
2655883729
75775462379
2368012611049
80440106764817
2951219812933057
116299525867995629
4899240744635092571
219705395187452015923
10449948501874965563651
525445257345556693801913
27848959374722952425334841
1551723179991864497606172809

Again, for the same reasons mentioned earlier, the process fails to go further, no better example was found. The method used is a homemade Monte-Carlo method that uses Simulated Annealing (principe du recuit simulé in French).

If a function grows too slowly, eventually the average gap between primes increases and the process ceases to give any more primes. The next step was to consider formulas like Mills or Wright. The question was then : is there a way to get a useful formula that grows just enough to produce primes ?

If we consider the recurrence $a_{n+1} = {a_n}^2 - a_n + 1$ that arises in the context of Sylvester sequence. The Sylvester sequence is A000058 of the OEIS catalogue and begins with $a_0 = 2$, like this : (2, 3, 7, 43, 1807, 3263443, 10650056950807,), that sequence has the property that

$$1 = \frac{1}{2} + \frac{1}{3} + \frac{1}{7} + \frac{1}{43} + \frac{1}{1807} + \frac{1}{3263443} + \cdots$$

The natural extension that comes next is: can we choose a_0 so that a_n will always produce primes ? The answer is yes, when $a(0) = 1.6181418093242092 \ldots$ and by using the $\lfloor x \rfloor$ function we get,

$$a(n) = 2, 3, 7, 43, 1811, 3277913, 10744710357637, \ldots$$

The sequence and formula are interesting for one reason the growth rate is quite smaller than the one of Mills and Wright.

A Formula for primes

What if we choose the exponent to be as small as possible? The problem with that last one is that it is stilll growing too fast, $\mathrm{a}(14) = 9.838 \ldots \times 10^{1667}$. The size of primes doubles in length at each step.

The simplest found was a_0= 43.80468771580293481… and using the { x } , rounding to the nearest integer, we get

$$a_{n+1} = a_n^{\frac{5}{4}}$$

Now, what if we carefully choose a_0 so that the exponent is smaller, would it work ? Let's try with 11/10 and start with a larger number.

$$a_{n+1} = a_n^{\frac{11}{10}}$$

For example when $S_0 = 1000000000000000000000000000049.31221074776345 \ldots$ and the exponent beeing 11/10 then we get the primes :

1000000000000000000000000000049
1584893192461113485202101373392367 53
52480746024977259736431215702272589 4401
39084089579240203009194723709573563459 33709
7099046158552872493128982511805942200534009 5813
343811184035069918804446105763101544331290090895 2333
4897246900042000942655570714250230366715503641784965 40501
…

If we want a smaller starting value then a_0 has to be bigger, I could get a series of primes when $a_0 = 10^{64} + 57 + \varepsilon$, where $0 < \varepsilon < 0.5$ chosen at random. In this case the exponent is

$$a_{n+1} = a_n^{\frac{21}{20}}$$

If we choose $a_0 = 10^{600} + 543 + \varepsilon$ then we get our formula to be.

$$a_{n+1} = a_n^{\frac{101}{100}}$$

If a(0) = 2.038239154782068767463490862609548251448624778443173 61… and the exponent 3/2 then the sequence of primes is :

3
5
11
37
223
3331
192271
84308429
774116799347
681098209317971743
562101323304225290104514179
1332667822014585978282511662572214575909
1538448162271607869601834587431948506238982765193425993274489

The natural question that comes next is : Can we generate all the primes with one single exponent ? Here is the tree graph of primes with the exponent 3/2.

Other ways of generating primes

Three other methods are presented. The first is based on a very high degree polynomial equation. We begin by the known Euler polynomialn^2+n+41 that produces 40 primes in a row when n goes from 0 to 39. In 2010, François Dress and Bernard Landreau found a 6^{th} degree polynomial that gives 56 primes values. It was obtained after several months of computation. The polynomial is

$$\frac{n^6}{72}-\frac{5n^5}{24}+\frac{n^6}{72}-\frac{1493n^4}{72}+\frac{1493n^3}{72}+\frac{100471n^2}{18}-\frac{11971n}{6}-57347\,.$$

All other attempts failed to go further. If we take another approach to it there is more.

If c is a real positive constant then

$$[cn^k+1/2]$$

Where [.] denotes the integer part function. For a fixed k much larger than 2 it is possible to calculate c using the simulated annealing and Monte-Carlo method used before. For example, if $k = 64$ then by using the formula

$$p = nextprime(n^k)$$

$$c = \frac{p}{n^k}$$

Then the value of the new c is calculated by rounding properly

$$p = nextprime\left(\left[cn^k+\frac{1}{2}\right]\right)$$

$$c_{k+1} = \frac{p}{(n+1)^k}$$

The hypothesis is : for a large k, c is constant and will produce an arbitrary number of primes. When k = 64, the method quickly finds 8 primes values but if k = 599 then 70 primes are produced. The value of c found is :

```
1.00000000000000000000000000000000000000000000000000000000000\
000000000000000000000000000000000000000000000000000000000000\
000000000000000000000000000000000000000000000000001556808232\
856463140060524522424280772934429200613394089802373785244587288\
947728630209290312111238140972072983869588571830870498647900351\
444778386216146076636515297094962921999030102707899655875083040\
665145525741443694679398437806398341541385226365101056503219529\
413038357761603719148231623498218255233850813147568998744093447\
755692241415245289626420200959624751354414736085310934015664680\
524107510074349656822480162844410900000000000000000000006345174\
532890027939294089368385587928671551372984708501441585620418908\
704346095361979911978459546006902805287612053866464609018035122\
943293118288150470557741929174121124290158771911527759861726269\
584165754859588606741441445142853262133802425477033893100601759\
975564856010884639189352574717896440249667645304974867391425986\
281906375105044471660347105158250104363991834652263421461752834\
975705269597512556117680247632410969440028501070105083651713662\
403142442324953222225461614957943694687384105246348009815340760\
743692542284884699931895728286412542979603098044884198867262644\
327255465732046886424762403984621514353794473387936916290029911\
549331529954509625933626714214956674379391784430822602413846681\
316909794084400273637943424856690207465817148161052758912779700\
994744869116506018623229585002806564436851770494944385807359855\
293872240395791047188062073418411994344296873461271952410865026\
038448423723192039177383879070574428090707165141606873722573750\
361413628118939617030810117859967828092642486835827304995753842\
763548295039042203138474167878091109846423833375302339937989596\
168347847772485744109449547853544023508997274222527142776574434\
316734525917951119963956001777673781864527502261570125603162371\
258224293816247417631732394744100802942101015408972089551902455\
081699086722777589571816095705739686859606854254953133788523318\
      655544571643416019528317460445413625249324735829
```

When k = 1001 I find this value of c that produces 104 prime values.

```
1.0000000000000000000000000000000000000000000000000000000000000000000000000000000000000000\
000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000\
000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000\
000000000000000000000000000000000125990588497934548663662088537715317898307345260180583\
239314358724555329817297106513494441582139488185453236405937727542690908153827943733036 05\
310322530652091645439928673895339886453788692930545215355404376402649247172183377018615 82\
638636102002300761706857737942057248158616439351733615199836131168826902841442481019202 08\
065101822106435737963163809108635396061203708668932324960514850864974679907635781146790 66\
435113450813623644227691271940216492351536624649137032946188221668879838703155884411062 14\
813409623059821730429508312713597404138622930061038299621315223049018118543690792855608 01\
561273955940556352623163356765982351399366005194717524465338841277751368008203218970348 33\
692521631496015366196367000000000000000000000000000000000000000285692246353792958227722 7375\
345732094591555555350790166247626322909668882728710513033698020930989044591593202532043 7\
918652649805344433445246098289991107619332925895040307879828755674986532299084889785545 66\
673431971836083350967255132622112448190360819980557364781081669492843970768782656892517 79\
501654371694034171210939040257416005104266296652730377726248639357420238848425131005515 63\
133861989316486362099221987059093082608423421762885883850112569626002972347670596885812 15\
421283991007161607605174972488236532528226966395535882753943591238829695331790564932076 92\
390580729768767039185512105650979471018088025717809344853975486750969077190034224330554 54\
364328928961965322410170679580912802265891581924347800526675954971660723444825630943031 32\
077499696188318461582326221834348501484027569112113989357765460731255649682630586748253 26\
783859946967176752499817099659580009874439472122446447633795154069385146723461311212946 11\
463218443692954932494828950877297532608497866937057280926988514654923921737307068499624 92\
312165961662529820707731485622820497974355592727116488110466260319286985502288618373349 23\
474452821218488363554133325708988523247428131322031893170970969313370902493644733761882 54\
803333723304904602543991005562225061836781939723955841735959523325147966650242848454884 85\
644568880465462425461519996887400689912955194273736103712603928610453499609186143051876 06\
011999465054526694148203400925668619247482616724474888438292775133460891630045075713137 65\
067924468480159509860873098258796564844912087707112995085361384831564050052891258792150 48\
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26258088459413010473109268527768400545228463809891383098883080726850216779393886850653390\
0282953922794033452518766212590

Which is a record since the polynomial [$cn^{1001} + {}^{1}/_{2}$] will produce 104 primes when n = 2..105.

The second formula is based on an observation about primes in arithmetic progression. The idea is to change the base asumption. Instead of an arithmetic progression, I use a geometric progression. If f is defined as :

$$f(n) = [c^n + 1/2]$$

Then when c = 2.553854696… , 7 primes are generated : 3, 7, 17, 43, 109, 277 and 709. But what if c >> 1 ? Again by using the previous method I can produce this table of values for c

Constant c such that $[c^n + 1/2]$ is prime	Values	Number of primes produced
2.553854696…	3, 7, 17, 43, 109, 277, 709	7
2027.1671684764912194343956	n=1..97	97
577.181936975247888…	n=1..22	22
593.46526943871…	n=2..48	47
31622.7767185595693…	n=2..388	387
55237.07504296764715433124781528617	n=2..633	632
999982.6807693608…	n=1..899	899

The record number of values is 899 with c = 999982.6807693608… From which we could conjecture that for sufficiently large c, an arbitrary number of primes can be produced.

The 3rd formula is based on Fermat primes. Fermat primes are the primes of the form : $2^{2^n} + 1$. What is known so far is that for n = 1 to 4 we have the primes 2, 5, 17, 65537 and the pattern stops here apparently. Again, by modifying the premise we just have to write

$$F(n) = [r^{2^n} + {}^{1}/_{2}]$$

And to search for a suitable real value of r that will produce the first Fermat primes and the ones after (modified Fermat primes). If r = 2 then we get the usual Fermat numbers after which n > 4 fails to produce more primes. But if r = 1.0905077… then we get this series of primes:

2, 3, 5, 17, 257, 65537, 4294967311, 18446744193968636141,
340282371357715587431288126011714099603,
115792092256830257597513487698137234684227436353307972878385071833485576558709, …
(see appendix for more prime values and the precise value of r).

Description of the algorithm and method

There are 3 steps

1) First we choose a starting value and exponent (preferably a rational fraction for technical reasons).
2) Use Monte-Carlo method with the Simulated Annealing, in plain english we keep only the values that show primes and ignore the rest. Once we have a series of 4-5 primes we are ready for the next step.
3) We use a formula for forward calculation and backward. The forward calculation is

   Forward : Next smallest prime to { $a(n)^e$ }.

   It is easy to find a probable prime up to thousands of digits. Maple has a limit of about 10000 digits on a Intel core i7 6700K, if I use PFGW I can get a probable prime of 1000000 digits in a matter of minutes.

   Backward : (to check if the formula works)

   Previous prime = solve for x in $x^e - S(n+1)$. Where $S(n+1)$ is the next prime candidate. This is where e needs to be in rational form in order to solve easily in floating point to high precision using Newton-like methods.

Conclusions

There are no proofs of all this, just empirical results. In practical terms, we have now a way to generate an arbitrary series of primes with (so far) a minimal growth function. The formulas are much smaller in growth rate than of the 2 historical results of Mills and Wright. Perhaps there is even a simpler formulation, I did not find anything simpler. In the appendix, the 50'th term of the sequence beginning with $10^{500} + 961$ is given, breaking the record of known series of primes in either a polynomial (46 values) or primes in arithmetic progression (26 values).

Appendix

Value of a(0) for $a_{n+1} = \ a_n{}^{\frac{5}{4}}$.

a(0) = (2600 digits)

43.804687715802934818596645625690894950810370871374951840740613287526704195060\
366496333710781644573956794355807574665393001463642473571079065207223321969922\
326494552387132067683753960164070302790690162723433581603760191002753078666774\
469946232497117701938607432869957196648511059166975420773341676861028138823707\
145708991647291076721005353676471481568302151534888878241346300219659089891215\
780781876827661832857108335468360232717755741928208235635733800860513772442062\
635595955070876840969137876654827677627140855187201242505001367812919936457278\
681348489670886436487861801073801226232634055126530099559493255624932776432552\
819474610435954018952569956027804788810438427180867593350995548365511057097553\
078554907720252882377686408554606097825641952371450537976431101758354711734008\
411947508130804223989560771735318948100069661049983567447403715688569612514858\
845657617953174046955389995901747768439405384567213891121279848782014529155104\
762944095592501085547572538562641744623272171448569221345575666891390474294002\
850913797454553749686615314701489947123703486927933780200879792730832786803750\
888773473411013259989624300666869363484432710398119568157264257588349720524610\
724950300561199331161448271858746051867038409517235853216892492983271791674287\
592808379854507150680319033191053916646608417677828927515630775969847441245544\
423584026051368572757642718277566432037026758283071283990356856317699564626274\
141668226527755609491863884378866239825492922461105400756900478891920861098228\
819905683224721442158661002137912306130798847105167149728311015686742896068018\
550396662614552146567091338925704955085781257617120202395748735733978849371267\
942622500001488791100476056516835505380255746631227807052972606079110664459745\
766180349330513035089116848652537022141697254064935243516349196181106691168487\
829457572952139696867092560917096482738315096882000734661646200875466712591216\
988739323450547076413473147462443730390869791803764287897015457090312441400397\
035727236780869036666409143377213276642966534966635531648174167527244529903914\
228689377194990458562026548370540809026564047544800854856902269641927853701527\
905251566604661324828400744171499811210910655283172912855463849940560379083667\
405586467283254508314622550771124665670893874389752184393456176782493932252715\
944665640637738534342298630458648025198013277482984689488630293472751232085566\
731105782657070205597874756526478693575883866091883579095671672689596878189398\
619757708983553342825007007646386167273823668982442934126614300824963048052924\
514224472135474312310496397493614629336444086765617451388176181121619586135540\
722659098283245516053234889227085303252621795327120375393624184469944478085265\
926712756105661

50’th prime in the formula $a_{n+1} = a_n{}^{\frac{101}{100}}$ and $a_0 = 10^{500} + 961 + \epsilon, 0 < \ \epsilon < 0.5$

S(50)= (807 digits).

129729528971426122166658259081315435974871367309456840812055525509563976052536464197\
821936120784492089449745630948278142648656401758919926499683620493424145145363861773\
044716845814540511418289754542689191694327904116242782241131052138054549585683795895\
226460529926493834263717492409387560259409231253958370245042303023794648019244182073\
576593618946511947995963350548413770285593359081097306798650486731513585054871329096\
194202981055877907668708729761964242992640744211230936407662435884639367683685800000\
716124853576007781499789743771269181463159253173337794440878414346193538514506034277\
502087533266305538298562224619861085522581430515597209416207494298867400378422593043\
260350351208262898632520628116793338057678207643439460644660886621181985756002255888\
2590435234023721689322609979064776193485350033987563

Primes generated by $\{c^n\}$, $n > 1$, will produce 387 consecutive primes. { } is the nearest integer.

c = 31622.77671855595693411819787061428809211960501987066916850926043972317850362074\
0864754895287267371526503218374753273756995351706550976656637342668814334968666\
7101253409003404462577908490644122849185117910040141357956901545395665144254926\
0368733397963027260382237357834177624731483528295517610175141894026910108773091\
9102265276155731349632552235646978900096647191268683240475690792929946636829523\
4537889911695242127537567542061489388029765753953635000339676196085538110591110\
4585141090122658582123787181713420542296072149094895524374290222132677205020669\
4445695634733625075384860598871603780178145604934183538244723142272254031969991\
1200615639448998851471276974790874283370668013877448445827189836933336932748683\
7096031634918922648353422318092987818160592340775276127611835724115334227731181\
2382996597965951469528485537607311159096901622414506311547051905763888568046565\
1049166427400916041959872354321110387678917719194813843955193371342978053712862\
0453524374293366590642009861486573168440136568062290968471195962360800157373661\
7143373070165944776420832972019845288922599364819623823506773804799606015475838\
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Value of r for the Modified Fermat primes :

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Modifies Fermat primes:

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