２０２０年度　東京大学大学院

数理科学研究科　数理科学専攻　修士課程

英　語　（筆記試験）

２０１９年　８月　２６日（月）
１０：４０　～　１２：００

問題は全部で２題ある．２題とも解答すること．

（１）　解答しようとする各問ごとに解答用紙を１枚使用すること．
各解答用紙の所定欄に各自の**氏名**，**受験番号**と解答する**問題の番号**を記入すること．

（２）　草稿用紙の上部に各自の**受験番号**を明記すること．ただし氏名を記入してはならない．

（３）　試験終了後に提出するものは，１題につき１枚，計**２枚の答案**，および**草稿用紙**である．着手した問題数が２題にみたない場合でも，氏名と受験番号のみを記入した白紙の答案を補い，２枚とすること．
指示に反したもの，**答案が２枚でないものは無効**とする．

（４）　解答用紙の裏面を使用する場合は，表面の右下に「裏面使用」と明記すること．

E 第1問

(1) 次の英文の下線部を和訳せよ．ただし，数学記号はそのまま訳文に用いても良い．

One of Euler's most sensational early discoveries, perhaps the one which established his growing reputation most firmly, was his summation of the series $\sum_1^\infty n^{-2}$ and more generally of $\sum_1^\infty n^{-2\nu}$, i.e. in modern notation $\zeta(2\nu)$, for all positive even integers 2ν. This was a famous problem, first formulated by P. Mengoli in 1650; it had resisted the efforts of all earlier analysts, including Leibniz and the Bernoullis. <u>Characteristically, before solving it, Euler had engaged in extensive numerical calculations in order to get good approximate values for these sums; it is largely with this intention, it seems, that he had developed the method traditionally known as "the Euler-Maclaurin summation formula", and in so doing had re-discovered the "Bernoulli numbers", whose true importance for number theory was not to emerge until the next century.</u>

What Euler found in 1735 is that $\zeta(2) = \pi^2/6$, and more generally, for $\nu \geq 1$, $\zeta(2\nu) = r_\nu \pi^{2\nu}$, where the r_ν are rational numbers which eventually turned out to be closely related to the Bernoulli numbers. <u>At first Euler obtained the value of $\zeta(2)$, and at least the next few values of $\zeta(2\nu)$, by a somewhat reckless application of Newton's algebraic results, on the sums of powers of the roots for an equation of finite degree, to transcendental equations of the type $1 - \sin(x/a) = 0$. With this procedure he was treading on thin ice, and of course he knew it.</u>

[注] Euler: オイラー（人名），Maclaurin: マクローリン（人名），Bernoulli: ベルヌーイ（人名），Newton: ニュートン（人名）．

[出典] A. Weil, Number Theory: an approach through history; From Hammurapi to Legendre, Birkhäuser, 1984, p. 184 （一部改変）．

(2) 次の英文を和訳せよ．ただし，数学記号はそのまま訳文に用いても良い．

A system of partial differential equations is, informally speaking, a collection of several partial differential equations for several unknown functions.

Definition. An expression of the form

$$\mathbf{F}(D^k\mathbf{u}(x), D^{k-1}\mathbf{u}(x), \ldots, D\mathbf{u}(x), \mathbf{u}(x), x) = \mathbf{0} \quad (x \in U)$$

is called a k^{th}-order system of partial differential equations, where

$$\mathbf{F} : \mathbb{R}^{mn^k} \times \mathbb{R}^{mn^{k-1}} \times \cdots \times \mathbb{R}^{mn} \times \mathbb{R}^m \times U \to \mathbb{R}^m$$

is given and

$$\mathbf{u} : U \to \mathbb{R}^m, \quad \mathbf{u} = (u^1, \ldots, u^m)$$

is the unknown.

（中略）

There is no general theory known concerning the solvability of all partial differential equations. Such a theory is extremely unlikely to exist, given the rich variety of physical, geometric, and probabilistic phenomena which can be modeled by partial differential equations. Instead, research focuses on various particular partial differential equations that are important for applications within and outside of mathematics, with the hope that insight from the origins of these partial differential equations can give clues as to their solutions.

[出典] L. C. Evans, Partial Differential Equations, Graduate Studies in Mathematics, AMS, 1998, vol. 19, pp. 2–3 （一部改変）．

E 第2問

次の和文を英訳せよ．ただし，数学記号はそのまま訳文に用いても良い．

$\{f_1, \ldots, f_m\}$ を正規直交系，すなわち，$(f_i, f_j) = \delta_{ij}$ $(1 \leq i, j \leq m)$ なるベクトルの集合とすれば，次のようなことが成立する：

$$x = c_1 f_1 + \cdots + c_m f_m, \quad y = d_1 f_1 + \cdots + d_m f_m$$

に対して，$(x, y) = c_1 d_1 + \cdots + c_m d_m$．特に，$(x, f_i) = c_i$ $(1 \leq i \leq m)$．

逆にこのような性質をもつベクトルの集合 $\{f_1, \ldots, f_m\}$ は正規直交系である．

正規直交系である基底を正規直交基底という．一般に $\mathbf{R}^n$ の任意の部分空間 W が与えられたとき，W の基底として正規直交系がえらべるであろうか？ それはつねに可能である．実際，次の定理が成立する．

定理　W を r 次元の部分空間とする．$\{f_1, \ldots, f_p\}$ を W に含まれる正規直交系とすれば，$p \leq r$ であり，適当に $(r-p)$ 個の W のベクトル f_i $(p+1 \leq i \leq r)$ をつけ加えて $\{f_1, \ldots, f_p, f_{p+1}, \ldots, f_r\}$ がまた正規直交系になる（従って W の基底になる）ようにすることができる．

[注]（一つの）正規直交系：(an) orthonormal system
（一つの）基底：(a) basis

[出典] 佐武一郎『線型代数学』裳華房 (1974)，p. 99（一部改変）．