

# Minimal elements for the base $b$ representations of the primes which are $> b$

## Keywords

[prime number](#), [number theory](#), [minimal element](#), [partially ordered set](#), [subsequence](#), [formal language theory](#), [positional notation](#), [radix](#), [algorithm](#), [computer science](#), [primality test](#), [Miller–Rabin primality test](#), [Baillie–PSW primality test](#), [sieving](#), [heuristic algorithm](#), [conjecture](#), [open problem](#), [mathematical proof](#)

## Introduction

A [string](#)  $x$  is a [subsequence](#) of another string  $y$ , if  $x$  can be obtained from  $y$  by deleting zero or more of the [characters](#) (in this article, [digits](#)) in  $y$ . For example, 514 is a subsequence of 352148, “*string*” is a subsequence of “*Meistersinger*”. In contrast, 758 is not a subsequence of 378259, since the [characters](#) (in this article, [digits](#)) must be in the same order. The [empty string](#)  $\lambda$  is a subsequence of every string. There are  $2^n$  subsequences of a string with length  $n$ , e.g. the subsequences of 123456 are (totally  $2^6 = 64$  subsequences):

$\lambda$ , 1, 2, 3, 4, 5, 6, 12, 13, 14, 15, 16, 23, 24, 25, 26, 34, 35, 36, 45, 46, 56, 123, 124, 125, 126, 134, 135, 136, 145, 146, 156, 234, 235, 236, 245, 246, 256, 345, 346, 356, 456, 1234, 1235, 1236, 1245, 1246, 1256, 1345, 1346, 1356, 1456, 2345, 2346, 2356, 2456, 3456, 12345, 12346, 12356, 12456, 13456, 23456, 123456

(In this article, we only consider the subsequences with length  $\geq 2$ , and not consider the subsequences [beginning with 0](#) and/or [ending with 0](#), e.g. for the string 123456, we have these subsequences: 12, 13, 14, 15, 16, 23, 24, 25, 26, 34, 35, 36, 45, 46, 56, 123, 124, 125, 126, 134, 135, 136, 145, 146, 156, 234, 235, 236, 245, 246, 256, 345, 346, 356, 456, 1234, 1235, 1236, 1245, 1246, 1256, 1345, 1346, 1356, 1456, 2345, 2346, 2356, 2456, 3456, 12345, 12346, 12356, 12456, 13456, 23456, 123456, totally 57 subsequences, and for a string with length  $n$  with no character 0, there are  $2^n - n - 1$  subsequences)

[Subsequence](#) should not to be confused with [substring](#), a substring is a contiguous sequence of characters within a string, they are related to two hard problems: [longest common subsequence problem](#) and [longest common substring problem](#), respectively, e.g. 397 is a subsequence of 163975, “*string*” is a substring of “*substring*”. In contrast, 514 is a subsequence of 352148, but not a substring. The [empty string](#)  $\lambda$  is a substring of every string. There are  $n*(n+1)/2+1$  substrings of a string with length  $n$ , e.g. the substrings of 123456 are (totally  $6*(6+1)/2+1 = 22$  substrings):

$\lambda$ , 1, 2, 3, 4, 5, 6, 12, 23, 34, 45, 56, 123, 234, 345, 456, 1234, 2345, 3456, 12345, 23456, 123456

There are  $64 - 22 = 42$  subsequences of 123456 which are not substrings:

13, 14, 15, 16, 24, 25, 26, 35, 36, 46, 124, 125, 126, 134, 135, 136, 145, 146, 156, 235, 236, 245, 246, 256, 346, 356, 1235, 1236, 1245, 1246, 1256, 1345, 1346, 1356, 1456, 2346, 2356, 2456, 12346, 12356, 12456, 13456

(For the references of the difference of “subsequence” and “substring”, see [this post](#) and [this post](#), and see the list below)

subsequence	substring
<a href="#">A071062</a>	<a href="#">A033274</a>
<a href="#">A130448</a>	<a href="#">A238334</a>
<a href="#">A039995</a>	<a href="#">A039997</a>
<a href="#">A039994</a>	<a href="#">A039996</a>
<a href="#">A094535</a>	<a href="#">A093301</a>

(In this article, we only research [subsequence](#) and not research [substring](#), the reason is the minimal set of [subsequence ordering](#) must be [finite](#) even if the set is [infinite](#) (by the theorem that there are no [infinite antichains](#) for the [subsequence ordering](#)), and hence we may find this set, but the minimal set of [substring ordering](#) may be [infinite](#), and it is highly possible that we cannot find this set, e.g. the minimal set of subsequence ordering of the set of prime number digit strings with length  $\geq 2$  in decimal ([proofs for that this set is infinite](#)) is known to be finite and contain exactly 77 elements, and the largest element is  $50^{28}27$ , where  $0^{28}$  means the string with 28 0's, but the minimal set of substring ordering of the set of prime number digit strings with length  $\geq 2$  in decimal is very likely to be infinite, since all primes of the form  $1\{0\}3$  ( $10^n+3$ , [A159352](#)) or  $3\{0\}1$  ( $3 \cdot 10^n+1$ , [A259866](#)) are minimal elements of substring ordering of the set of prime number digit strings with length  $\geq 2$  in decimal, and there is likely infinitely many primes of the form  $1\{0\}3$  and infinitely many primes of the form  $3\{0\}1$  (see the “Proof” section of this article, also [see this reference](#)))

The [set](#) of all [strings](#) ordered by [subsequence](#) (i.e. under the [binary relation](#) “is a subsequence of”) is a [partially ordered set](#) (since this binary relation is [reflexive](#), [antisymmetric](#), and [transitive](#)), hence, any given ([finite](#) or [infinite](#)) set (e.g. the set of the “[prime numbers](#)  $> b$ ” [strings](#) in [base](#)  $b$ , for  $2 \leq b \leq 36$ ), which is the target of this article) of strings ordered by subsequence is also a partially ordered set, and thus we can draw its [Hasse diagram](#) and find its [greatest element](#), [least element](#), [maximal elements](#), and [minimal](#)



6568969, 6620329, 6901129, 7006609, 7011904, 7033104, 7096896, 7177041, 7474756, 7551504, 7557001, 7573504, 7941124, 8020224, 8054244, 8282884, 8340544, 8508889, 8538084, 8620096, 8809024, 9229444, 9535744, 9809424, 9847044, 9935104, 9998244, 13118884, 13337104, 15038884, 15578809, 18939904, 19775809, 20903184, 20912329, 20994724, 23902321, 27709696, 29833444, 31102929, 31899904, 33039504, 33085504, 33315984, 33500944, 35533521, 35545444, 37797904, 38093584, 39980329, 40755456, 45535504, 47073321, 47444544, 50098084, 50566321, 50580544, 50608996, 50808384, 51151104, 53333809, 53993104, 55011889, 55517401, 55666521, 57501889, 57775201, 58247424, 58339044, 58859584, 59089969, 60575089, 60590656, 61199329, 65658609, 66650896, 66863329, 69072721, 69338929, 70006689, 70543201, 70997476, 71351809, 72233001, 73153809, 73994404, 74407876, 74632321, 75968656, 77668969, 77686596, 77757124, 77898276, 78907689, 78960996, 78978769, 79869969, 84052224, 85507009, 86992929, 88059456, 88096996, 88585744, 88868329, 89056969, 91833889, 94303521, ...}, although this set seems to be endless, but by the theorem that there are no [infinite antichains](#) for the [subsequence ordering](#), this set must be finite, but this set is extremely difficult to found ([reference](#)), and it is also difficult to determine the number of elements in this set, and is much more difficult than that of the first set in every base  $2 \leq b \leq 36$  (to find these two sets in bases  $2 \leq b \leq 36$  (the prime or square =  $b$  (i.e. the prime or square "10") is also excluded when the base ( $b$ ) is itself prime or square), we can use some [theorems](#) in [number theory](#), e.g. a digit in base  $b$  can be the last digit of a prime number  $> b$  if and only if this digit is [coprime](#) to  $b$  (i.e. this digit is in the [reduced residue system](#) mod  $b$ , there are [eulerphi](#)( $b$ ) such digits), and a digit in base  $b$  can be the last digit of a square number  $> b$  if and only if this digit is a [quadratic residue mod  \$b\$](#) ). For example, it is not even known whether there is a square composed of digits 6, 7, 8 (except  $676 = 26^2$ ) ([reference](#) and [reference](#) and [reference](#)), also, it is not even known whether the non-simple family  $3^m 5^n 9^4 4$  contain a square or not, this situation usually not occur for primes in any base, i.e. every non-simple family which can not be ruled out as containing no primes  $>$  base usually contain a small prime  $>$  base, thus although the problem in this article (i.e. finding the minimal set of the primes  $> b$  in base  $b$ , for  $2 \leq b \leq 36$ ) is hard, it is much easier than finding the minimal set of the squares  $> 10$  in decimal (also finding the minimal set of the squares  $> b$  in base  $b$  for any base  $b > 4$ ), thus the latter set is not discussed in this article.

In this article, we want to find the [set](#) of the minimal strings of the "[prime number  \$> b\$](#) " [digit strings](#) in [bases](#)  $2 \leq b \leq 36$ , since [decimal](#) (base 10) is not special in [mathematics](#), there is no reason to only find this set in decimal (base 10), also, finding this set in decimal (base 10) is too easy to be researched in an article (only harder than bases 2, 3, 4, 6), thus it is necessary to research this set in other bases  $b$ .

Equivalently, a string  $x$  in a set of strings  $S$  is a minimal string [if and only if](#) any proper subsequence of  $x$  (subsequence of  $x$  which is unequal to  $x$ , like [proper subset](#)) is not in  $S$ .

The minimal set  $M(L)$  of a [language](#)  $L$  is interesting, this is because it allows us to compute two natural and related languages, defined as follows:

$sub(L) := \{x \in \Sigma^* : \text{there exists } y \in L \text{ such that } x \text{ is a subsequence of } y\};$

$sup(L) := \{x \in \Sigma^* : \text{there exists } y \in L \text{ such that } y \text{ is a subsequence of } x\}.$

An amazing fact is that  $sub(L)$  and  $sup(L)$  are always regular. This follows from the following classical theorem:

Theorem: For every language  $L$ , there are only finitely many minimal strings.

Indeed, we have  $sup(L) = sup(M(L))$  and  $\Sigma^* - sub(L) = sup(M(\Sigma^* - sub(L)))$ , and the superword language of a finite language is regular, since  $sup(\{w_1, \dots, w_n\}) =$

$\bigcup_{i=1}^n \Sigma^* w_{i,1} \Sigma^* \dots \Sigma^* w_{i,|w_i|} \Sigma^*$  where  $w_i = w_{i,1} \dots w_{i,|w_i|}$  with  $w_{i,j} \in \Sigma$ .

Since there are no [infinite antichains](#) for the [subsequence ordering](#) of [strings](#) whose [characters](#) are in a fixed [finite set](#) (note that there can be [infinite antichains](#) for general [ordering](#), e.g. the set of [primes](#) is an infinite antichain for the [divisibility](#) ordering ([proofs for that this set is infinite](#)), also, the set of strings  $\{abc, abbc, abbbc, abbbbc, \dots\}$  is an infinite antichain for the [substring](#) ordering of strings whose characters are in a fixed finite set  $\{a, b, c\}$ ), the set  $M(S)$  of minimal strings of any set  $S$  of strings must be [finite](#).

Although the set  $M(S)$  of minimal strings is necessarily [finite](#), determining it explicitly for a given  $S$  can be a difficult computational problem. We use some [numbertheoretic heuristics](#) to [compute](#)  $M(L_b)$ , where  $L_b$  is the language of [base- \$b\$](#)  representations of the [prime numbers](#) which are  $> b$ , for  $2 \leq b \leq 16$ . (Also, I left as a challenge to readers the task of computing  $M(L_b)$  for  $17 \leq b \leq 36$ ) (we stop at base 36 since this base is a maximum base for which it is possible to [write](#) the [numbers](#) with the [symbols](#) 0, 1, ..., 9 (the 10 [Arabic numerals](#)) and A, B, ..., Z (the 26 [Latin letters](#)) of the Latin alphabet, references: <http://www.tonymarston.net/php-mysql/converter.html> <https://www.dcode.fr/base-36-cipher> <https://docs.python.org/3/library/functions.html#int> <https://reference.wolfram.com/language/ref/BaseForm.html> <https://baseconvert.com/> <https://www.calculand.com/unit-converter/zahlen.php?og=Base+2-36&ug=1>, also see <https://primes.utm.edu/notes/words.html> for English words which are prime numbers when viewed as a number base 36)

This problem is very hard, since determining  $M(L)$  for arbitrary  $L$  is in general [unsolvable](#) and can be difficult even when  $L$  is relatively simple, the set  $M(L)$  is an [antichain](#) of  $L$  for the subsequence ordering (although may not be the “maximum antichain” (an antichain that has cardinality at least as large as every other antichain), which may not exist even for the subsequence ordering, although there cannot be an infinite antichains for the subsequence ordering), the problems in this article (i.e. determining  $M(L_b)$  for  $2 \leq b \leq 36$ ) are very hard [open problems](#) in [number theory](#) when  $b$  is large (say  $> 16$ ) and may be [NP-complete](#) or an [undecidable problem](#), or an example of [Gödel's incompleteness theorems](#) (like the [continuum hypothesis](#) and the [halting problem](#), in fact, if the halting problem can be solved, then the problem in this article can also be solved (we only need to write a [computer](#)

[program](#) for this problem, since this problem is [discrete](#)), however, the halting problem is known to be undecidable, i.e. a general [algorithm](#) to solve the halting problem for all possible program-input pairs cannot exist) (even in the weaker case that [probable primes](#) are allowed in place of [proven primes](#), i.e. not including [primality proving](#) of the probable primes in  $M(L_b)$ ), or as hard as [the unsolved problems in mathematics](#), such as the [Riemann hypothesis](#) and the [abc conjecture](#), determining  $M(L_b)$  is much harder when  $b > 24$  and/or  $eulerphi(b)$  is larger, since  $eulerphi(b)$  is the number of possible last digits of a prime number  $> b$  in base  $b$  (these digits are exactly the base  $b$  digits [coprime](#) to  $b$ , all these bases are possible and for all such digits, there are [infinitely many](#) such primes (by [Dirichlet's theorem](#)), and for digits not coprime to  $b$  (let  $d$  be the [greatest common divisor \(GCD\)](#) of the digit and  $b$ ), all such numbers are [divisible](#) by  $d$  and  $d \leq b$ , thus cannot be primes  $> b$ ). We can imagine an alien force, vastly more powerful than us, landing on Earth and demanding  $M(L_b)$  for  $b = 17$  (or 18, 19, 20, 21, 22, 23, 24, 28, 30, 36) (including [primality proving](#) of all primes in this set) or they will destroy our planet. In that case, I claim, we should marshal all our computers and all our mathematicians and attempt to find the set and to prove the primality of all numbers in this set. But suppose, instead, that they ask for  $M(L_b)$  for  $b = 25$  (or 26, 27, 29, 31, 32, 33, 34, 35). In that case, I believe, we should attempt to destroy the aliens.

## Notation

In what follows, if  $x$  is a [string](#) of [symbols](#) over the [alphabet](#)  $\Sigma_b := \{0, 1, \dots, b - 1\}$  (the set of the base- $b$  [digits](#)) we let  $[x]_b$  denote the evaluation of  $x$  in the [positional numeral system](#) with [base \(or radix\)](#)  $b$  (starting with the [most significant digit](#)), and  $[\lambda]_b := 0$  where  $\lambda$  is the [empty string](#). This is extended to languages as follows:  $[L]_b := \{ [x]_b : x \in L \}$ . We use [the convention](#) that  $A := 10, B := 11, C := 12, \dots, Z := 35$ , to conveniently represent strings of symbols in base  $b > 10$ . We let  $(x)_b$  be the [canonical representation](#) of  $x$  in base  $b$ , that is, the representation without [leading zeros](#). Finally, as usual, for a language  $L$  we let  $L^n := LLL\dots LLL$  with  $n$   $L$ s and  $L^* := \cup_{i \geq 0} L^i$ .

This is a list for  $L_b$  for bases  $2 \leq b \leq 36$ .

$b$	$L_b$ (using A – Z to represent digit values 10 to 35)
<u>2</u>	11, 101, 111, 1011, 1101, 10001, 10011, 10111, 11101, 11111, 100101, 101001, 101011, 101111, 110101, 111011, 111101, 1000011, 1000111, 1001001, 1001111, 1010011, 1011001, 1100001, 1100101, 1100111, 1101011, 1101101, 1110001, 1111111, 10000011, 10001001, 10001011, 10010101, 10010111, 10011101, 10100011, 10100111, 10101101, 10110011, 10110101, 10111111, 11000001, 11000101, 11000111, 11010011, 11011111, 11100011, 11100101, 11101001, 11101111, 11110001, 11111011, 100000001, 100000111, 100001101, 100001111, 100010101, 100011001, 100011011, 100100101, 100110011, 100110111, 100111001, 100111101, 101001011, 101010001, 101011011, 101011101, 101100001, 101100111, 101101111, 101110101, 101111011, 101111111, 110000101, 110001101, 110010001, 110011001, 110100011, 110100101, 110101111, 110110001,



	110110111, 110111011, 111000001, 111001001, 111001101, 111001111, 111010011, 111011111, 111100111, 111101011, 111110011, 111110111, 111111101, 1000001001, 1000001011, 1000011101, 1000100011, ...
<a href="#">3</a>	12, 21, 102, 111, 122, 201, 212, 1002, 1011, 1101, 1112, 1121, 1202, 1222, 2012, 2021, 2111, 2122, 2201, 2221, 10002, 10022, 10121, 10202, 10211, 10222, 11001, 11012, 11201, 11212, 12002, 12011, 12112, 12121, 12211, 20001, 20012, 20102, 20122, 20201, 21002, 21011, 21022, 21101, 21211, 22021, 22102, 22111, 22122, 22212, 22221, 100022, 100112, 100202, 100222, 101001, 101021, 101102, 101111, 101212, 102101, 102112, 102121, 102202, 110021, 110111, 110212, 110221, 111002, 111022, 111121, 111211, 112001, 112012, 112102, 112201, 112212, 120011, 120112, 120121, 120222, 121001, 121021, 121102, 121122, 121221, 122002, 122011, 122022, 122202, 200001, 200012, 200111, 200122, 200212, 201022, 201101, 202001, 202021, 202122, ...
<a href="#">4</a>	11, 13, 23, 31, 101, 103, 113, 131, 133, 211, 221, 223, 233, 311, 323, 331, 1003, 1013, 1021, 1033, 1103, 1121, 1201, 1211, 1213, 1223, 1231, 1301, 1333, 2003, 2021, 2023, 2111, 2113, 2131, 2203, 2213, 2231, 2303, 2311, 2333, 3001, 3011, 3013, 3103, 3133, 3203, 3211, 3221, 3233, 3301, 3323, 10001, 10013, 10031, 10033, 10111, 10121, 10123, 10211, 10303, 10313, 10321, 10331, 11023, 11101, 11123, 11131, 11201, 11213, 11233, 11311, 11323, 11333, 12011, 12031, 12101, 12121, 12203, 12211, 12233, 12301, 12313, 12323, 13001, 13021, 13031, 13033, 13103, 13133, 13213, 13223, 13303, 13313, 13331, 20021, 20023, 20131, 20203, 20231, ...
<a href="#">5</a>	12, 21, 23, 32, 34, 43, 104, 111, 122, 131, 133, 142, 203, 214, 221, 232, 241, 243, 304, 313, 324, 342, 401, 403, 412, 414, 423, 1002, 1011, 1022, 1024, 1044, 1101, 1112, 1123, 1132, 1143, 1204, 1211, 1231, 1233, 1242, 1244, 1321, 1343, 1402, 1404, 1413, 1424, 1431, 2001, 2012, 2023, 2034, 2041, 2102, 2111, 2113, 2133, 2212, 2221, 2223, 2232, 2311, 2322, 2342, 2344, 2403, 2414, 2432, 2443, 3004, 3013, 3024, 3042, 3101, 3114, 3134, 3141, 3211, 3213, 3224, 3233, 3244, 3312, 3321, 3323, 3332, 3404, 3422, 3431, 3444, 4003, 4014, 4041, 4043, 4131, 4142, 4212, 4223, ...
<a href="#">6</a>	11, 15, 21, 25, 31, 35, 45, 51, 101, 105, 111, 115, 125, 135, 141, 151, 155, 201, 211, 215, 225, 241, 245, 251, 255, 301, 305, 331, 335, 345, 351, 405, 411, 421, 431, 435, 445, 455, 501, 515, 521, 525, 531, 551, 1011, 1015, 1021, 1025, 1035, 1041, 1055, 1105, 1115, 1125, 1131, 1141, 1145, 1151, 1205, 1231, 1235, 1241, 1245, 1311, 1321, 1335, 1341, 1345, 1355, 1411, 1421, 1431, 1435, 1445, 1501, 1505, 1521, 1535, 1541, 1555, 2001, 2011, 2015, 2025, 2041, 2045, 2051, 2055, 2115, 2131, 2135, 2151, 2155, 2205, 2225, 2231, 2301, 2311, 2325, 2335, ...
<a href="#">7</a>	14, 16, 23, 25, 32, 41, 43, 52, 56, 61, 65, 104, 113, 115, 124, 131, 133, 142, 146, 155, 166, 203, 205, 212, 214, 221, 241, 245, 254, 256, 302, 304, 313, 322, 326, 335, 344, 346, 362, 364, 401, 403, 421, 436, 443, 445, 452, 461, 463, 506, 515, 524, 533, 535, 544, 551, 553, 566, 616, 623, 625, 632, 652, 661, 1004, 1006, 1013, 1022, 1033, 1042, 1051, 1055, 1064, 1105, 1112, 1123, 1136, 1141, 1154, 1156, 1165, 1202, 1211, 1222, 1226, 1231, 1235, 1253, 1264, 1301, 1312, 1316, 1325, 1343, 1345, 1402, 1411, 1424, 1433, 1442, ...
<a href="#">8</a>	13, 15, 21, 23, 27, 35, 37, 45, 51, 53, 57, 65, 73, 75, 103, 107, 111, 117, 123,

	131, 141, 145, 147, 153, 155, 161, 177, 203, 211, 213, 225, 227, 235, 243, 247, 255, 263, 265, 277, 301, 305, 307, 323, 337, 343, 345, 351, 357, 361, 373, 401, 407, 415, 417, 425, 431, 433, 445, 463, 467, 471, 475, 513, 521, 533, 535, 541, 547, 557, 565, 573, 577, 605, 615, 621, 631, 643, 645, 657, 661, 667, 673, 701, 711, 715, 717, 723, 737, 747, 753, 763, 767, 775, 1011, 1013, 1035, 1043, 1055, 1063, 1071, ...
<a href="#"><u>9</u></a>	12, 14, 18, 21, 25, 32, 34, 41, 45, 47, 52, 58, 65, 67, 74, 78, 81, 87, 102, 108, 117, 122, 124, 128, 131, 135, 151, 155, 162, 164, 175, 177, 184, 201, 205, 212, 218, 221, 232, 234, 238, 241, 254, 267, 272, 274, 278, 285, 287, 308, 315, 322, 328, 331, 337, 342, 344, 355, 371, 375, 377, 382, 407, 414, 425, 427, 432, 438, 447, 454, 461, 465, 472, 481, 485, 504, 515, 517, 528, 531, 537, 542, 548, 557, 562, 564, 568, 582, 601, 605, 614, 618, 625, 638, 641, 661, 667, 678, 685, 702, ...
<a href="#"><u>10</u></a>	11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199, 211, 223, 227, 229, 233, 239, 241, 251, 257, 263, 269, 271, 277, 281, 283, 293, 307, 311, 313, 317, 331, 337, 347, 349, 353, 359, 367, 373, 379, 383, 389, 397, 401, 409, 419, 421, 431, 433, 439, 443, 449, 457, 461, 463, 467, 479, 487, 491, 499, 503, 509, 521, 523, 541, 547, 557, 563, 569, ...
<a href="#"><u>11</u></a>	12, 16, 18, 21, 27, 29, 34, 38, 3A, 43, 49, 54, 56, 61, 65, 67, 72, 76, 81, 89, 92, 94, 98, 9A, A3, 106, 10A, 115, 117, 126, 128, 133, 139, 142, 148, 153, 155, 164, 166, 16A, 171, 182, 193, 197, 199, 1A2, 1A8, 1AA, 209, 214, 21A, 225, 227, 232, 236, 238, 247, 25A, 263, 265, 269, 281, 287, 296, 298, 2A1, 2A7, 304, 30A, 315, 319, 324, 331, 335, 342, 351, 353, 362, 364, 36A, 373, 379, 386, 38A, 391, 395, 3A6, 403, 407, 414, 418, 423, 434, 436, 452, 458, 467, 472, 478, 47A, ...
<a href="#"><u>12</u></a>	11, 15, 17, 1B, 25, 27, 31, 35, 37, 3B, 45, 4B, 51, 57, 5B, 61, 67, 6B, 75, 81, 85, 87, 8B, 91, 95, A7, AB, B5, B7, 105, 107, 111, 117, 11B, 125, 12B, 131, 13B, 141, 145, 147, 157, 167, 16B, 171, 175, 17B, 181, 18B, 195, 19B, 1A5, 1A7, 1B1, 1B5, 1B7, 205, 217, 21B, 221, 225, 237, 241, 24B, 251, 255, 25B, 267, 271, 277, 27B, 285, 291, 295, 2A1, 2AB, 2B1, 2BB, 301, 307, 30B, 315, 321, 325, 327, 32B, 33B, 347, 34B, 357, 35B, 365, 375, 377, 391, 397, 3A5, 3AB, 3B5, 3B7, ...
<a href="#"><u>13</u></a>	14, 16, 1A, 23, 25, 2B, 32, 34, 38, 41, 47, 49, 52, 56, 58, 61, 65, 6B, 76, 7A, 7C, 83, 85, 89, 9A, A1, A7, A9, B6, B8, C1, C7, CB, 104, 10A, 10C, 119, 11B, 122, 124, 133, 142, 146, 148, 14C, 155, 157, 164, 16A, 173, 179, 17B, 184, 188, 18A, 197, 1A8, 1AC, 1B1, 1B5, 1C6, 1CC, 209, 20B, 212, 218, 223, 229, 232, 236, 23C, 247, 24B, 256, 263, 265, 272, 274, 27A, 281, 287, 292, 296, 298, 29C, 2AB, 2B6, 2BA, 2C5, 2C9, 302, 311, 313, 328, 331, 33B, 344, 34A, 34C, 355, ...
<a href="#"><u>14</u></a>	13, 15, 19, 21, 23, 29, 2D, 31, 35, 3B, 43, 45, 4B, 51, 53, 59, 5D, 65, 6D, 73, 75, 79, 7B, 81, 91, 95, 9B, 9D, A9, AB, B3, B9, BD, C5, CB, CD, D9, DB, 101, 103, 111, 11D, 123, 125, 129, 131, 133, 13D, 145, 14B, 153, 155, 15B, 161, 163, 16D, 17D, 183, 185, 189, 199, 1A1, 1AB, 1AD, 1B3, 1B9, 1C3, 1C9, 1D1, 1D5, 1DB, 205, 209, 213, 21D, 221, 22B, 22D, 235, 239, 241, 249, 24D, 251, 255, 263, 26B, 271, 279, 27D, 285, 293, 295, 2A9, 2B1, 2BB, 2C3, 2C9, 2CB, 2D3, ...



<a href="#">15</a>	12, 14, 18, 1E, 21, 27, 2B, 2D, 32, 38, 3E, 41, 47, 4B, 4D, 54, 58, 5E, 67, 6B, 6D, 72, 74, 78, 87, 8B, 92, 94, 9E, A1, A7, AD, B2, B8, BE, C1, CB, CD, D2, D4, E1, ED, 102, 104, 108, 10E, 111, 11B, 122, 128, 12E, 131, 137, 13B, 13D, 148, 157, 15B, 15D, 162, 171, 177, 182, 184, 188, 18E, 197, 19D, 1A4, 1A8, 1AE, 1B7, 1BB, 1C4, 1CE, 1D1, 1DB, 1DD, 1E4, 1E8, 1EE, 207, 20B, 20D, 212, 21E, 227, 22B, 234, 238, 23E, 24B, 24D, 261, 267, 272, 278, 27E, 281, 287, ...
<a href="#">16</a>	11, 13, 17, 1D, 1F, 25, 29, 2B, 2F, 35, 3B, 3D, 43, 47, 49, 4F, 53, 59, 61, 65, 67, 6B, 6D, 71, 7F, 83, 89, 8B, 95, 97, 9D, A3, A7, AD, B3, B5, BF, C1, C5, C7, D3, DF, E3, E5, E9, EF, F1, FB, 101, 107, 10D, 10F, 115, 119, 11B, 125, 133, 137, 139, 13D, 14B, 151, 15B, 15D, 161, 167, 16F, 175, 17B, 17F, 185, 18D, 191, 199, 1A3, 1A5, 1AF, 1B1, 1B7, 1BB, 1C1, 1C9, 1CD, 1CF, 1D3, 1DF, 1E7, 1EB, 1F3, 1F7, 1FD, 209, 20B, 21D, 223, 22D, 233, 239, 23B, 241, ...
17	12, 16, 1C, 1E, 23, 27, 29, 2D, 32, 38, 3A, 3G, 43, 45, 4B, 4F, 54, 5C, 5G, 61, 65, 67, 6B, 78, 7C, 81, 83, 8D, 8F, 94, 9A, 9E, A3, A9, AB, B4, B6, BA, BC, C7, D2, D6, D8, DC, E1, E3, ED, F2, F8, FE, FG, G5, G9, GB, 104, 111, 115, 117, 11B, 128, 12E, 137, 139, 13D, 142, 14A, 14G, 155, 159, 15F, 166, 16A, 171, 17B, 17D, 186, 188, 18E, 191, 197, 19F, 1A2, 1A4, 1A8, 1B3, 1BB, 1BF, 1C6, 1CA, 1CG, 1DB, 1DD, 1EE, 1F3, 1FD, 1G2, 1G8, 1GA, 1GG, 209, ...
<a href="#">18</a>	11, 15, 1B, 1D, 21, 25, 27, 2B, 2H, 35, 37, 3D, 3H, 41, 47, 4B, 4H, 57, 5B, 5D, 5H, 61, 65, 71, 75, 7B, 7D, 85, 87, 8D, 91, 95, 9B, 9H, A1, AB, AD, AH, B1, BD, C7, CB, CD, CH, D5, D7, DH, E5, EB, EH, F1, F7, FB, FD, G5, H1, H5, H7, HB, 107, 10D, 115, 117, 11B, 11H, 127, 12D, 131, 135, 13B, 141, 145, 14D, 155, 157, 15H, 161, 167, 16B, 16H, 177, 17B, 17D, 17H, 18B, 191, 195, 19D, 19H, 1A5, 1AH, 1B1, 1C1, 1C7, 1CH, 1D5, 1DB, 1DD, 1E1, 1EB, ...
19	14, 1A, 1C, 1I, 23, 25, 29, 2F, 32, 34, 3A, 3E, 3G, 43, 47, 4D, 52, 56, 58, 5C, 5E, 5I, 6D, 6H, 74, 76, 7G, 7I, 85, 8B, 8F, 92, 98, 9A, A1, A3, A7, A9, B2, BE, BI, C1, C5, CB, CD, D4, DA, DG, E3, E5, EB, EF, EH, F8, G3, G7, G9, GD, H8, HE, I5, I7, IB, IH, 106, 10C, 10I, 113, 119, 11H, 122, 12A, 131, 133, 13D, 13F, 142, 146, 14C, 151, 155, 157, 15B, 164, 16C, 16G, 175, 179, 17F, 188, 18A, 199, 19F, 1A6, 1AC, 1AI, 1B1, 1B7, 1BH, 1C4, ...
<a href="#">20</a>	13, 19, 1B, 1H, 21, 23, 27, 2D, 2J, 31, 37, 3B, 3D, 3J, 43, 49, 4H, 51, 53, 57, 59, 5D, 67, 6B, 6H, 6J, 79, 7B, 7H, 83, 87, 8D, 8J, 91, 9B, 9D, 9H, 9J, AB, B3, B7, B9, BD, BJ, C1, CB, CH, D3, D9, DB, DH, E1, E3, ED, F7, FB, FD, FH, GB, GH, H7, H9, HD, HJ, I7, ID, IJ, J3, J9, JH, 101, 109, 10J, 111, 11B, 11D, 11J, 123, 129, 12H, 131, 133, 137, 13J, 147, 14B, 14J, 153, 159, 161, 163, 171, 177, 17H, 183, 189, 18B, 18H, 197, 19D, ...
21	12, 18, 1A, 1G, 1K, 21, 25, 2B, 2H, 2J, 34, 38, 3A, 3G, 3K, 45, 4D, 4H, 4J, 52, 54, 58, 61, 65, 6B, 6D, 72, 74, 7A, 7G, 7K, 85, 8B, 8D, 92, 94, 98, 9A, A1, AD, AH, AJ, B2, B8, BA, BK, C5, CB, CH, CJ, D4, D8, DA, DK, ED, EH, EJ, F2, FG, G1, GB, GD, GH, H2, HA, HG, I1, I5, IB, IJ, J2, JA, JK, K1, KB, KD, KJ, 102, 108, 10G, 10K, 111, 115, 11H, 124, 128, 12G, 12K, 135, 13H, 13J, 14G, 151, 15B, 15H, 162, 164, 16A, 16K, 175, ...
22	11, 17, 19, 1F, 1J, 1L, 23, 29, 2F, 2H, 31, 35, 37, 3D, 3H, 41, 49, 4D, 4F, 4J, 4L, 53, 5H, 5L, 65, 67, 6H, 6J, 73, 79, 7D, 7J, 83, 85, 8F, 8H, 8L, 91, 9D, A3, A7, A9, AD, AJ, AL, B9, BF, BL, C5, C7, CD, CH, CJ, D7, DL, E3, E5, E9, F1, F7, FH, FJ, G1, G7, GF, GL, H5, H9, HF, I1, I5, ID, J1, J3, JD, JF, JL, K3, K9, KH, KL, L1, L5, LH, 103, 107, 10F, 10J, 113, 11F, 11H, 12D, 12J, 137, 13D,

	13J, 13L, 145, 14F, 14L, ...
23	16, 18, 1E, 1I, 1K, 21, 27, 2D, 2F, 2L, 32, 34, 3A, 3E, 3K, 45, 49, 4B, 4F, 4H, 4L, 5C, 5G, 5M, 61, 6B, 6D, 6J, 72, 76, 7C, 7I, 7K, 87, 89, 8D, 8F, 94, 9G, 9K, 9M, A3, A9, AB, AL, B4, BA, BG, BI, C1, C5, C7, CH, D8, DC, DE, DI, E9, EF, F2, F4, F8, FE, FM, G5, GB, GF, GL, H6, HA, HI, I5, I7, IH, IJ, J2, J6, JC, JK, K1, K3, K7, KJ, L4, L8, LG, LK, M3, MF, MH, 10C, 10I, 115, 11B, 11H, 11J, 122, 12C, 12I, 131, ...
<a href="#">24</a>	15, 17, 1D, 1H, 1J, 1N, 25, 2B, 2D, 2J, 2N, 31, 37, 3B, 3H, 41, 45, 47, 4B, 4D, 4H, 57, 5B, 5H, 5J, 65, 67, 6D, 6J, 6N, 75, 7B, 7D, 7N, 81, 85, 87, 8J, 97, 9B, 9D, 9H, 9N, A1, AB, AH, AN, B5, B7, BD, BH, BJ, C5, CJ, CN, D1, D5, DJ, E1, EB, ED, EH, EN, F7, FD, FJ, FN, G5, GD, GH, H1, HB, HD, HN, I1, I7, IB, IH, J1, J5, J7, JB, JN, K7, KB, KJ, KN, L5, LH, LJ, MD, MJ, N5, NB, NH, NJ, 101, 10B, 10H, 10N, ...
25	14, 16, 1C, 1G, 1I, 1M, 23, 29, 2B, 2H, 2L, 2N, 34, 38, 3E, 3M, 41, 43, 47, 49, 4D, 52, 56, 5C, 5E, 5O, 61, 67, 6D, 6H, 6N, 74, 76, 7G, 7I, 7M, 7O, 8B, 8N, 92, 94, 98, 9E, 9G, A1, A7, AD, AJ, AL, B2, B6, B8, BI, C7, CB, CD, CH, D6, DC, DM, DO, E3, E9, EH, EN, F4, F8, FE, FM, G1, G9, GJ, GL, H6, H8, HE, HI, HO, I7, IB, ID, IH, J4, JC, JG, JO, K3, K9, KL, KN, LG, LM, M7, MD, MJ, ML, N2, NC, NI, NO, ...
<a href="#">26</a>	13, 15, 1B, 1F, 1H, 1L, 21, 27, 29, 2F, 2J, 2L, 31, 35, 3B, 3J, 3N, 3P, 43, 45, 49, 4N, 51, 57, 59, 5J, 5L, 61, 67, 6B, 6H, 6N, 6P, 79, 7B, 7F, 7H, 83, 8F, 8J, 8L, 8P, 95, 97, 9H, 9N, A3, A9, AB, AH, AL, AN, B7, BL, BP, C1, C5, CJ, CP, D9, DB, DF, DL, E3, E9, EF, EJ, EP, F7, FB, FJ, G3, G5, GF, GH, GN, H1, H7, HF, HJ, HL, HP, IB, IJ, IN, J5, J9, JF, K1, K3, KL, L1, LB, LH, LN, LP, M5, MF, ML, N1, ...
<a href="#">27</a>	12, 14, 1A, 1E, 1G, 1K, 1Q, 25, 27, 2D, 2H, 2J, 2P, 32, 38, 3G, 3K, 3M, 3Q, 41, 45, 4J, 4N, 52, 54, 5E, 5G, 5M, 61, 65, 6B, 6H, 6J, 72, 74, 78, 7A, 7M, 87, 8B, 8D, 8H, 8N, 8P, 98, 9E, 9K, 9Q, A1, A7, AB, AD, AN, BA, BE, BG, BK, C7, CD, CN, CP, D2, D8, DG, DM, E1, E5, EB, EJ, EN, F4, FE, FG, FQ, G1, G7, GB, GH, GP, H2, H4, H8, HK, I1, I5, ID, IH, IN, J8, JA, K1, K7, KH, KN, L2, L4, LA, LK, LQ, M5, ...
28	11, 13, 19, 1D, 1F, 1J, 1P, 23, 25, 2B, 2F, 2H, 2N, 2R, 35, 3D, 3H, 3J, 3N, 3P, 41, 4F, 4J, 4P, 4R, 59, 5B, 5H, 5N, 5R, 65, 6B, 6D, 6N, 6P, 71, 73, 7F, 7R, 83, 85, 89, 8F, 8H, 8R, 95, 9B, 9H, 9J, 9P, A1, A3, AD, AR, B3, B5, B9, BN, C1, CB, CD, CH, CN, D3, D9, DF, DJ, DP, E5, E9, EH, ER, F1, FB, FD, FJ, FN, G1, G9, GD, GF, GJ, H3, HB, HF, HN, HR, I5, IH, IJ, J9, JF, JP, K3, K9, KB, KH, KR, L5, LB, ...
29	12, 18, 1C, 1E, 1I, 1O, 21, 23, 29, 2D, 2F, 2L, 2P, 32, 3A, 3E, 3G, 3K, 3M, 3Q, 4B, 4F, 4L, 4N, 54, 56, 5C, 5I, 5M, 5S, 65, 67, 6H, 6J, 6N, 6P, 78, 7K, 7O, 7Q, 81, 87, 89, 8J, 8P, 92, 98, 9A, 9G, 9K, 9M, A3, AH, AL, AN, AR, BC, BI, BS, C1, C5, CB, CJ, CP, D2, D6, DC, DK, DO, E3, ED, EF, EP, ER, F4, F8, FE, FM, FQ, FS, G3, GF, GN, GR, H6, HA, HG, HS, I1, IJ, IP, J6, JC, JI, JK, JQ, K7, KD, KJ, KL, ...
30	11, 17, 1B, 1D, 1H, 1N, 1T, 21, 27, 2B, 2D, 2J, 2N, 2T, 37, 3B, 3D, 3H, 3J, 3N, 47, 4B, 4H, 4J, 4T, 51, 57, 5D, 5H, 5N, 5T, 61, 6B, 6D, 6H, 6J, 71, 7D, 7H, 7J, 7N, 7T, 81, 8B, 8H, 8N, 8T, 91, 97, 9B, 9D, 9N, A7, AB, AD, AH, B1, B7, BH, BJ, BN, BT, C7, CD, CJ, CN, CT, D7, DB, DJ, DT, E1, EB, ED, EJ, EN, ET, F7,

	FB, FD, FH, FT, G7, GB, GJ, GN, GT, HB, HD, I1, I7, IH, IN, IT, J1, J7, JH, JN, JT, K1, ...
31	16, 1A, 1C, 1G, 1M, 1S, 1U, 25, 29, 2B, 2H, 2L, 2R, 34, 38, 3A, 3E, 3G, 3K, 43, 47, 4D, 4F, 4P, 4R, 52, 58, 5C, 5I, 5O, 5Q, 65, 67, 6B, 6D, 6P, 76, 7A, 7C, 7G, 7M, 7O, 83, 89, 8F, 8L, 8N, 8T, 92, 94, 9E, 9S, A1, A3, A7, AL, AR, B6, B8, BC, BI, BQ, C1, C7, CB, CH, CP, CT, D6, DG, DI, DS, DU, E5, E9, EF, EN, ER, ET, F2, FE, FM, FQ, G3, G7, GD, GP, GR, HE, HK, HU, I5, IB, ID, IJ, IT, J4, JA, JC, JI, ...
<a href="#">32</a>	15, 19, 1B, 1F, 1L, 1R, 1T, 23, 27, 29, 2F, 2J, 2P, 31, 35, 37, 3B, 3D, 3H, 3V, 43, 49, 4B, 4L, 4N, 4T, 53, 57, 5D, 5J, 5L, 5V, 61, 65, 67, 6J, 6V, 73, 75, 79, 7F, 7H, 7R, 81, 87, 8D, 8F, 8L, 8P, 8R, 95, 9J, 9N, 9P, 9T, AB, AH, AR, AT, B1, B7, BF, BL, BR, BV, C5, CD, CH, CP, D3, D5, DF, DH, DN, DR, E1, E9, ED, EF, EJ, EV, F7, FB, FJ, FN, FT, G9, GB, GT, H3, HD, HJ, HP, HR, I1, IB, IH, IN, IP, IV, ...
33	14, 18, 1A, 1E, 1K, 1Q, 1S, 21, 25, 27, 2D, 2H, 2N, 2V, 32, 34, 38, 3A, 3E, 3S, 3W, 45, 47, 4H, 4J, 4P, 4V, 52, 58, 5E, 5G, 5Q, 5S, 5W, 61, 6D, 6P, 6T, 6V, 72, 78, 7A, 7K, 7Q, 7W, 85, 87, 8D, 8H, 8J, 8T, 9A, 9E, 9G, 9K, A1, A7, AH, AJ, AN, AT, B4, BA, BG, BK, BQ, C1, C5, CD, CN, CP, D2, D4, DA, DE, DK, DS, DW, E1, E5, EH, EP, ET, F4, F8, FE, FQ, FS, GD, GJ, GT, H2, H8, HA, HG, HQ, HW, I5, I7, ID, ...
34	13, 17, 19, 1D, 1J, 1P, 1R, 1X, 23, 25, 2B, 2F, 2L, 2T, 2X, 31, 35, 37, 3B, 3P, 3T, 41, 43, 4D, 4F, 4L, 4R, 4V, 53, 59, 5B, 5L, 5N, 5R, 5T, 67, 6J, 6N, 6P, 6T, 71, 73, 7D, 7J, 7P, 7V, 7X, 85, 89, 8B, 8L, 91, 95, 97, 9B, 9P, 9V, A7, A9, AD, AJ, AR, AX, B5, B9, BF, BN, BR, C1, CB, CD, CN, CP, CV, D1, D7, DF, DJ, DL, DP, E3, EB, EF, EN, ER, EX, FB, FD, FV, G3, GD, GJ, GP, GR, GX, H9, HF, HL, HN, HT, ...
35	12, 16, 18, 1C, 1I, 1O, 1Q, 1W, 21, 23, 29, 2D, 2J, 2R, 2V, 2X, 32, 34, 38, 3M, 3Q, 3W, 3Y, 49, 4B, 4H, 4N, 4R, 4X, 54, 56, 5G, 5I, 5M, 5O, 61, 6D, 6H, 6J, 6N, 6T, 6V, 76, 7C, 7I, 7O, 7Q, 7W, 81, 83, 8D, 8R, 8V, 8X, 92, 9G, 9M, 9W, 9Y, A3, A9, AH, AN, AT, AX, B4, BC, BG, BO, BY, C1, CB, CD, CJ, CN, CT, D2, D6, D8, DC, DO, DW, E1, E9, ED, EJ, EV, EX, FG, FM, FW, G3, G9, GB, GH, GR, GX, H4, H6, HC, ...
<a href="#">36</a>	11, 15, 17, 1B, 1H, 1N, 1P, 1V, 1Z, 21, 27, 2B, 2H, 2P, 2T, 2V, 2Z, 31, 35, 3J, 3N, 3T, 3V, 45, 47, 4D, 4J, 4N, 4T, 4Z, 51, 5B, 5D, 5H, 5J, 5V, 67, 6B, 6D, 6H, 6N, 6P, 6Z, 75, 7B, 7H, 7J, 7P, 7T, 7V, 85, 8J, 8N, 8P, 8T, 97, 9D, 9N, 9P, 9T, 9Z, A7, AD, AJ, AN, AT, B1, B5, BD, BN, BP, BZ, C1, C7, CB, CH, CP, CT, CV, CZ, DB, DJ, DN, DV, DZ, E5, EH, EJ, F1, F7, FH, FN, FT, FV, G1, GB, GH, GN, GP, GV, ...

The primes in  $M(L_b)$  are called **minimal prime base  $b$**  in this article, although in fact this name should be used for  $L_b$  is the language of base- $b$  representations of the prime numbers, where primes  $> b$  is not required ([reference](#)), this problem is an extension of the [original minimal prime problem](#) to include [Conjectures 'R Us Sierpinski/Riesel](#) conjectures base  $b$  with  $k$ -values  $< b$ , i.e. the smallest prime of the form  $k^*b^n+1$  and  $k^*b^n-1$  for all  $k < b$ . The original minimal prime base  $b$  puzzle does not cover CRUS Sierpinski/Riesel conjectures base  $b$  with [CK](#)  $< b$ , since in Riesel side, the prime is not minimal prime in original definition



For example, 857 is a minimal prime in decimal because there is no prime  $> 10$  among the shorter subsequences of the digits: 8, 5, 7, 85, 87, 57. The subsequence does not have to consist of consecutive digits, so 149 is not a minimal prime in decimal (because 19 is prime and  $19 > 10$ ). But it does have to be in the same order; so, for example, 991 is still a minimal prime in decimal even though a subset of the digits can form the shorter prime  $19 > 10$  by changing the order.

A summary of the results of our [algorithm](#) is presented in the table in the next section, I completely solved all bases up to 16 except for bases 14, 16, and the odd bases  $> 6$  (the [proofs](#) are at the end of this article), for bases 14, 16, and the odd bases  $> 6$ , I only found all minimal primes up to certain limit (about  $2^{32}$ ) and some larger minimal primes (such as  $3^{161}$  in base 7 and  $54^{11}$  in base 9). I left as a challenge to readers the task of solving (finding all minimal primes and proving that these are all such primes) bases 7, 9, 11, 13, 14, 15, 16, and bases 17 through 36 (this will be a hard problem, e.g. base 23 has a minimal prime  $9E^{800873}$ , and base 30 has a minimal prime  $OT^{34205}$ ).

[Prime numbers](#) are central in [number theory](#) because of the [fundamental theorem of arithmetic](#): every natural number greater than 1 is either a prime itself or can be [factorized](#) as a [product](#) of primes that is unique [up to](#) their order. Besides, “the [sets](#) in this article” to “the prime numbers (except  $b$  itself) [digit strings](#) with length  $> 1$  in [base  \$b\$](#) ” to “the [partially ordered binary relation](#) by [subsequence](#)” is “the [sets](#) of prime numbers” to “the integers  $> 1$ ” to “the [partially ordered binary relation](#) by [divisibility](#)” (and indeed, the “ $> 1$ ” in “the prime numbers (except  $b$  itself) [digit strings](#) with length  $> 1$  in [base  \$b\$](#) ” can be corresponded to the “ $> 1$ ” in “the integers  $> 1$ ”) (for the reason why  $b$  itself is excluded, see the sections above and [this forum post](#)), thus the problem in this article is very important and beautiful.

[Recreations](#) involving the [decimal digits](#) of [primes](#) have a long history. To give just a few examples, without trying to be exhaustive, Yates studied the “[repunits](#)”, which are primes of the form 111...111. Caldwell and Dubner studied the “[near-repdigits](#)”, which are primes with all like or repeated digits but one (e.g. 7877 and 333337). Card introduced prime numbers such as 37337999, in which every [nonempty prefix](#) is also a prime; he called them “snowball” primes. These were later studied by Angell & Godwin and Caldwell, who called them “[right-truncatable](#)” primes. They also studied the “[left-truncatable](#)” primes, such as 4632647, in which every [nonempty suffix](#) is prime (the left-truncatable primes are called “Russian doll primes” like that the right-truncatable primes are called “snowball primes”, see [this page](#)). Kahan and Weintraub gave a list of all the left-truncatable primes (The list of all left-truncatable primes and right-truncatable primes are in <http://primerecords.dk/left-truncatable.txt> and <http://primerecords.dk/right-truncatable.txt>, respectively, also see OEIS sequences [A024785](#) and [A024770](#)). Huestis introduced the “recursively laminar primes”. In this note, I discuss an apparently new problem on the decimal digits of primes, but one inspired from a classical theorem in [formal language theory](#), i.e. there are only [finitely many minimal elements](#) for the [subsequence ordering](#) of any given set of [strings](#) (in fact, every set of pairwise [incomparable](#) strings (for the [subsequence ordering](#)) is finite).

However, there is no reason to only study these classes of primes in decimal, since the number 10 is not special in [mathematics](#), decimal ([base 10](#)) is not special in [mathematics](#), thus, we had better study about the [base  \$b\$  digits](#) of [primes](#) for other bases  $b$ . For the repunit

primes, there are [a list](#) of repunit primes or [PRPs](#) in all bases  $2 \leq b \leq 160$  and length  $\leq 32803$ , and [a list](#) of repunit primes or [PRPs](#) in all bases  $2 \leq b \leq 999$  and length  $\leq 3571$ , also see OEIS sequences [A084740](#) and [A084738](#) for the smallest repunit (probable) primes in base  $b$ ; for the near-repdigit primes, there was no list of the smallest such primes (only [a list](#) of [factorization](#) of such numbers in decimal (base 10)), but recently I built [a list](#) of the smallest primes or [PRPs](#) in given near-repdigit form  $x\{y\}$  (i.e.  $xyyy\dots yyy$ ) or  $\{x\}y$  (i.e.  $xxx\dots xxy$ ) (where  $x$  and  $y$  are digits in base  $b$ ) in bases  $2 \leq b \leq 36$  (I stop at base 36 since this base is a maximum base for which it is possible to [write](#) the [numbers](#) with the [symbols](#) 0, 1, ..., 9 (the 10 [Arabic numerals](#)) and A, B, ..., Z (the 26 [Latin letters](#)) of the Latin alphabet, references: <http://www.tonymarston.net/php-mysql/converter.html> <https://www.dcode.fr/base-36-cipher> <https://docs.python.org/3/library/functions.html#int> <https://reference.wolfram.com/language/ref/BaseForm.html> <https://baseconvert.com/> <https://www.calculand.com/unit-converter/zahlen.php?og=Base+2-36&ug=1>); for the left-truncatable primes, there is a [graph](#) of the actual values and estimation formulas for bases  $3 \leq b \leq 120$  (no such prime exists for  $b = 2$ ), also see OEIS sequences [A103443](#) and [A103463](#) and [A076623](#) for the largest left-truncatable primes in base  $b$  and the total number of left-truncatable primes in base  $b$ ; for the right-truncatable primes, there is [data](#) for bases  $3 \leq b \leq 90$  (no such prime exists for  $b = 2$ ), also see OEIS sequences [A023107](#) and [A103483](#) and [A076586](#) for the largest right-truncatable primes in base  $b$  and the total number of right-truncatable primes in base  $b$ . Thus, this new problem on the digits of primes (i.e. the problem on the digits of primes inspired from a classical theorem in [formal language theory](#)) should also be generalized to other bases, and this problem in various bases is exactly the target of this article (in this article we aim to solve this problem in bases  $2 \leq b \leq 36$  (I stop at base 36 since this base is a maximum base for which it is possible to [write](#) the [numbers](#) with the [symbols](#) 0, 1, ..., 9 (the 10 [Arabic numerals](#)) and A, B, ..., Z (the 26 [Latin letters](#)) of the Latin alphabet, references: <http://www.tonymarston.net/php-mysql/converter.html> <https://www.dcode.fr/base-36-cipher> <https://docs.python.org/3/library/functions.html#int> <https://reference.wolfram.com/language/ref/BaseForm.html> <https://baseconvert.com/> <https://www.calculand.com/unit-converter/zahlen.php?og=Base+2-36&ug=1>), but since this problem (finding all minimal primes) is much harder than finding all left-truncatable primes or all right-truncatable primes for the same base, in this article we only solve this problem in bases  $2 \leq b \leq 16$ , and I left as a challenge to readers the task of solving this problem in bases  $17 \leq b \leq 36$ , of course, you can also try to solve this problem in bases  $2 \leq b \leq 120$  as the same problem for the left-truncatable primes, but this will be extremely difficult).

There is a [conjecture](#) that there are [infinitely many](#) repunit primes in all bases  $b$  which are not [perfect powers](#) (if  $b$  is a perfect power, then it can be shown that there is at most one



repunit prime in base  $b$ , since the numbers with all digits 1 in these bases can be factored algebraically (this strategy (algebraic factorization) will also be used in our problem, to show that some families (such as  $10^n - 1$  in base 8 and  $38^n - 1$  in base 9) contains no primes  $>$  base), and it is also conjectured that there are also [infinitely many](#) primes in any given near-repdigit form  $x\{y\}$  (i.e.  $xyyy\dots yyy$ ) or  $\{x\}y$  (i.e.  $xxx\dots xxy$ ) (where  $x$  and  $y$  are digits in base  $b$ ) if this form cannot be proven as only contain composites or only contain finitely many primes, also, it is conjectured that there are finitely many left-truncatable primes and finitely many right-truncatable primes in any given base  $b$ , however, unlike minimal primes (which can be proven to be finite in any given base  $b$  by using the theorem that there are no [infinite antichains](#) for the [subsequence ordering](#)), none of these conjectures are proven.

These classes of primes are related to the class of primes in this article (i.e. minimal primes) and hence related to the problem in this article (i.e. finding  $M(L_b)$  for bases  $2 \leq b \leq 36$ ), since the smallest [repunit prime](#) (if exists) is always a minimal prime to the same base  $b$ , and the smallest [near-repdigit prime](#) with a given form  $x\{y\}$  (i.e.  $xyyy\dots yyy$ ) or  $\{x\}y$  (i.e.  $xxx\dots xxy$ ) (where  $x$  and  $y$  are digits in base  $b$ ) (if exists) is also always a minimal prime to the same base  $b$ , also, since all [suffixes](#) and all [prefixes](#) are also [substrings](#), hence also [subsequences](#), a [left-truncatable prime](#) or [right-truncatable prime](#) with length  $\geq 3$  cannot be a minimal prime to the same base  $b$ , and left-truncatable primes or right-truncatable primes can be regarded the opposite of minimal primes ([reference](#)).

Problems about the digits of prime numbers have a long history, and many of them are still [unsolved](#). For example, are there infinitely many primes, all of whose base-10 digits are 1? Currently, there are only five such "[repunits](#)" known, corresponding to  $(10^p - 1)/9$  for  $p \in \{2, 19, 23, 317, 1031\}$ . It seems likely that four more are given by  $p \in \{49081, 86453, 109297, 270343, 5794777, 8177207\}$ , but this has not yet been [rigorously proven](#). This problem also exists for other bases, e.g. for base 12, there are only nine proven such numbers, corresponding to  $(12^p - 1)/11$  for  $p \in \{2, 3, 5, 19, 97, 109, 317, 353, 701\}$ . It seems likely that five more are given by  $p \in \{9739, 14951, 37573, 46889, 769543\}$ , but this has not yet been [rigorously proven](#). However, for some bases there exists no such primes, these bases are 9, 25, 32, 49, 64, 81, 121, 125, 144, ..., (<https://oeis.org/A096059>) this is because the numbers with all digits 1 in these bases can be factored algebraically (this strategy (algebraic factorization) will also be used in our problem, to show that some families (such as  $10^n - 1$  in base 8 and  $38^n - 1$  in base 9) contains no primes  $>$  base). Some positive integers  $n$  are repunit in some base  $2 \leq b \leq n - 2$  (every integer  $n \geq 3$  are trivially repunit in base  $b = n - 1$  since  $n$  is written "11" in base  $b = n - 1$ , but every integer  $n \geq 2$  are not repunit in any base  $b \geq n$  since  $n$  is written "10" in base  $b = n$  and  $n$  is single-digit number (and this digit is not 1) in any base  $b > n$ ), they are called [Brazilian numbers](#), all integers  $>6$  which are neither primes nor squares of primes are Brazilian numbers, but it is unknown whether there are infinitely many primes which are also Brazilian numbers (however, it is known that every squares of primes except  $121 = "11111"$  in base 3 are not Brazilian numbers). Another unsolved problem about the digits of prime numbers is whether

there are infinitely many [palindromic primes](#) (primes which remain the same when their digits are reversed, such as 151 and 94849) in base 10? So far, the largest known such prime is [10<sup>1234567</sup> - 20342924302 \\* 10<sup>617278</sup> - 1](#), this number has 1234567 digits, can also be written as [9<sup>617278</sup>796570756979<sup>617278</sup>](#), and the largest 20 known such primes are listed in [this page](#). Of course, this problem also exists for other bases, there is no single bases for which it is known whether there are infinitely many [palindromic primes](#). Some positive integers  $n$  are not palindromic in any base  $2 \leq b \leq n-2$  (every integer  $n \geq 3$  are trivially palindromic in base  $b = n-1$  since  $n$  is written "11" in base  $b = n-1$ , also every positive integer  $n$  are trivially palindromic in any base  $b > n$  since  $n$  is single-digit number in any base  $b > n$ , but every integer  $n \geq 2$  are not palindromic in base  $b = n$  since  $n$  is written "10" in base  $b = n$ ), they are called [strictly non-palindromic numbers](#), all such integers  $> 6$  are primes, since all composites  $n > 6$  is either "product of two numbers  $k$  and  $m$  with  $m-k \geq 2$ " (in this case,  $n$  is written " $kk$ " in base  $b = m-1$ ) or "square of prime  $p$ " (in this case,  $n$  is written "121" in base  $b = p-1$  if  $p > 3$ , or written "1001" in base  $b = 2$  if  $p = 3$ ), it is also unknown whether there are infinitely many such integers, but it is [known](#) that in every base, [almost all](#) palindromic numbers are [composite](#) (neither 1 nor prime).

## Table

$|x|$  is the length of  $x$ , and in the " $\max(x, x \in L_b)$ " column,  $xy^n z$  means  $xyyy\dots yyyz$  with  $n$   $y$ 's (the  $n$ -value is written in decimal), not  $y$  to the  $n$ th power.

$b$	$ M(L_b) $	$\max(x, x \in M(L_b))$	$\max( x , x \in M(L_b))$	Algebraic form of $\max(x, x \in M(L_b))$
2	1	11	2	3
3	3	111	3	13
4	5	221	3	41
5	22	10 <sup>93</sup> 13	96	5 <sup>95</sup> +8
6	11	40041	5	5209
7	71	3 <sup>16</sup> 1	17	$\frac{7^{17} - 5}{2}$
8	75	4 <sup>2207</sup>	221	$\frac{4 \cdot 8^{221} + 17}{7}$
9 <sup>①</sup>	$\geq 149$	30 <sup>1158</sup> 11	1161	3*9 <sup>1160</sup> +10
10	77	50 <sup>28</sup> 27	31	5*10 <sup>30</sup> +27

11 <sup>①</sup>	≥914	557 <sup>1011</sup> or 57 <sup>n</sup> with n>50000	1013	$\frac{607 \cdot 11^{1011} - 7}{10}$
12	106	40 <sup>3977</sup>	42	4*12 <sup>41</sup> +91
13 <sup>①②</sup>	≥2497	80 <sup>32017</sup> 111 or 95 <sup>n</sup> with n>50000 or A3 <sup>n</sup> A with n>50000	32021	8*13 <sup>32020</sup> +183
14 <sup>①</sup>	≥606	4D <sup>19698</sup>	19699	5*14 <sup>19698</sup> - 1
15 <sup>①</sup>	≥1212	7 <sup>15597</sup>	157	$\frac{15^{157} + 59}{2}$
16 <sup>①②</sup>	≥2045	DB <sup>32234</sup>	32235	$\frac{206 \cdot 16^{32234} - 1}{15}$

Notes:

① I have not proved these bases, these are the largest elements in  $M(L_b)$  known to me, and they are just the [lower bounds](#).

② Data based on results of strong [probable primality tests](#), i.e. at least one element in the set  $M(L_b)$  is only [strong probable prime](#) (i.e. numbers which passed the [Miller–Rabin primality tests](#) to first few prime bases, for the smallest *composite* number which passed the Miller–Rabin primality test to first  $n$  prime bases, see <https://oeis.org/A014233>) and not [provable prime](#), thus we cannot definitely say that the corresponding families can be removed from the list of unsolved families, and we cannot definitely [compute](#) this part of the [set](#)  $M(L_b)$ , e.g. since 80<sup>32017</sup>111 (base 13) is only strong probable prime and it is the smallest (probable) prime in family 8{0}111 in base 13, we cannot definitely say that the family 8{0}111 (base 13) can be removed from the list of unsolved families, and since DB<sup>32234</sup> (base 16) is only strong probable prime and it is the smallest (probable) prime in family D{B} in base 16, we cannot definitely say that the family D{B} (base 16) can be removed from the list of unsolved families.

It is found that both  $|M(L_b)|$  and  $\max(|x|, x \in M(L_b))$  are [roughly](#)  $e^{(b-1) \cdot \text{eulerphi}(b)}$ , the value  $(b-1) \cdot \text{eulerphi}(b)$  is the number of possible (first digit,last digit) combos ([ordered pair](#)) of a minimal prime in base  $b$  (these (first digit,last digit) combos are also all possible (first digit,last digit) combos ([ordered pair](#)) of a prime  $> b$  in base  $b$ ) (they are only all “possible” (first digit,last digit) combos ([ordered pair](#)) of a minimal prime in base  $b$ , this does not mean that they must be realized, e.g. there are no minimal primes with (first digit,last digit) = (2,2) in base 3), since the first digit has  $b-1$  choices (all digits except 0 can be the first digit), and the last digit has [eulerphi](#)( $b$ ) choices (only digits [coprime](#) to  $b$  (i.e. the digits in the [reduced residue system](#) mod  $b$ ) can be the last digit), by the [rule of product](#), there are  $(b-1) \cdot \text{eulerphi}(b)$  choices of the (first digit,last digit) combo. Thus,  $(b-1) \cdot \text{eulerphi}(b)$  is also the relative hardness for (finding and proving the set  $M(L_b)$  in) base  $b$ , there is exactly a sequence of  $(b-1) \cdot \text{eulerphi}(b)$  in OEIS: [A062955](#), for these  $(b-1) \cdot \text{eulerphi}(b)$  possible (first digit,last digit) combos, we want to find all minimal primes with such (first digit,last digit)

combo, if the string “first digit, last digit” represents a prime in base  $b$ , then this prime will be the only minimal prime with this (first digit,last digit) combo (since the string “first digit, last digit” is a [subsequence](#) of all numbers with this (first digit,last digit) combo), otherwise, we should find all digits which can be inserted to this (first digit,last digit) combo, i.e. the string “first digit, such digit, last digit” is neither prime nor have a [subsequence](#) which represents a prime, then do this repeatedly (find the possible (first digit,last digit) combos for the string which inserted to the starting (first digit,last digit) combo, etc.), then do [program loops](#), these program loops must be finite by the theorem that there are no [infinite antichains](#) for the [subsequence ordering](#), see the “proof” section and [this forum post](#) and [this article](#).

The [probability](#) for a [random](#) prime to have a given (first digit,last digit) combo ([ordered pair](#)) which is a possible (first digit,last digit) combo ([ordered pair](#)) of a prime  $> b$  in base  $b$  (i.e. “first digit” is not 0, and “last digit” is [coprime](#) to  $b$ ) are all the same, i.e. they are all  $1/((b-1)^* \text{eulerphi}(b))$  no matter which (first digit,last digit) combo ([ordered pair](#)) is given, the only condition is that “first digit” is not 0, and “last digit” is [coprime](#) to  $b$ , for the first digit, there is a [reference](#) about this, the primes do not follow the [Benford's law](#) ([reference of Benford's law to other bases](#)) (only the prime factors of the numbers with [exponential growth](#) (such as the [repunits](#) and the [Fibonacci numbers](#)) follow), instead, all nonzero digits have the same probability (i.e. probability  $1/(b-1)$ ) for a random prime in base  $b$ , just like a positive integer in base  $b$ , for the last digit, by the [prime number theorem](#) (extended to [arithmetic progression](#)), all digits coprime to  $b$  have the same probability (i.e. probability  $1/\text{eulerphi}(b)$ ) for a random prime in base  $b$ , however, according to [Chebyshev's bias](#), if  $d_1$  is a [quadratic residue mod](#)  $b$ ,  $d_2$  is a quadratic nonresidue mod  $b$  (i.e.  $d_1$  can be the last digit of a [square number](#), while  $d_2$  cannot be), then for the primes  $\leq N$  for a random positive integer  $N$ , the probability for the number of primes end with  $d_2$  in base  $b$  is more than the number of primes end with  $d_1$  in base  $b$  is larger than [50%](#), e.g. the smallest  $N$  such that the number of primes end with 1 in base 4 is more than the number of primes end with 3 in base 4 is 12203231 (26861 in decimal), and the smallest  $N$  such that the number of primes end with 1 in base 3 is more than the number of primes end with 2 in base 3 is 2011012212222201102200001 (608981813029 in decimal), references: <https://oeis.org/A007350> <https://oeis.org/A007352> <https://oeis.org/A199547> <https://oeis.org/A306891> <https://oeis.org/A038698> <https://oeis.org/A112632> <https://oeis.org/A275939> <https://oeis.org/A306499> <https://oeis.org/A306500>, this is a classic example of [the strong law of small numbers](#), another classic example is it appears that the sum of the [Liouville function](#) (which is an important function in [number theory](#), defined as  $(-1)^{\text{bigomega}(n)}$ , which is [A008836\(n\)](#)) of the positive integers  $\leq N$  is  $\leq 0$  if  $N > 1$ , is it always true? (the [Pólya conjecture](#)), the smallest  $N$  such that this conjecture is false is 906150257 (this conjecture is important in [number theory](#) since if this conjecture is true, then the [Riemann hypothesis](#) can be proved, and hence many conjectures in number theory can also be proved), for more examples, see <https://primes.utm.edu/glossary/xpage/LawOfSmall.html> and <https://oeis.org/A005165/a005165.pdf>.

Excluding the primes  $\leq b$  (i.e. only counting the primes  $> b$ ) makes the [formula](#) of the number of possible (first digit,last digit) combo of a minimal prime in base  $b$  more simple and



Our results assume that a number which has passed [Miller–Rabin primality tests](#) to the first 13 prime bases (for the composite numbers which pass this test to the first  $n$  prime bases (i.e. numbers which are [strong pseudoprimes](#) to the first  $n$  prime bases), see <https://oeis.org/A014233>, we use  $n = 13$  for the primality tests, i.e. test the prime bases  $p \leq 41$ ) and the [strong Lucas primality test](#) with parameters  $(P, Q)$  defined by Selfridge's Method A (for the composite numbers which pass this test (i.e. numbers which are [strong Lucas pseudoprimes](#) with parameters  $(P, Q)$  defined by Selfridge's Method A), see <https://oeis.org/A217255>) is in fact prime, since in some cases (e.g.  $b = 13$  and  $b = 16$ ) some candidate elements of  $M(L_b)$  are too long to be [proven prime](#) rigorously (and neither  $N-1$  nor  $N+1$  can be  $\geq 33.3333\%$  [factored](#)), and the [probability](#) that such a number is in fact composite is very low, e.g. for such a number with 5000 decimal digits, the probability is less than  $7.6 \cdot 10^{-680}$ , and for such a number with 100000 decimal digits, the probability is less than  $1.3 \cdot 10^{-10584}$ , both of them are “almost” zero, i.e. we can “almost surely” (99.9999...% (with more than 10000 9's) surely, but not 100% surely) that they are primes, and the numbers which currently cannot be [proven prime](#) rigorously are usually very large (usually  $> 10^{5000}$ , see [top 20 ECPP proving page](#) and [top 20 Primo proving page](#), the largest prime which is proven by ECPP is [p\(1289844341\)](#), where  $p(n)$  is the [integer partition function](#), this number has 40000 decimal digits, and this number is the largest known [ordinary prime](#), i.e. none of  $p^n \pm 1$  (for small  $n$ ) [factor](#) enough to make the number easily provable using the [classical methods of primality proof](#)), and if such a number is larger, then probability that this number is in fact composite is lower, thus the probability is much less than  $7.6 \cdot 10^{-680}$ , see [this page](#), also, our tests (combine of the [Miller–Rabin primality tests](#) to the first 13 prime bases and the [strong Lucas primality test](#) with parameters  $(P, Q)$  defined by Selfridge's Method A) cover the [Baillie–PSW primality test](#) (which is only combine of the [Miller–Rabin primality tests](#) to base 2 and the [strong Lucas primality test](#) with parameters  $(P, Q)$  defined by Selfridge's Method A, i.e. (let  $D$  be the first number in the sequence 5,  $-7$ , 9,  $-11$ , 13,  $-15$  ... such that  $\left(\frac{D}{N}\right) = -1$  ( $N$  is the number which we want to test primality), where  $\left(\frac{m}{n}\right)$  is the [Jacobi symbol](#)), set  $P = 1$  and  $Q = (1 - D)/4$ ), and no known composites which pass the Baillie–PSW test, and no composites  $< 2^{64}$  pass the Baillie–PSW test ([reference](#) and [reference](#)), although it is still conjectured that there exist infinitely many “Baillie–PSW [pseudoprimes](#)”, i.e. composites which pass the Baillie–PSW test, thus if a such number is in fact composite, it will be a pseudoprime to the Baillie–PSW test, which currently no single example is known!

There are five unsolved families (families for which not even a probable prime is known nor can be ruled out as only contain composites (only count the numbers  $> b$ )) for bases  $2 \leq b \leq 16$  found by me and searched to length 50000 with no (probable) prime found.

$b$	Unsolved family	Algebraic form
11	$57^n$	$\frac{57 \cdot 11^n - 7}{10}$









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base 15 (not proved, only checked to the prime 55555557)

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practice,  $L'$  is usually chosen to be a finite [union](#) of sets of the form  $L_1L_2^*L_3$ , where each of  $L_1, L_2, L_3$  is finite. In the case we consider in this paper, we then have to determine whether such a language contains a prime or not.

However, it is not even known if the following simpler [decision problem](#) is recursively solvable:

Problem: Given strings  $x, y, z$ , and a base  $b$ , does there exist a prime number whose base- $b$  expansion is of the form  $xy^n z$  for some  $n \geq 0$ ?

An algorithm to solve this problem, for example, would allow us to decide if there are any additional [Fermat primes](#) (of the form  $2^{2^n} + 1$ ) other than the known ones (corresponding to  $n = 0, 1, 2, 3, 4$ ). To see this, take  $b = 2, x = 1, y = 0$ , and  $z = 0^{16}1$ . Since if  $2^n + 1$  is prime then  $n$  must be a power of two, a prime of the form  $(xy^*z)_b$  must be a new Fermat prime. Besides, it would allow us to decide if there are infinitely many [Mersenne primes](#) (of the form  $2^p - 1$  with prime  $p$ ). To see this, take  $b = 2, x = \lambda$  (the empty string),  $y = 1$ , and  $z = 1^{n+1}$ , where  $n$  is the exponent of the Mersenne prime which we want to know whether it is the largest Mersenne prime or not. Since if  $2^n - 1$  is prime then  $n$  must be a prime, a prime of the form  $(xy^*z)_b$  must be a new Mersenne prime. Also, it would allow us to decide if 21181 is a [Sierpinski number](#) (take  $b = 2, x = 101001010111101, y = 0$ , and  $z = 1$ ) and if 23669 is a [Riesel number](#) (take  $b = 2, x = 101110001110100, y = 1$ , and  $z = \lambda$  (the empty string)).

Therefore, in practice, we are forced to try to rule out prime representations based on [heuristics](#) such as [modular techniques](#) and [factorizations](#).

It will be necessary for our algorithm to determine if families of the form  $(xy^*z)_b$  contain a prime  $> b$  or not. We use two different heuristic strategies to show that such families contain no primes  $> b$ .

In the first strategy, we mimic the well-known technique of "[covering congruences](#)", by finding some [finite set](#)  $S$  of [primes](#)  $p$  such that every number in a given family is [divisible by](#) some [element](#) of  $S$  (this is equivalent to finding an integer  $N$  such that all numbers in a given family are not [coprime](#) to  $N$ ). In the second strategy, we attempt to find an [algebraic factorization](#), such as [difference-of-two-squares factorization](#), [sum/difference-of-two-cubes factorization](#), and [Aurifeuillian factorization](#) for  $x^4 + 4y^4$ .

Examples of the first strategy: (we can show that the corresponding numbers are  $>$  all elements in  $S$ , if  $n$  makes corresponding numbers  $> b$  (i.e.  $n \geq 1$  for  $51^n$  in base 9 and  $25^n$  in base 11 and  $4^n D$  in base 16 and  $8^n F$  in base 16,  $n \geq 0$  for other examples), thus these factorizations are nontrivial)

- \* In base 10, all numbers of the form  $46^n 9$  are divisible by 7
- \* In base 6, all numbers of the form  $40^n 1$  are divisible by 5
- \* In base 15, all numbers of the form  $96^n 8$  are divisible by 11



- \* In base 9, all numbers of the form  $51^n$  are divisible by some element of  $\{2,5\}$  (note: the prime 5 is not allowed since the prime must be  $>$  base)
- \* In base 11, all numbers of the form  $25^n$  are divisible by some element of  $\{2,3\}$  (note: the prime 2 is not allowed since the prime must be  $>$  base)
- \* In base 14, all numbers of the form  $B0^n1$  are divisible by some element of  $\{3,5\}$
- \* In base 8, all numbers of the form  $64^n7$  are divisible by some element of  $\{3,5,13\}$
- \* In base 13, all numbers of the form  $30^n95$  are divisible by some element of  $\{5,7,17\}$
- \* In base 16, all numbers of the form  $4^nD$  are divisible by some element of  $\{3,7,13\}$  (note: the prime D is not allowed since the prime must be  $>$  base)
- \* In base 16, all numbers of the form  $8^nF$  are divisible by some element of  $\{3,7,13\}$

Examples of the second strategy: (we can show that both factors are  $> 1$ , if  $n$  makes corresponding numbers  $> b$  (i.e.  $n \geq 2$  for  $1^n$  in base 9,  $n \geq 0$  for  $10^n1$  in base 8 and  $B4^n1$  in base 16,  $n \geq 1$  for other examples), thus these factorizations are nontrivial)

- \* In base 9, all numbers of the form  $1^n$  factored as  $(3^n - 1) * (3^n + 1) / 8$
- \* In base 8, all numbers of the form  $10^n1$  factored as  $(2^{n+1} + 1) * (4^{n+1} - 2^{n+1} + 1)$
- \* In base 9, all numbers of the form  $38^n$  factored as  $(2 * 3^n - 1) * (2 * 3^n + 1)$  (note: the prime 3 is not allowed since the prime must be  $>$  base)
- \* In base 16, all numbers of the form  $8F^n$  factored as  $(3 * 4^n - 1) * (3 * 4^n + 1)$
- \* In base 16, all numbers of the form  $F^n7$  factored as  $(4^{n+1} - 3) * (4^{n+1} + 3)$  (note: the prime 7 is not allowed since the prime must be  $>$  base)
- \* In base 9, all numbers of the form  $31^n$  factored as  $(5 * 3^n - 1) * (5 * 3^n + 1) / 8$  (note: the prime 3 is not allowed since the prime must be  $>$  base)
- \* In base 16, all numbers of the form  $4^n1$  factored as  $(2 * 4^{n+1} - 7) * (2 * 4^{n+1} + 7) / 15$
- \* In base 16, all numbers of the form  $15^n$  factored as  $(2 * 4^n - 1) * (2 * 4^n + 1) / 3$
- \* In base 16, all numbers of the form  $C^nD$  factored as  $(2 * 4^{n+1} - 2 * 2^{n+1} + 1) * (2 * 4^{n+1} + 2 * 2^{n+1} + 1) / 5$  (note: the prime D is not allowed since the prime must be  $>$  base)
- \* In base 16, all numbers of the form  $B4^n1$  factored as  $(13 * 4^{n+1} - 7) * (13 * 4^{n+1} + 7) / 15$

Examples of combine of the two strategies: (we can show that for the part of the first strategy, the corresponding numbers are  $>$  all elements in  $S$ , and for the part of the second strategy, both factors are  $> 1$ , if  $n$  makes corresponding numbers  $> b$  (i.e.  $n \geq 0$  for  $B^n9B$  in base 12,  $n \geq 1$  for other examples), thus these factorizations are nontrivial)

- \* In base 14, numbers of the form  $8D^n$  are divisible by 5 if  $n$  is odd and factored as  $(3 * 14^{n/2} - 1) * (3 * 14^{n/2} + 1)$  if  $n$  is even
- \* In base 12, numbers of the form  $B^n9B$  are divisible by 13 if  $n$  is odd and factored as  $(12^{(n+2)/2} - 5) * (12^{(n+2)/2} + 5)$  if  $n$  is even

\* In base 14, numbers of the form  $D^{n/5}$  are divisible by 5 if  $n$  is even and factored as  $(14^{(n+1)/2} - 3) * (14^{(n+1)/2} + 3)$  if  $n$  is odd (note: the prime 5 is not allowed since the prime must be  $>$  base)

\* In base 17, numbers of the form  $19^n$  are divisible by 2 if  $n$  is odd and factored as  $(5 * 17^{n/2} - 3) * (5 * 17^{n/2} + 3) / 16$  if  $n$  is even

\* In base 19, numbers of the form  $16^n$  are divisible by 5 if  $n$  is odd and factored as  $(2 * 19^{n/2} - 1) * (2 * 19^{n/2} + 1) / 3$  if  $n$  is even

As previously mentioned, in practice to [compute](#)  $M(L_b)$  one works with an underapproximation  $M$  of  $M(L_b)$  and an overapproximation  $L$  of  $L_b - \text{sup}(M)$ . One then refines such approximations until  $L = \emptyset$  from which it follows that  $M = M(L_b)$ .

For the initial approximation, note that every minimal prime in base  $b$  with at least 4 digits is of the form  $xY^*z$ , where  $x \in \{x \mid x \text{ is base-}b \text{ digit, } x \neq 0\}$ ,  $z \in \{z \mid z \text{ is base-}b \text{ digit, } \text{gcd}(z,b) = 1\}$ , and  $Y^*$  (for this  $(x,z)$  pair) =  $\{y \mid xy, xz, yz, xyz \text{ are all composites}\}$ . (Of course, if  $xz$  is prime, then the  $Y^*$  set for this  $(x,z)$  pair is  $\emptyset$ )

Making use of this, our algorithm sets  $M$  to be the set of base- $b$  representations of the minimal primes with at most 3 digits (which can be found simply by brute force) and  $L$  to be  $\cup_{x,z} (xY^*z)$  as described above.

All remaining minimal primes are members of  $L$ , so to find them we explore the families in  $L$ . During this process, each family will be decomposed into possibly multiple other families. For example, a simple way of exploring the family  $xY^*z$  where  $Y = \{y_1, \dots, y_n\}$  is to decompose it into the families  $xY^*y_1z, \dots, xY^*y_nz$ . If the smallest member (say  $xy_iz$ ) of any such family happens to be prime, it can be added to  $M$  and the family  $xY^*y_iz$  removed from consideration. Furthermore, once  $M$  has been updated it may be possible to simplify some families in  $L$ . In this case,  $xY^*y_jz$  (for  $j \neq i$ ) can be simplified to  $x(Y-y_i)^*y_jz$  since no minimal prime contains  $xy_iz$  as a proper subsequence.

We call families of the form  $xy^*z$  (where  $x, z \in \Sigma_b^*$  and  $y \in \Sigma_b$ ) *simple families*. Our algorithm then proceeds as follows:

1. Let

$M := \{\text{minimal primes in base } b \text{ of length } \leq 3\}$

$L := \cup_{x,z \in \Sigma_b^*} (xY^*z)$

where  $x \neq 0$  and  $Y$  is the set of digits  $y$  such that  $xyz$  has no subword in  $M$ .

2. While  $L$  contains non-simple families:

(a) Explore each family of  $L$ , and update  $L$ .

(b) Examine each family of  $L$ :

- i. Let  $w$  be the shortest string in the family. If  $w$  has a subword in  $M$ , then remove the family from  $L$ . If  $w$  represents a prime, then add  $w$  to  $M$  and remove the family from  $L$ .
- ii. If possible, simplify the family.
- iii. Check if the family can be proven to contain no primes  $>$  base, and if so then remove the family from  $L$ .

(c) As much as possible and update  $L$ ; after each split examine the new families as in (b).

At the conclusion of the algorithm described,  $L$  will consist of simple families (of the form  $xy^*z$ ) which have not yet yielded a prime, but for which there is no obvious reason why there can't be a prime of such a form. In such a case, the only way to proceed is to test the primality of larger and larger numbers of such form and hope a prime is eventually discovered (we usually [conjecture](#) that there must be a prime at some point if it cannot be [proved](#) that there can't be a prime, by covering congruence, algebra factorization, or combine of them, since there is a [heuristic argument](#) that there are [infinitely many](#) such primes ([reference](#)), since by the [prime number theorem](#), the [chance](#) that a [random](#)  $n$ -digit base  $b$  number is prime is [approximately](#)  $1/n$  ([reference](#) [reference](#)) (also see [this page](#) and [this page](#), the chance is approximately  $\frac{b-1}{\ln(b)} \cdot \frac{b^{n-1}}{n}$ , where  $\ln$  is the [natural logarithm](#)). If one conjectures the numbers  $xy^*z$  behave similarly (i.e. " $N$  of the form  $xy^*z$ " and " $N$  is prime" are [independent events](#)) you would [expect](#)  $\sum_{n=2}^{\infty} \frac{1}{n} = \infty$  ([harmonic series](#) is [divergent](#)) primes of the form  $xy^*z$ , of course, this does not always happen, since some  $xy^*z$  families can be proven to contain no primes  $>$  base, and every  $xy^*z$  family has its own [Nash weight](#) (or [difficulty](#)),  $xy^*z$  families which can be proven to contain no primes  $>$  base have Nash weight (or difficulty) 0, thus  $xy^*z$  families are not "completely" random. They are random enough that the prime number theorem can be used to predict their primality, but divisibility by small primes is not as random and can easily be predicted: Once one candidate is found to be divisible by a prime  $p$  or to have an algebraic factorization (e.g. difference-of-two-squares factorization, sum/difference-of-two-cubes factorization, Aurifeuillian factorization for  $x^4+4y^4$ ), another predictable candidate will also be divisible by  $p$  or also have the same algebraic factorization. This decreases the probability of expected primes. Sometimes though, the candidates will never be divisible by a prime  $p$ , which increases the probability of expected primes. However, it is at least a reasonable conjecture in the absence of evidence to the contrary, the numbers in simple families are of the form  $\frac{a \cdot b^{n+c}}{\gcd(a+c, b-1)}$  for some fixed integer [triple](#)  $(a, b, c)$ , where  $a \geq 1$ ,  $b \geq 2$  ( $b$  is the base),  $c \neq 0$ ,  $\gcd(a, c) = 1$ ,  $\gcd(b, c) = 1$ , this is an [exponential sequence](#), there is also a similar conjecture for [polynomial sequence](#): the [Bunyakovsky conjecture](#), the condition is similar to our conjecture in this article, both are the small prime factors and the algebraic factors, the main difference is that polynomial sequence cannot have a covering set with  $>1$  primes, however, unlike our conjecture (the analog of [Bunyakovsky conjecture](#) for [exponential sequences](#)), the analog of [Dickson's conjecture](#) and [Schinzel's hypothesis H](#) for [exponential sequences](#) is widely believed to be false, e.g. for all integer  $k$  divisible by 3, it is widely believed that there are only finitely many integers  $n \geq 1$  such that  $k \cdot 2^n \pm 1$  are [twin primes](#) (see [this page](#) and [this page](#), the conjecture that 237 is the smallest odd number divisible by 3 such that  $k \cdot 2^n \pm 1$  are never

twin primes will never be proven), another example is that it is widely believed that 127 is the largest number  $n$  such that the [Mersenne number](#)  $2^n - 1$  and the [Wagstaff number](#)  $(2^n + 1)/3$  are both primes (see [New Mersenne Conjecture](#) and its [status page](#), such primes are listed in <https://oeis.org/A107360>) (in fact, if  $n$  is [even number](#), then  $(2^n + 1)/3$  is not integer, thus we only need to consider [odd](#)  $n$ , and for odd number  $n = 2^*m + 1$ ,  $(2^n + 1)/3 = (2^*4^m + 1)/3$ , thus it can be written as the form  $\frac{a \cdot b^{n+c}}{\gcd(a+c, b-1)}$ , with  $(a, b, c) = (2, 4, 1)$ , thus is included in this conjecture, also, if  $n$  is odd composite, then  $2^n - 1$  and  $(2^n + 1)/3$  are both composites, thus we only need to consider odd prime  $n$ ), another example is that it is widely believed that there are only finitely many integers  $n$  such that  $n$  and  $n \pm 1$  all have [primitive roots](#), and  $3^{541} - 1$  may be the largest such  $n$ , since it is widely believed that there are only finitely many integers  $n \geq 1$  such that the given pair of [exponential sequences](#) both produce primes:  $(2^*3^n - 1, 2^*3^n + 1)$ ,  $(3^n + 1)/2, 3^n + 2)$ ,  $(3^n - 1)/2, 3^n - 2)$ , see <https://oeis.org/A305237>, also it is widely believed that for any [polynomial sequence](#) and any [exponential sequence](#), there are only finitely many  $n$  such that both sequences produce primes, e.g. it is widely believed that only finitely many [Mersenne exponents](#) (i.e. primes  $p$  such that  $2^p - 1$  is also prime) are [Sophie Germain primes](#) (such primes  $p$  are listed in <https://oeis.org/A065406>), i.e. the number of primes  $p$  such that  $2^*p + 1$  and  $2^p - 1$  are both prime is expected to be finite, also it is widely believed that only finitely many [Mersenne exponents](#) (i.e. primes  $p$  such that  $2^p - 1$  is also prime) are members of [twin primes pair](#) (such primes  $p$  are listed in <https://oeis.org/A346645>), see [this post](#) and [this thread](#)). For example, the base 11 family  $57^n$ , this family have already been searched to length 50000 with no prime or [PRP](#) found, however the algebraic form of this family is  $(57^*11^n - 7)/10$ , and there is no  $n$  satisfying that  $57^*11^n$  and 7 are both  $r$ -th powers for some  $r > 1$  to make this number have [difference-of-two-r-th powers factorization](#) (since 7 is not [perfect power](#)), nor there is  $n$  satisfying that  $57^*11^n$  and  $-7$  are (one is 4th power, another is of the form  $4^*m^4$ ) to make this number have [Aurifeuillian factorization](#) for  $x^4 + 4y^4$  (since  $-7$  is neither [4th power](#) nor of the form  $4^*m^4$ ), thus, base 11 family  $57^n$  has no algebraic factorization for any  $n$ , thus  $57^n$  eventually should yield a prime unless it can be proven to contain no primes  $>$  base using covering congruence, and we have:

$57^n$  is divisible by 2 for  $n \equiv 1 \pmod 2$   
 $57^n$  is divisible by 13 for  $n \equiv 2 \pmod{12}$   
 $57^n$  is divisible by 17 for  $n \equiv 4 \pmod{16}$   
 $57^n$  is divisible by 5 for  $n \equiv 0 \pmod 5$   
 $57^n$  is divisible by 23 for  $n \equiv 6 \pmod{22}$   
 $57^n$  is divisible by 601 for  $n \equiv 8 \pmod{600}$   
 $57^n$  is divisible by 97 for  $n \equiv 12 \pmod{48}$   
 $57^n$  is divisible by 1279 for  $n \equiv 16 \pmod{426}$   
 ...

and it does not appear to be any covering set of primes (and its Nash weight (or difficulty) is positive, and it has prime candidate), so there must be a prime at some point.

The [multiplicative order](#) of  $b \bmod p$  is important in this problem, since if a prime  $p$  divides the number with  $n$  digits in a family in base  $b$ , then  $p$  also divides the number with  $k \cdot r + n$  digits in the same family in base  $b$  for all nonnegative integer  $k$ , where  $r$  is the multiplicative order of  $b \bmod p$  (unless the multiplicative order of  $b \bmod p$  is 1, i.e.  $p$  divides  $b - 1$ , in this case  $p$  also divides the number with  $k \cdot p + n$  digits in the same family in base  $b$  for all [nonnegative integer](#)  $k$ ), the primes  $p$  such that the multiplicative order of  $b \bmod p$  is  $n$  are exactly the primes  $p$  dividing  $Z_s(n, b, 1)$ , where  $Z_s$  is the [Zsigmondy number](#), i.e.  $Z_s(n, b, 1)$  is the greatest divisor of  $b^n - 1$  that is coprime to  $b^m - 1$  for all positive integers  $m < n$ , with  $b \geq 2$  and  $n \geq 1$ , if (and only if) there is only one such prime, then this prime is [unique prime](#) in base  $b$ , see [list of the multiplicative order of  \$b \bmod p\$  for  \$b \leq 128\$  and primes  \$p \leq 4096\$](#) , [list of primes  \$p\$  such that the multiplicative order of  \$b \bmod p\$  is  \$n\$  for  \$2 \leq b \leq 64\$  and  \$1 \leq n \leq 64\$](#) , [smallest prime  \$p\$  such that  \$\text{znorder}\(\text{Mod}\(m, p\)\) = \(p-1\)/n\$  for  \$2 \leq m \leq 128\$  and  \$1 \leq n \leq 128\$](#) , [bases  \$b\$  such that  \$\Phi\(n, b\)\$  \(where  \$\Phi\$  is cyclotomic polynomial\) has algebra factors or small prime factors](#), [bases  \$b\$  such that there is unique prime with period length  \$n\$](#) , [unique period length in base  \$b\$](#) , also see factorization of  $b^n \pm 1$  (which is equivalent to factorization of  $Z_s(n, b, 1)$ ) with  [\$b \leq 12\$](#)   [\$13 \leq b \leq 99\$](#)   [\$b = 10\$](#)  [any  \$b\$](#)  [any  \$b\$](#) , also see [this page](#).

The numbers in simple families are of the form  $\frac{a \cdot b^{n+c}}{\text{gcd}(a+c, b-1)}$  for some fixed integers  $a, b, c$  where  $a \geq 1, b \geq 2$  ( $b$  is the base),  $c \neq 0, \text{gcd}(a, c) = 1, \text{gcd}(b, c) = 1$  (thus, all large minimal primes base  $b$  (but possible not all minimal primes base  $b$  if  $b$  is large, e.g.  $b = 25, 29, 31, 35$ ) have a nice short algebraic description (see [this page](#) and [this page](#), the prime numbers in these two pages do *not* have nice short algebraic descriptions, also see [this page](#)) and have simple expression ([expression](#) with  $\leq 40$  [characters](#), all taken from “[0](#)” “[1](#)” “[2](#)” “[3](#)” “[4](#)” “[5](#)” “[6](#)” “[7](#)” “[8](#)” “[9](#)” “[+](#)” “[-](#)” “[\\*](#)” “[/](#)” “[^](#)” “[\(](#)” “[\)](#)”, [factorial](#) (!), [double factorial](#) (!!), and [primorial](#) (#) are not allowed since they can be used to ensure many small factors, see [this page](#)). Except in the special case  $c = \pm 1$  and  $\text{gcd}(a+c, b-1) = 1$ , when  $n$  is large the known [primality tests](#) for such a number are too inefficient to run (since this special case  $c = \pm 1$  and  $\text{gcd}(a+c, b-1) = 1$  is the only case which  $N-1$  and/or  $N+1$  is [smooth](#), i.e. the case  $c = 1$  and  $\text{gcd}(a+c, b-1) = 1$  (corresponding to generalized [Proth prime](#) base  $b: a \cdot b^n + 1$ , they are related to [generalized Sierpinski conjecture base  \$b\$](#) ) can be easily proven prime using Pocklington [N-1 method](#), and the case  $c = -1$  and  $\text{gcd}(a+c, b-1) = 1$  (corresponding to generalized [Riesel prime](#) base  $b: a \cdot b^n - 1$ , they are related to [generalized Riesel conjecture base  \$b\$](#) ) can be easily proven prime using Morrison [N+1 method](#)). In this case one must resort to a [probable primality test](#) such as a [Miller–Rabin primality test](#) or a [Baillie–PSW primality test](#), unless a [divisor](#) of the number can be found, and thus these numbers cannot be [proven primes](#) and can only be [probable primes](#), and we cannot definitely say that the corresponding families can be removed from the list of unsolved families, and we cannot definitely [compute](#) this part of the [set](#)  $M(L_b)$ . Since we are testing many numbers in an [exponential sequence](#), it is possible to use a [sieving process](#) (such as [srsieve](#) software) to find divisors rather than using [trial division](#), i.e. we will remove the integers  $n$  such that  $\frac{a \cdot b^{n+c}}{\text{gcd}(a+c, b-1)}$  either has a [prime factor](#)

less than certain limit (say  $2^{32}$ ) or has algebraic factorization, and [test the primality](#) of  $\frac{a \cdot b^{n+c}}{\gcd(a+c, b-1)}$  for other integers  $n$ .

To do this, we made use of Geoffrey Reynolds' [srsieve](#) software. This program uses the [baby-step giant-step algorithm](#) to find all primes  $p$  which divide  $a \cdot b^n + c$  where  $p$  and  $n$  lie in a specified [range](#). Since this program cannot handle the [general case](#)  $\frac{a \cdot b^{n+c}}{\gcd(a+c, b-1)}$  when  $\gcd(a+c, b-1) > 1$  we only used it to sieve the sequence  $a \cdot b^n + c$  for primes  $p$  not dividing  $\gcd(a+c, b-1)$ , and initialized the list of candidates to not include  $n$  for which there is some prime  $p$  dividing  $\gcd(a+c, b-1)$  for which  $p$  divides  $\frac{a \cdot b^{n+c}}{\gcd(a+c, b-1)}$ . The program had to be modified slightly to remove a check which would prevent it from running in the case when  $a$ ,  $b$ , and  $c$  were all [odd](#) (since then 2 divides  $a \cdot b^n + c$ , but 2 may not divide  $\frac{a \cdot b^{n+c}}{\gcd(a+c, b-1)}$ ).

Once the numbers with small divisors had been removed, it remained to test the remaining numbers using a probable primality test. For this we used the software [LLR](#) by Jean Penné. Although undocumented, it is possible to run this program on numbers of the form  $\frac{a \cdot b^{n+c}}{\gcd(a+c, b-1)}$  when  $\gcd(a+c, b-1) > 1$ , so this program required no modifications (also, [LLR](#) can do a proven primality test (i.e. [prove the primality](#)) for numbers of the form  $a \cdot b^n \pm 1$  (i.e. the special case  $c = \pm 1$  and  $\gcd(a+c, b-1) = 1$ ) with  $b^n > a$ ). A script was also written which allowed one to run [srsieve](#) while [LLR](#) was testing the remaining candidates, so that when a divisor was found by [srsieve](#) on a number which had not yet been tested by [LLR](#) it would be removed from the list of candidates. In the cases where the elements of  $M(L_b)$  could be proven prime rigorously, we employed [PRIMO](#) by Marcel Martin, an [elliptic curve primality proving](#) implementation (for the primes of the form  $\frac{a \cdot b^{n+c}}{\gcd(a+c, b-1)}$ , with  $c \neq \pm 1$  and/or  $\gcd(a+c, b-1) \neq 1$ , we cannot use the [classical tests](#) (including the tests of  $N-1$ ,  $N+1$ ,  $N^2+1$ ,  $N^2+N+1$ ,  $N^2-N+1$  (all such [polynomials](#) are [cyclotomic polynomials](#) of  $N$ , and such tests are called [cyclotomy proofs](#), see [this page](#)), and the [combined tests](#)), since for these primes, none of them is at least 1/3 [factorable](#) ([Brillhart-Lehmer-Selfridge primality test](#)) (see [this page](#)) (if we want to use the [classical tests](#) to prove the primality of  $N$ , then we must [factor](#) at least one of  $N-1$ ,  $N+1$ ,  $N^2+1$ ,  $N^2+N+1$ ,  $N^2-N+1$  to the factored part  $\geq 33.3333\%$  (i.e. [product](#) of known [prime factors](#)  $\geq$  the [cube root](#) of  $N$ ), and except [trial division](#) with the primes up to certain limit (say  $2^{64}$ ) and the [algebra factors](#) (e.g. [difference-of-two-squares factorization](#), [sum/difference-of-two-cubes factorization](#), [Aurifeuillian factorization](#), and algebra factors of the [Cunningham number](#)  $b^n \pm 1$  ( $b^n - 1$  can be factored to product of  $\Phi_d(b)$  with  $d$  dividing  $n$ , and  $b^n + 1$  can be factored to product of  $\Phi_d(b)$  with  $d$  dividing  $2n$  but not dividing  $n$ , where  $\Phi$  is the [cyclotomic polynomial](#)), see [this reference](#)), we can use [elliptic-curve factorization method \(ECM\)](#), Pollard [P-1 method](#), Williams [P+1 method](#), [special number field sieve \(SNFS\)](#), [general number field sieve \(GNFS\)](#), etc. to factor the numbers, however, all these factorization algorithms take long time, i.e. they cannot be done in [polynomial time](#) (unlike primality proving, when the numbers are sufficiently large, no efficient, [non-quantum](#) integer factorization [algorithm](#) is known, i.e. integer factorization may



be [NP-complete](#). However, it has not been proven that no efficient algorithm exists. The presumed [difficulty](#) of this problem is at the heart of widely used algorithms in [cryptography](#) such as [RSA](#). Many areas of [mathematics](#) and [computer science](#) have been brought to bear on the problem, including [elliptic curves](#), [algebraic number theory](#), and [quantum computing](#)), and hence to do this is impractically), i.e. they are [ordinary primes](#), and if the prime is not large (say less than  $10^{25000}$ ), we can use [elliptic curve primality proving \(ECP\)](#) to proof (see [PRIMO top 20 records](#) and [elliptic curve primality proving top 20 records](#) and [top primes proven by Francois Morain's programs](#)) and make [primality certificate](#), but if the prime is very large (say  $> 10^{25000}$ ), the known [primality tests](#) for such a number are too inefficient to run (although there is [AKS primality test](#), which can prove the primality of any general prime in [polynomial time](#), but since its [time complexity](#) is  $O(\ln(N)^{12})$ , and if we can do  $10^9$  [bitwise operations](#) per second, use this test to prove the primality of a 5000-digit prime need  $5.422859049 \times 10^{39}$  [seconds](#), or  $1.719577324 \times 10^{32}$  [years](#), much longer than [the age of the universe](#), thus to do this test is still impractically), thus we can only resort to a [probable primality test](#) such as [Miller–Rabin primality test](#) and [Baillie–PSW primality test](#), unless a [divisor](#) of the number can be found, and hence we cannot [prove the primality](#) of this number, thus we cannot definitely say that the corresponding families can be removed from the list of unsolved families, and we cannot definitely [compute](#) this part of the [sets](#)  $M(L_b)$ .

<a href="#">Fermat pseudoprime</a> (to base $b = 2$ : <a href="https://oeis.org/A001567">https://oeis.org/A001567</a> , and see <a href="#">this data</a> )	<a href="#">Lucas pseudoprime</a> (to parameters $(P, Q) = (1, -1)$ : <a href="https://oeis.org/A081264">https://oeis.org/A081264</a> union <a href="https://oeis.org/A141137">https://oeis.org/A141137</a> , and see <a href="#">this data</a> ) (to parameters $(P, Q)$ defined by Selfridge's Method A: <a href="https://oeis.org/A217120">https://oeis.org/A217120</a> , and see <a href="#">this data</a> )
<a href="#">Strong Fermat pseudoprime</a> (to base $b = 2$ : <a href="https://oeis.org/A001262">https://oeis.org/A001262</a> , and see <a href="#">this data</a> )	<a href="#">Strong Lucas pseudoprime</a> (to parameters $(P, Q)$ defined by Selfridge's Method A: <a href="https://oeis.org/A217255">https://oeis.org/A217255</a> , and see <a href="#">this data</a> )
Over Fermat pseudoprime (to base $b = 2$ : composite factors of <a href="#">A019320</a> ( $n$ ) / $\gcd(\text{A019320}(n), n)$ for some $n$ , there is an OEIS sequence: <a href="https://oeis.org/A141232">https://oeis.org/A141232</a> )	Over Lucas pseudoprime (to parameters $(P, Q) = (1, -1)$ : composite factors of <a href="#">A061446</a> ( $n$ ) / $\gcd(\text{A061446}(n), n)$ for some $n$ )
<a href="#">Baillie–PSW</a> pseudoprime (none are known, none $< 2^{64}$ exist)	
<a href="#">Carmichael number</a> ( <a href="https://oeis.org/A002997">https://oeis.org/A002997</a> )	<a href="#">Lucas–Carmichael number</a> ( <a href="https://oeis.org/A006972">https://oeis.org/A006972</a> )
<a href="#">Pépin primality test</a> (for <a href="#">Fermat numbers</a> , i.e. numbers of the form $2^{2^n} + 1$ ( <a href="#">A000051</a> ), if $2^{2^n} + 1$ is prime, then $n$ must be power of 2, such primes are <a href="https://oeis.org/A019434">https://oeis.org/A019434</a> )	<a href="#">Lucas–Lehmer primality test</a> (for <a href="#">Mersenne numbers</a> , i.e. numbers of the form $2^n - 1$ ( <a href="https://oeis.org/A000225">https://oeis.org/A000225</a> ), if $2^n - 1$ is prime, then $n$ must be prime, such primes are <a href="https://oeis.org/A000668">https://oeis.org/A000668</a> )
<a href="#">Proth primality test</a> (for numbers of the form	<a href="#">Lucas–Lehmer–Riesel primality test</a> (for

$k2^n+1$ with $k$ odd and $k < 2^n$ , i.e. <a href="https://oeis.org/A080075">Proth numbers</a> , <a href="https://oeis.org/A080075">https://oeis.org/A080075</a> , such primes are <a href="https://oeis.org/A080076">https://oeis.org/A080076</a> , also there is <a href="#">a list</a> of such primes sorted by $k$ )	numbers of the form $k2^n - 1$ with $k$ odd and $k < 2^n$ , i.e. <a href="https://oeis.org/A112714">Proth numbers of the second kind</a> , <a href="https://oeis.org/A112714">https://oeis.org/A112714</a> , such primes are <a href="https://oeis.org/A112715">https://oeis.org/A112715</a> , also there is <a href="#">a list</a> of such primes sorted by $k$ )
Pocklington <a href="#">N-1 primality test</a> (for numbers $n$ such that $n-1$ can be trivially fully factored)	Morrison <a href="#">N+1 primality test</a> (for numbers $n$ such that $n+1$ can be trivially fully factored)
<a href="#">Combined N-1 / N+1 primality test</a> (and other <a href="#">cyclotomy tests</a> , i.e. $\Phi_r(N)$ for small $r$ (where $\Phi$ is the <a href="#">cyclotomic polynomial</a> ), including $N^2+1$ , $N^2+N+1$ , $N^2 - N+1$ )	
Pollard <a href="#">P-1 integer factorization method</a>	Williams <a href="#">P+1 integer factorization method</a>

Some families  $xy^*z$  could not be ruled out as containing no primes  $>$  base, but no primes  $>$  base could be found in the family, even after searching through numbers with over 50000 digits. Many  $xy^*z$  families contain no small primes even though they do contain very large primes, for example:

- \* In base 5, the smallest prime in the family  $10^n13$  is  $10^{93}13$
- \* In base 8, the smallest prime in the family  $4^n7$  is  $4^{220}7$  (the prime 7 is not counted since the prime must be  $>$  base)
- \* In base 9, the smallest prime in the family  $30^n11$  is  $30^{1158}11$
- \* In base 9, the smallest prime in the family  $27^n07$  is  $27^{686}07$
- \* In base 11, family  $57^n$  can not be ruled out as containing no primes  $>$  base but no primes  $>$  base found in the family after searching to length 50000 (the prime 5 is not counted since the prime must be  $>$  base)
- \* In base 13, the smallest prime in the family  $80^n111$  is  $80^{32017}111$  (this prime is only a probable prime, i.e. not proven prime)
- \* In base 13, the smallest prime in the family  $2B30^n1$  is  $2B3^{15197}1$
- \* In base 13, the smallest prime in the family  $B0^nBBA$  is  $B0^{6540}BBA$  (this prime is only a probable prime, i.e. not proven prime)
- \* In base 13, the smallest prime in the family  $390^n1$  is  $390^{6266}1$
- \* In base 13, the smallest prime in the family  $720^n2$  is  $720^{2297}2$
- \* In base 13, family  $95^n$  can not be ruled out as containing no primes  $>$  base but no primes  $>$  base found in the family after searching to length 50000
- \* In base 13, family  $A3^nA$  can not be ruled out as containing no primes  $>$  base but no primes  $>$  base found in the family after searching to length 50000
- \* In base 14, the smallest prime in the family  $4D^n$  is  $4D^{19698}$
- \* In base 16, family  $3^nAF$  can not be ruled out as containing no primes  $>$  base but no primes  $>$  base found in the family after searching to length 50000
- \* In base 16, family  $4^nDD$  can not be ruled out as containing no primes  $>$  base but no primes  $>$  base found in the family after searching to length 50000
- \* In base 16, the smallest prime in the family  $DB^n$  is  $DB^{32234}$  (this prime is only a probable prime, i.e. not proven prime) (the prime D is not counted since the prime must be  $>$  base)



For any given base  $b$ , we find all  $(x,z)$  digits-pair such that  $x \neq 0$  and  $\gcd(z,b) = 1$ , and find the corresponding sets  $Y^*$ , see below.

**Bold** for minimal primes in base  $b$ , i.e. elements of the set  $M(L_b)$

## base 2

The possible (first digit,last digit) for an element with  $\geq 3$  digits in the minimal set of the strings for primes with at least two digits are:

(1,1)

\* Case (1,1):

\*\* **11** is prime, and thus the only minimal prime in this family.

## base 3

The possible (first digit,last digit) for an element with  $\geq 3$  digits in the minimal set of the strings for primes with at least two digits are:

(1,1), (1,2), (2,1), (2,2)

\* Case (1,1):

\*\* Since 12, 21, **111** are primes, we only need to consider the family  $1\{0\}1$  (since any digits 1, 2 between them will produce smaller primes)

\*\*\* All numbers of the form  $1\{0\}1$  are divisible by 2, thus cannot be prime.

\* Case (1,2):

\*\* **12** is prime, and thus the only minimal prime in this family.

\* Case (2,1):

\*\* **21** is prime, and thus the only minimal prime in this family.

\* Case (2,2):

\*\* Since 21, 12 are primes, we only need to consider the family  $2\{0,2\}2$  (since any digits 1 between them will produce smaller primes)

\*\*\* All numbers of the form  $2\{0,2\}2$  are divisible by 2, thus cannot be prime.

## base 4

The possible (first digit,last digit) for an element with  $\geq 3$  digits in the minimal set of the strings for primes with at least two digits are:

(1,1), (1,3), (2,1), (2,3), (3,1), (3,3)

\* Case (1,1):

\*\* **11** is prime, and thus the only minimal prime in this family.

\* Case (1,3):

\*\* **13** is prime, and thus the only minimal prime in this family.

\* Case (2,1):

\*\* Since 23, 11, 31, **221** are primes, we only need to consider the family  $2\{0\}1$  (since any digits 1, 2, 3 between them will produce smaller primes)

\*\*\* All numbers of the form  $2\{0\}1$  are divisible by 3, thus cannot be prime.

\* Case (2,3):

\*\* **23** is prime, and thus the only minimal prime in this family.

\* Case (3,1):

\*\* **31** is prime, and thus the only minimal prime in this family.

\* Case (3,3):

\*\* Since 31, 13, 23 are primes, we only need to consider the family  $3\{0,3\}3$  (since any digits 1, 2 between them will produce smaller primes)

\*\*\* All numbers of the form  $3\{0,3\}3$  are divisible by 3, thus cannot be prime.

## base 5

The possible (first digit,last digit) for an element with  $\geq 3$  digits in the minimal set of the strings for primes with at least two digits are:

(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)

\* Case (1,1):

\*\* Since 12, 21, **111**, **131** are primes, we only need to consider the family  $1\{0,4\}1$  (since any digits 1, 2, 3 between them will produce smaller primes)



\* Case (2,4):

\*\* Since 21, 23, 34 are primes, we only need to consider the family  $2\{0,2,4\}4$  (since any digits 1, 3 between them will produce smaller primes)

\*\*\* All numbers of the form  $2\{0,2,4\}4$  are divisible by 2, thus cannot be prime.

\* Case (3,1):

\*\* Since 32, 34, 21 are primes, we only need to consider the family  $3\{0,1,3\}1$  (since any digits 2, 4 between them will produce smaller primes)

\*\*\* Since 313, 111, 131, **3101** are primes, we only need to consider the families  $3\{0,3\}1$  and  $3\{0,3\}11$  (since any digit combo 10, 11, 13 between (3,1) will produce smaller primes)

\*\*\*\* For the  $3\{0,3\}1$  family, we can separate this family to four families:

\*\*\*\*\* For the  $30\{0,3\}01$  family, we have the prime **30301**, and the remain case is the family  $30\{0\}01$ .

\*\*\*\*\* All numbers of the form  $30\{0\}01$  are divisible by 2, thus cannot be prime.

\*\*\*\*\* For the  $30\{0,3\}31$  family, note that there must be an even number of 3's between (30,31), or the result number will be divisible by 2 and cannot be prime.

\*\*\*\*\* Since 33331 is prime, any digit combo 33 between (30,31) will produce smaller primes.

\*\*\*\*\* Thus, the only possible prime is the smallest prime in the family  $30\{0\}31$ , and this prime is **300031**.

\*\*\*\*\* For the  $33\{0,3\}01$  family, note that there must be an even number of 3's between (33,01), or the result number will be divisible by 2 and cannot be prime.

\*\*\*\*\* Since 33331 is prime, any digit combo 33 between (33,01) will produce smaller primes.

\*\*\*\*\* Thus, the only possible prime is the smallest prime in the family  $33\{0\}01$ , and this prime is **33001**.

\*\*\*\*\* For the  $33\{0,3\}31$  family, we have the prime **33331**, and the remain case is the family  $33\{0\}31$ .

\*\*\*\*\* All numbers of the form  $33\{0\}31$  are divisible by 2, thus cannot be prime.

\*\*\*\* All numbers of the form  $3\{0,3\}11$  are divisible by 3, thus cannot be prime.

\* Case (3,2):

\*\* **32** is prime, and thus the only minimal prime in this family.

\* Case (3,3):

\*\* Since 32, 34, 23, 43, **313** are primes, we only need to consider the family  $3\{0,3\}3$  (since any digits 1, 2, 4 between them will produce smaller primes)

\*\*\* All numbers of the form  $3\{0,3\}3$  are divisible by 3, thus cannot be prime.

\* Case (3,4):

\*\* **34** is prime, and thus the only minimal prime in this family.

\* Case (4,1):

\*\* Since 43, 21, **401** are primes, we only need to consider the family  $4\{1,4\}1$  (since any digits 0, 2, 3 between them will produce smaller primes)

\*\*\* Since 414, 111 are primes, we only need to consider the family  $4\{4\}1$  and  $4\{4\}11$  (since any digit combo 14 or 11 between them will produce smaller primes)

\*\*\*\* The smallest prime of the form  $4\{4\}1$  is **44441**.

\*\*\*\* All numbers of the form  $4\{4\}11$  are divisible by 2, thus cannot be prime.

\* Case (4,2):

\*\* Since 43, 12, 32 are primes, we only need to consider the family  $4\{0,2,4\}2$  (since any digits 1, 3 between them will produce smaller primes)

\*\*\* All numbers of the form  $4\{0,2,4\}2$  are divisible by 2, thus cannot be prime.

\* Case (4,3):

\*\* **43** is prime, and thus the only minimal prime in this family.

\* Case (4,4):

\*\* Since 43, 34, **414** are primes, we only need to consider the family  $4\{0,2,4\}4$  (since any digits 1, 3 between them will produce smaller primes)

\*\*\* All numbers of the form  $4\{0,2,4\}4$  are divisible by 2, thus cannot be prime.

## base 6

The possible (first digit,last digit) for an element with  $\geq 3$  digits in the minimal set of the strings for primes with at least two digits are:

(1,1), (1,5), (2,1), (2,5), (3,1), (3,5), (4,1), (4,5), (5,1), (5,5)

\* Case (1,1):

\*\* **11** is prime, and thus the only minimal prime in this family.

\* Case (1,5):

\*\* **15** is prime, and thus the only minimal prime in this family.

\* Case (2,1):

\*\* **21** is prime, and thus the only minimal prime in this family.

\* Case (2,5):

\*\* **25** is prime, and thus the only minimal prime in this family.

\* Case (3,1):

\*\* **31** is prime, and thus the only minimal prime in this family.

\* Case (3,5):

\*\* **35** is prime, and thus the only minimal prime in this family.

\* Case (4,1):

\*\* Since 45, 11, 21, 31, 51 are primes, we only need to consider the family  $4\{0,4\}1$  (since any digits 1, 2, 3, 5 between them will produce smaller primes)

\*\*\* Since **4401** and **4441** are primes, we only need to consider the families  $4\{0\}1$  and  $4\{0\}41$  (since any digits combo 40 and 44 between them will produce smaller primes)

\*\*\*\* All numbers of the form  $4\{0\}1$  are divisible by 5, thus cannot be prime.

\*\*\*\* The smallest prime of the form  $4\{0\}41$  is **40041**

\* Case (4,5):

\*\* **45** is prime, and thus the only minimal prime in this family.

\* Case (5,1):

\*\* **51** is prime, and thus the only minimal prime in this family.

\* Case (5,5):

\*\* Since 51, 15, 25, 35, 45 are primes, we only need to consider the family  $5\{0,5\}5$  (since any digits 1, 2, 3, 4 between them will produce smaller primes)

\*\*\* All numbers of the form  $5\{0,5\}5$  are divisible by 5, thus cannot be prime.

## base 7

The possible (first digit,last digit) for an element with  $\geq 3$  digits in the minimal set of the strings for primes with at least two digits are:

\* Case (1,1):

\*\* Since 14, 16, 41, 61, **131** are primes, we only need to consider the family  $1\{0,1,2,5\}1$  (since any digits 3, 4, 6 between them will produce smaller primes)

\*\*\* Since the digit sum of primes must be odd (otherwise the number will be divisible by 2, thus cannot be prime), there is an odd total number of 1 and 5 in the  $\{\}$

\*\*\*\* If there are  $\geq 3$  number of 1 and 5 in the  $\{\}$ :

\*\*\*\*\* If there is 111 in the  $\{\}$ , then we have the prime **11111**

\*\*\*\*\* If there is 115 in the  $\{\}$ , then the prime 115 is a subsequence

\*\*\*\*\* If there is 151 in the  $\{\}$ , then the prime 115 is a subsequence

\*\*\*\*\* If there is 155 in the  $\{\}$ , then the prime 155 is a subsequence

\*\*\*\*\* If there is 511 in the  $\{\}$ , then the current number is 15111, which has digit sum = 12, but digit sum divisible by 3 will cause the number divisible by 3 and cannot be prime, and we cannot add more 1 or 5 to this number (to avoid 11111, 155, 515, 551 as subsequence), thus we must add at least one 2 to this number, but then the number has both 2 and 5, and will have either 25 or 52 as subsequence, thus cannot be minimal prime

\*\*\*\*\* If there is 515 in the  $\{\}$ , then the prime 515 is a subsequence

\*\*\*\*\* If there is 551 in the  $\{\}$ , then the prime 551 is a subsequence

\*\*\*\*\* If there is 555 in the  $\{\}$ , then the prime 551 is a subsequence

\*\*\*\* Thus there is only one 1 (and no 5) or only one 5 (and no 1) in the  $\{\}$ , i.e. we only need to consider the families  $1\{0,2\}1\{0,2\}1$  and  $1\{0,2\}5\{0,2\}1$

\*\*\*\*\* For the  $1\{0,2\}1\{0,2\}1$  family, since **1211** is prime, we only need to consider the family  $1\{0\}1\{0,2\}1$

\*\*\*\*\* Since all numbers of the form  $1\{0\}1\{0\}1$  are divisible by 3 and cannot be prime, we only need to consider the family  $1\{0\}1\{0\}2\{0\}1$

\*\*\*\*\* Since **11201** is prime, we only need to consider the family  $1\{0\}1\{0\}21$

\*\*\*\*\* The smallest prime of the form  $11\{0\}21$  is **1100021**

\*\*\*\*\* All numbers of the form  $101\{0\}21$  are divisible by 5, thus cannot be prime

\*\*\*\*\* The smallest prime of the form  $1001\{0\}21$  is **100121**

\*\*\*\*\* Since this prime has no 0 between  $1\{0\}1$  and 21, we do not need to consider more families

\*\*\*\* For the  $1\{0,2\}5\{0,2\}1$  family, since 25 and 52 are primes, we only need to consider the family  $1\{0\}5\{0\}1$

\*\*\*\*\* Since **1051** is prime, we only need to consider the family  $15\{0\}1$

\*\*\*\*\* The smallest prime of the form  $15\{0\}1$  is **150001**

\* Case (1,2):

\*\* Since 14, 16, 32, 52 are primes, we only need to consider the family  $1\{0,1,2\}2$  (since any digits 3, 4, 5, 6 between them will produce smaller primes)

\*\*\* Since **1112** and **1222** are primes, there is at most one 1 and at most one 2 in  $\{ \}$

\*\*\*\* If there are one 1 and one 2 in  $\{ \}$ , then the digit sum is 6, and the number will be divisible by 6 and cannot be prime.

\*\*\*\* If there is one 1 but no 2 in  $\{ \}$ , then the digit sum is 4, and the number will be divisible by 2 and cannot be prime.

\*\*\*\* If there is no 1 but one 2 in  $\{ \}$ , then the form is  $1\{0\}2\{0\}2$

\*\*\*\*\* Since **1022** and **1202** are primes, we only need to consider the number 122

\*\*\*\*\* 122 is not prime.

\*\*\*\* If there is no 1 and no 2 in  $\{ \}$ , then the digit sum is 3, and the number will be divisible by 3 and cannot be prime.

\* Case (1,3):

\*\* Since 14, 16, 23, 43, **113**, **133** are primes, we only need to consider the family  $1\{0,5\}3$  (since any digits 1, 2, 3, 4, 6 between them will produce smaller primes)

\*\*\* Since 155 is prime, we only need to consider the family  $1\{0\}3$  and  $1\{0\}5\{0\}3$

\*\*\*\* All numbers of the form  $1\{0\}3$  are divisible by 2, thus cannot be prime.

\*\*\*\* All numbers of the form  $1\{0\}5\{0\}3$  are divisible by 3, thus cannot be prime.

\* Case (1,4):

\*\* **14** is prime, and thus the only minimal prime in this family.

\* Case (1,5):

\*\* Since 14, 16, 25, 65, **115**, **155** are primes, we only need to consider the family  $1\{0,3\}5$  (since any digits 1, 2, 4, 5, 6 between them will produce smaller primes)

\*\*\* All numbers of the form  $1\{0,3\}5$  are divisible by 3, thus cannot be prime.

\* Case (1,6):

\*\* **16** is prime, and thus the only minimal prime in this family.

\* Case (2,1):



\*\* Since 23, 25, 41, 61, **221** are primes, we only need to consider the family  $2\{0,1\}1$  (since any digits 2, 3, 4, 5, 6 between them will produce smaller primes)

\*\*\* Since **2111** is prime, we only need to consider the families  $2\{0\}1$  and  $2\{0\}1\{0\}1$

\*\*\*\* All numbers of the form  $2\{0\}1$  are divisible by 3, thus cannot be prime.

\*\*\*\* All numbers of the form  $2\{0\}1\{0\}1$  are divisible by 2, thus cannot be prime.

\* Case (2,2):

\*\* Since 23, 25, 32, 52, **212** are primes, we only need to consider the family  $2\{0,2,4,6\}2$  (since any digits 1, 3, 5 between them will produce smaller primes)

\*\*\* All numbers of the form  $2\{0,2,4,6\}2$  are divisible by 2, thus cannot be prime.

\* Case (2,3):

\*\* **23** is prime, and thus the only minimal prime in this family.

\* Case (2,4):

\*\* Since 23, 25, 14 are primes, we only need to consider the family  $2\{0,2,4,6\}4$  (since any digits 1, 3, 5 between them will produce smaller primes)

\*\*\* All numbers of the form  $2\{0,2,4,6\}4$  are divisible by 2, thus cannot be prime.

\* Case (2,5):

\*\* **25** is prime, and thus the only minimal prime in this family.

\* Case (2,6):

\*\* Since 23, 25, 16, 56 are primes, we only need to consider the family  $2\{0,2,4,6\}6$  (since any digits 1, 3, 5 between them will produce smaller primes)

\*\*\* All numbers of the form  $2\{0,2,4,6\}6$  are divisible by 2, thus cannot be prime.

\* Case (3,1):

\*\* Since 32, 41, 61 are primes, we only need to consider the family  $3\{0,1,3,5\}1$  (since any digits 2, 4, 6 between them will produce smaller primes)

\*\*\* Since 551 is prime, we only need to consider the family  $3\{0,1,3\}1$  and  $3\{0,1,3\}5\{0,1,3\}1$  (since any digits combo 55 between (3,1) will produce smaller primes)

\*\*\*\* For the  $3\{0,1,3\}1$  family, since **3031** and 131 are primes, we only need to consider the families  $3\{0,1\}1$  and  $3\{3\}3\{0,1\}1$  (since any digits combo 03, 13 between (3,1) will produce smaller primes, thus for the digits between (3,1), all 3's must be before all 0's and 1's, and thus we can let the red 3 in  $3\{3\}3\{0,1\}1$  be the rightmost 3 between (3,1), all digits before this 3 must be 3's, and all digits after this 3 must be either 0's or 1's)

\*\*\*\* For the  $3\{0,1\}1$  family:

\*\*\*\*\* If there are  $\geq 2$  0's and  $\geq 1$  1's between (3,1), then at least one of **30011**, **30101**, **31001** will be a subsequence.

\*\*\*\*\* If there are no 1's between (3,1), then the form will be  $3\{0\}1$

\*\*\*\*\* All numbers of the form  $3\{0\}1$  are divisible by 2, thus cannot be prime.

\*\*\*\*\* If there are no 0's between (3,1), then the form will be  $3\{1\}1$

\*\*\*\*\* The smallest prime of the form  $3\{1\}1$  is **31111**

\*\*\*\*\* If there are exactly 1 0's between (3,1), then there must be  $< 3$  1's between (3,1), or **31111** will be a subsequence.

\*\*\*\*\* If there are 2 1's between (3,1), then the digit sum is 6, thus the number is divisible by 6 and cannot be prime.

\*\*\*\*\* If there are 1 1's between (3,1), then the number can only be either 3101 or 3011

\*\*\*\*\* Neither 3101 nor 3011 is prime.

\*\*\*\*\* If there are no 1's between (3,1), then the number must be 301

\*\*\*\*\* 301 is not prime.

\*\*\*\* For the  $3\{3\}3\{0,1\}1$  family:

\*\*\*\*\* If there are at least one 3 between (3,3 $\{0,1\}1$ ) and at least one 1 between (3 $\{3\}3$ ,1), then **33311** will be a subsequence.

\*\*\*\*\* If there are no 3 between (3,3 $\{0,1\}1$ ), then the form will be  $33\{0,1\}1$

\*\*\*\*\* If there are at least 3 1's between (33,1), then 31111 will be a subsequence.

\*\*\*\*\* If there are exactly 2 1's between (33,1), then the digit sum is 12, thus the number is divisible by 3 and cannot be prime.

\*\*\*\*\* If there are exactly 1 1's between (33,1), then the digit sum is 11, thus the number is divisible by 2 and cannot be prime.

\*\*\*\*\* If there are no 1's between (33,1), then the form will be  $33\{0\}1$

\*\*\*\*\* The smallest prime of the form  $33\{0\}1$  is **33001**

\*\*\*\*\* If there are no 1 between (3 $\{3\}3$ ,1), then the form will be  $3\{3\}3\{0\}1$

\*\*\*\*\* If there are at least 2 0's between (3 $\{3\}3$ ,1), then 33001 will be a subsequence.

\*\*\*\*\* If there are exactly 1 0's between (3 $\{3\}3$ ,1), then the form is  $3\{3\}301$

\*\*\*\*\* The smallest prime of the form  $3\{3\}301$  is **33333301**

\*\*\*\*\* If there are no 0's between  $(3\{3\}3,1)$ , then the form is  $3\{3\}31$

\*\*\*\*\* The smallest prime of the form  $3\{3\}31$  is **33333333333333331**

\*\*\*\* For the  $3\{0,1,3\}5\{0,1,3\}1$  family, since 335 is prime, we only need to consider the family  $3\{0,1\}5\{0,1,3\}1$

\*\*\*\* Numbers containing 3 between  $(3\{0,1\}5,1)$ :

\*\*\*\*\* The form is  $3\{0,1\}5\{0,1,3\}3\{0,1,3\}1$

\*\*\*\*\* Since 3031 and 131 are primes, we only need to consider the family  $35\{3\}3\{0,1,3\}1$  (since any digits combo 03, 13 between  $(3,1)$  will produce smaller primes)

\*\*\*\*\* Since 533 is prime, we only need to consider the family  $353\{0,1\}1$  (since any digits combo 33 between  $(35,1)$  will produce smaller primes)

\*\*\*\*\* Since 5011 is prime, we only need to consider the family  $353\{1\}\{0\}1$  (since any digits combo 01 between  $(353,1)$  will produce smaller primes)

\*\*\*\*\* If there are at least 3 1's between  $(353,\{0\}1)$ , then 31111 will be a subsequence.

\*\*\*\*\* If there are exactly 2 1's between  $(353,\{0\}1)$ , then the digit sum is 20, thus the number is divisible by 2 and cannot be prime.

\*\*\*\*\* If there are exactly 1 1's between  $(353,\{0\}1)$ , then the form is  $3531\{0\}1$

\*\*\*\*\* The smallest prime of the form  $3531\{0\}1$  is 3531001, but it is not minimal prime since 31001 is prime.

\*\*\*\*\* If there are no 1's between  $(353,\{0\}1)$ , then the digit sum is 15, thus the number is divisible by 3 and cannot be prime.

\*\*\*\* Numbers not containing 3 between  $(3\{0,1\}5,1)$ :

\*\*\*\*\* The form is  $3\{0,1\}5\{0,1\}1$

\*\*\*\*\* If there are  $\geq 2$  0's and  $\geq 1$  1's between  $(3,1)$ , then at least one of 30011, 30101, 31001 will be a subsequence.

\*\*\*\*\* If there are no 1's between  $(3,1)$ , then the form will be  $3\{0\}5\{0\}1$

\*\*\*\*\* All numbers of the form  $3\{0\}5\{0\}1$  are divisible by 3, thus cannot be prime.

\*\*\*\*\* If there are no 0's between  $(3,1)$ , then the form will be  $3\{1\}5\{1\}1$

\*\*\*\*\* If there are  $\geq 3$  1's between  $(3,1)$ , then 31111 will be a subsequence.

\*\*\*\*\* If there are exactly 2 1's between  $(3,1)$ , then the number can only be 31151, 31511, 35111

\*\*\*\*\* None of 31151, 31511, 35111 are primes.

\*\*\*\*\* If there are exactly 1 1's between (3,1), then the digit sum is 13, thus the number is divisible by 2 and cannot be prime.

\*\*\*\*\* If there are no 1's between (3,1), then the number is 351

\*\*\*\*\* 351 is not prime.

\*\*\*\*\* If there are exactly 1 0's between (3,1), then the form will be  $3\{1\}0\{1\}5\{1\}1$  or  $3\{1\}5\{1\}0\{1\}1$

\*\*\*\*\* No matter  $3\{1\}0\{1\}5\{1\}1$  or  $3\{1\}5\{1\}0\{1\}1$ , if there are  $\geq 3$  1's between (3,1), then 31111 will be a subsequence.

\*\*\*\*\* If there are exactly 2 1's between (3,1), then the number can only be 311051, 310151, 310511, 301151, 301511, 305111, 311501, 315101, 315011, 351101, 351011, 350111

\*\*\*\*\* Of these numbers, 311051, 301151, 311501, 351101, 350111 are primes.

\*\*\*\*\* However, 311051, 301151, 311501 have 115 as subsequence, and 350111 has 5011 as subsequence, thus only **351101** is minimal prime.

\*\*\*\*\* No matter  $3\{1\}0\{1\}5\{1\}1$  or  $3\{1\}5\{1\}0\{1\}1$ , if there are exactly 1 1's between (3,1), then the digit sum is 13, thus the number is divisible by 2 and cannot be prime.

\*\*\*\*\* If there are no 1's between (3,1), then the number is 3051 for  $3\{1\}0\{1\}5\{1\}1$  or 3501 for  $3\{1\}5\{1\}0\{1\}1$

\*\*\*\*\* Neither 3051 nor 3501 is prime.

\* Case (3,2):

\*\* **32** is prime, and thus the only minimal prime in this family.

\* Case (3,3):

\*\* Since 32, 23, 43, **313** are primes, we only need to consider the family  $3\{0,3,5,6\}3$  (since any digits 1, 2, 4 between them will produce smaller primes)

\*\*\* If there are  $\geq 2$  5's in {}, then 553 will be a subsequence.

\*\*\* If there are no 5's in {}, then the family will be  $3\{0,3,6\}3$

\*\*\*\* All numbers of the form  $3\{0,3,6\}3$  are divisible by 3, thus cannot be prime.

\*\*\* If there are exactly 1 5's in {}, then the family will be  $3\{0,3,6\}5\{0,3,6\}3$

\*\*\*\* Since 335, 65, **3503**, 533, 56 are primes, we only need to consider the family  $3\{0\}53$  (since any digit 3, 6 between  $(3,5\{0,3,6\}3)$  and any digit 0, 3, 6 between  $(3\{0,3,6\}5,3)$  will produce smaller primes)

\*\*\*\* The smallest prime of the form  $3\{0\}53$  is **300053**



\*\* Since 32, 16, 56, **346** are primes, we only need to consider the family  $3\{0,3,6\}6$  (since any digits 1, 2, 4, 5 between them will produce smaller primes)

\*\*\* All numbers of the form  $3\{0,3,6\}6$  are divisible by 3, thus cannot be prime.

\* Case (4,1):

\*\* **41** is prime, and thus the only minimal prime in this family.

\* Case (4,2):

\*\* Since 41, 43, 32, 52 are primes, we only need to consider the family  $4\{0,2,4,6\}2$  (since any digits 1, 3, 5 between them will produce smaller primes)

\*\*\* All numbers of the form  $4\{0,2,4,6\}2$  are divisible by 2, thus cannot be prime.

\* Case (4,3):

\*\* **43** is prime, and thus the only minimal prime in this family.

\* Case (4,4):

\*\* Since 41, 43, 14 are primes, we only need to consider the family  $4\{0,2,4,5,6\}4$  (since any digits 1, 3 between them will produce smaller primes)

\*\*\* If there is no 5's in {}, then the family will be  $4\{0,2,4,6\}4$

\*\*\*\* All numbers of the form  $4\{0,2,4,6\}4$  are divisible by 2, thus cannot be prime.

\*\*\* If there is at least one 5's in {}, then there cannot be 2 in {} (since if so, then either 25 or 52 will be a subsequence) and there cannot be 6 in {} (since if so, then either 65 or 56 will be a subsequence), thus the family is  $4\{0,4,5\}5\{0,4,5\}4$

\*\*\*\* Since 445, **4504**, 544 are primes, we only need to consider the family  $4\{0,5\}5\{5\}4$  (since any digit 4 between  $(4,5\{0,4,5\}4)$  and any digit 0, 4 between  $(4\{0,4,5\}5,4)$  will produce smaller primes)

\*\*\*\*\* If there are at least two 0's between  $(4,5\{0,4,5\}4)$ , then **40054** will be a subsequence.

\*\*\*\*\* If there is no 0's between  $(4,5\{0,4,5\}4)$ , then the family will be  $4\{5\}5\{5\}4$ , which is equivalent to  $4\{5\}4$

\*\*\*\*\* The smallest prime of the form  $4\{5\}4$  is 45555555555555554 (not minimal prime, since 4555 and 5554 are primes)

\*\*\*\*\* If there is exactly one 0's between  $(4,5\{0,4,5\}4)$ , then the family will be  $4\{5\}0\{5\}5\{5\}4$

\*\*\*\*\* Since 4504 is prime, we only need to consider the family  $40\{5\}5\{5\}4$  (since any digit 5 between  $(4,0\{5\}5\{5\}4)$  will produce small primes), which is equivalent to  $40\{5\}4$

\*\*\*\*\* The smallest prime of the form  $40\{5\}4$  is 40555555555555554 (not minimal prime, since 4555 and 5554 are primes)

\* Case (4,5):

\*\* Since 41, 43, 25, 65, **445** are primes, we only need to consider the family  $4\{0,5\}5$  (since any digits 1, 2, 3, 4, 6 between them will produce smaller primes)

\*\*\* If there are at least two 5's in {}, then **4555** will be a subsequence.

\*\*\* If there is exactly one 5's in {}, then the digit sum is 20, and the number will be divisible by 2 and cannot be prime.

\*\*\* If there is no 5's in {}, then the family will be  $4\{0\}5$

\*\*\*\* All numbers of the form  $4\{0\}5$  are divisible by 3, thus cannot be prime.

\* Case (4,6):

\*\* Since 41, 43, 16, 56 are primes, we only need to consider the family  $4\{0,2,4,6\}6$  (since any digits 1, 3, 5 between them will produce smaller primes)

\*\*\* All numbers of the form  $4\{0,2,4,6\}6$  are divisible by 2, thus cannot be prime.

\* Case (5,1):

\*\* Since 52, 56, 41, 61, **551** are primes, we only need to consider the family  $5\{0,1,3\}1$  (since any digits 2, 4, 5, 6 between them will produce smaller primes)

\*\*\* If there are at least two 3's in {}, then 533 will be a subsequence.

\*\*\* If there is no 3's in {}, then the family will be  $5\{0,1\}1$

\*\*\*\* Since **5011** is prime, we only need to consider the family  $5\{1\}\{0\}1$

\*\*\*\*\* Since 11111 is prime, we only need to consider the families  $5\{0\}1$ ,  $51\{0\}1$ ,  $511\{0\}1$ ,  $5111\{0\}1$  (since any digits combo 1111 between (5,1) will produce small primes)

\*\*\*\*\* All numbers of the form  $5\{0\}1$  are divisible by 6, thus cannot be prime.

\*\*\*\*\* The smallest prime of the form  $51\{0\}1$  is **510000001**

\*\*\*\*\* All numbers of the form  $511\{0\}1$  are divisible by 2, thus cannot be prime.

\*\*\*\*\* All numbers of the form  $5111\{0\}1$  are divisible by 3, thus cannot be prime.

\*\*\* If there is exactly one 3's in {}, then the family will be  $5\{0,1\}3\{0,1\}1$

\*\*\*\* If there is at least one 1's between  $(5,3\{0,1\}1)$ , then 131 will be a subsequence.

\*\*\*\*\* Thus we only need to consider the family  $5\{0\}3\{0,1\}1$

\*\*\*\*\* If there are no 1's between  $(5\{0\}3,1)$ , then the digit sum is 12, and the number will be divisible by 3 and cannot be prime.

\*\*\*\*\* If there are exactly one 1's between  $(5\{0\}3,1)$ , then the digit sum is 13, and the number will be divisible by 2 and cannot be prime.

\*\*\*\*\* If there are exactly three 1's between  $(5\{0\}3,1)$ , then the digit sum is 15, and the number will be divisible by 6 and cannot be prime.

\*\*\*\*\* If there are at least four 1's between  $(5\{0\}3,1)$ , then 11111 will be a subsequence.

\*\*\*\*\* If there are exactly two 1's between  $(5\{0\}3,1)$ , then the family will be  $5\{0\}3\{0\}1\{0\}1\{0\}1$

\*\*\*\*\* Since 5011 is prime, we only need to consider the family  $5311\{0\}1$  (since any digit 0 between  $(5,1\{0\}1)$  will produce small primes, this includes the leftmost three  $\{ \}$  in  $5\{0\}3\{0\}1\{0\}1\{0\}1$ , and thus only the rightmost  $\{ \}$  can contain 0)

\*\*\*\*\* The smallest prime of the form  $5311\{0\}1$  is **531101**

\* Case (5,2):

\*\* **52** is prime, and thus the only minimal prime in this family.

\* Case (5,3):

\*\* Since 52, 56, 23, 43, **533**, **553** are primes, we only need to consider the family  $5\{0,1\}3$  (since any digits 2, 3, 4, 5, 6 between them will produce smaller primes)

\*\*\* If there are at least two 1's in  $\{ \}$ , then 113 will be a subsequence.

\*\*\* If there is exactly one 1's in  $\{ \}$ , then the digit sum is 12, and the number will be divisible by 3 and cannot be prime.

\*\*\* If there is no 1's in  $\{ \}$ , then the digit sum is 11, and the number will be divisible by 2 and cannot be prime.

\* Case (5,4):

\*\* Since 52, 56, 14, **544** are primes, we only need to consider the family  $5\{0,3,5\}4$  (since any digits 1, 2, 4, 6 between them will produce smaller primes)

\*\*\* If there are no 5's in  $\{ \}$ , then the family will be  $5\{0,3\}4$

\*\*\*\* All numbers of the form  $5\{0,3\}4$  are divisible by 3, thus cannot be prime.

\*\*\* If there are at least one 5's and at least one 3's in  $\{ \}$ , then either 535 or 553 will be a subsequence.

\*\*\* If there are exactly one 5's and no 3's in  $\{ \}$ , then the digit sum is 20, and the number will be divisible by 2 and cannot be prime.

\*\*\* If there are at least two 5's in  $\{ \}$ , then **5554** will be a subsequence.

\* Case (5,5):



\*\* Since 52, 56, 25, 65, **515**, **535** are primes, we only need to consider the family  $5\{0,4,5\}5$  (since any digits 1, 2, 3, 6 between them will produce smaller primes)

\*\*\* If there are no 4's in {}, then the family will be  $5\{0,5\}5$

\*\*\*\* All numbers of the form  $5\{0,5\}5$  are divisible by 5, thus cannot be prime.

\*\*\* If there are no 5's in {}, then the family will be  $5\{0,4\}5$

\*\*\*\* All numbers of the form  $5\{0,4\}5$  are divisible by 2, thus cannot be prime.

\*\*\* If there are at least one 4's and at least one 5's in {}, then either **5455** or **5545** will be a subsequence.

\* Case (5,6):

\*\* **56** is prime, and thus the only minimal prime in this family.

\* Case (6,1):

\*\* **61** is prime, and thus the only minimal prime in this family.

\* Case (6,2):

\*\* Since 61, 65, 32, 52 are primes, we only need to consider the family  $6\{0,2,4,6\}2$  (since any digits 1, 3, 5 between them will produce smaller primes)

\*\*\* All numbers of the form  $6\{0,2,4,6\}2$  are divisible by 2, thus cannot be prime.

\* Case (6,3):

\*\* Since 61, 65, 23, 43 are primes, we only need to consider the family  $6\{0,3,6\}3$  (since any digits 1, 2, 4, 5 between them will produce smaller primes)

\*\*\* All numbers of the form  $6\{0,3,6\}3$  are divisible by 3, thus cannot be prime.

\* Case (6,4):

\*\* Since 61, 65, 14 are primes, we only need to consider the family  $6\{0,2,3,4,6\}4$  (since any digits 1, 5 between them will produce smaller primes)

\*\*\* If there is no 3's in {}, then the family will be  $6\{0,2,4,6\}4$

\*\*\*\* All numbers of the form  $6\{0,2,4,6\}4$  are divisible by 2, thus cannot be prime.

\*\*\* If there are exactly two 3's in {}, then the family will be  $6\{0,2,4,6\}3\{0,2,4,6\}3\{0,2,4,6\}4$

\*\*\*\* All numbers of the form  $6\{0,2,4,6\}3\{0,2,4,6\}3\{0,2,4,6\}4$  are divisible by 2, thus cannot be prime.

\*\*\* If there are at least three 3's in {}, then 3334 will be a subsequence.

\*\*\* If there is exactly one 3's in {}, then the family will be  $6\{0,2,4,6\}3\{0,2,4,6\}4$

\*\*\*\* If there is 0 between  $(6,3\{0,2,4,6\}4)$ , then **6034** will be a subsequence.

\*\*\*\* If there is 2 between  $(6,3\{0,2,4,6\}4)$ , then 23 will be a subsequence.

\*\*\*\* If there is 4 between  $(6,3\{0,2,4,6\}4)$ , then 43 will be a subsequence.

\*\*\*\* If there is 6 between  $(6,3\{0,2,4,6\}4)$ , then **6634** will be a subsequence.

\*\*\*\* If there is 0 between  $(6\{0,2,4,6\}3,4)$ , then 304 will be a subsequence.

\*\*\*\* If there is 2 between  $(6\{0,2,4,6\}3,4)$ , then 32 will be a subsequence.

\*\*\*\* If there is 4 between  $(6\{0,2,4,6\}3,4)$ , then 344 will be a subsequence.

\*\*\*\* If there is 6 between  $(6\{0,2,4,6\}3,4)$ , then 364 will be a subsequence.

\*\*\*\* Thus the number can only be 634

\*\*\*\*\* 634 is not prime.

\* Case (6,5):

\*\* **65** is prime, and thus the only minimal prime in this family.

\* Case (6,6):

\*\* Since 61, 65, 16, 56 are primes, we only need to consider the family  $6\{0,2,3,4,6\}6$  (since any digits 1, 5 between them will produce smaller primes)

\*\*\* If there is no 3's in {}, then the family will be  $6\{0,2,4,6\}6$

\*\*\*\* All numbers of the form  $6\{0,2,4,6\}6$  are divisible by 2, thus cannot be prime.

\*\*\* If there is no 2's and no 4's in {}, then the family will be  $6\{0,3,6\}6$

\*\*\*\* All numbers of the form  $6\{0,3,6\}6$  are divisible by 3, thus cannot be prime.

\*\*\* If there is at least one 3's and at least one 2's in {}, then either 32 or 23 will be a subsequence.

\*\*\* If there is at least one 3's and at least one 4's in {}, then either 346 or 43 will be a subsequence.

## base 8

The possible (first digit,last digit) for an element with  $\geq 3$  digits in the minimal set of the strings for primes with at least two digits are:

(1,1), (1,3), (1,5), (1,7), (2,1), (2,3), (2,5), (2,7), (3,1), (3,3), (3,5), (3,7), (4,1), (4,3), (4,5), (4,7), (5,1), (5,3), (5,5), (5,7), (6,1), (6,3), (6,5), (6,7), (7,1), (7,3), (7,5), (7,7)

\* Case (1,1):

\*\* Since 13, 15, 21, 51, **111**, **141**, **161** are primes, we only need to consider the family  $1\{0,7\}1$  (since any digits 1, 2, 3, 4, 5, 6 between them will produce smaller primes)

\*\*\* Since 107, 177, 701 are primes, we only need to consider the number 171 and the family  $1\{0\}1$  (since any digits combo 07, 70, 77 between them will produce smaller primes)

\*\*\*\* 171 is not prime.

\*\*\*\* All numbers of the form  $1\{0\}1$  factored as  $10^{n+1} = (2^{n+1}) * (4^n - 2^{n+1})$ , thus cannot be prime.

\* Case (1,3):

\*\* **13** is prime, and thus the only minimal prime in this family.

\* Case (1,5):

\*\* **15** is prime, and thus the only minimal prime in this family.

\* Case (1,7):

\*\* Since 13, 15, 27, 37, 57, **107**, **117**, **147**, **177** are primes, we only need to consider the family  $1\{6\}7$  (since any digits 0, 1, 2, 3, 4, 5, 7 between them will produce smaller primes)

\*\*\* The smallest prime of the form  $1\{6\}7$  is 16667 (not minimal prime, since 667 is prime)

\* Case (2,1):

\*\* **21** is prime, and thus the only minimal prime in this family.

\* Case (2,3):

\*\* **23** is prime, and thus the only minimal prime in this family.

\* Case (2,5):

\*\* Since 21, 23, 27, 15, 35, 45, 65, 75, **225**, **255** are primes, we only need to consider the family  $2\{0\}5$  (since any digits 1, 2, 3, 4, 5, 6, 7 between them will produce smaller primes)

\*\*\* All numbers of the form  $2\{0\}5$  are divisible by 7, thus cannot be prime.

\* Case (2,7):

\*\* **27** is prime, and thus the only minimal prime in this family.

\* Case (3,1):

\*\* Since 35, 37, 21, 51, **301**, **361** are primes, we only need to consider the family  $3\{1,3,4\}1$  (since any digits 0, 2, 5, 6, 7 between them will produce smaller primes)

\*\*\* Since 13, 343, 111, 131, 141, 431, **3331**, **3411** are primes, we only need to consider the families  $3\{3\}11$ ,  $33\{1,4\}1$ ,  $3\{3,4\}4\{4\}1$  (since any digits combo 11, 13, 14, 33, 41, 43 between them will produce smaller primes)

\*\*\*\* All numbers of the form  $3\{3\}11$  are divisible by 3, thus cannot be prime.

\*\*\*\* For the  $33\{1,4\}1$  family, since 111 and 141 are primes, we only need to consider the families  $33\{4\}1$  and  $33\{4\}11$  (since any digits combo 11, 14 between them will produce smaller primes)

\*\*\*\*\* The smallest prime of the form  $33\{4\}1$  is **3344441**

\*\*\*\*\* All numbers of the form  $33\{4\}11$  are divisible by 301, thus cannot be prime.

\*\*\*\* For the  $3\{3,4\}4\{4\}1$  family, since 3331 and 3344441 are primes, we only need to consider the families  $3\{4\}1$ ,  $3\{4\}31$ ,  $3\{4\}341$ ,  $3\{4\}3441$ ,  $3\{4\}34441$  (since any digits combo 33 or 34444 between (3,1) will produce smaller primes)

\*\*\*\*\* All numbers of the form  $3\{4\}1$  are divisible by 31, thus cannot be prime.

\*\*\*\*\* Since 4443 is prime, we only need to consider the numbers 3431, 34431, 34341, 344341, 343441, 3443441, 3434441, 34434441 (since any digit combo 444 between (3,3{4}1) will produce smaller primes)

\*\*\*\*\* None of 3431, 34431, 34341, 344341, 343441, 3443441, 3434441, 34434441 are primes.

\* Case (3,3):

\*\* Since 35, 37, 13, 23, 53, 73, **343** are primes, we only need to consider the family  $3\{0,3,6\}3$  (since any digits 1, 2, 4, 5, 7 between them will produce smaller primes)

\*\*\* All numbers of the form  $3\{0,3,6\}3$  are divisible by 3, thus cannot be prime.

\* Case (3,5):

\*\* **35** is prime, and thus the only minimal prime in this family.

\* Case (3,7):

\*\* **37** is prime, and thus the only minimal prime in this family.

\* Case (4,1):

\*\* Since 45, 21, 51, **401**, **431**, **471** are primes, we only need to consider the family  $4\{1,4,6\}1$  (since any digits 0, 2, 3, 5, 7 between them will produce smaller primes)

\*\*\* Since 111, 141, 161, 661, **4611** are primes, we only need to consider the families  $4\{4\}11$ ,  $4\{4,6\}4\{1,4,6\}1$ ,  $4\{4\}6\{4\}1$  (since any digits combo 11, 14, 16, 61, 66 between them will produce smaller primes)

\*\*\*\* The smallest prime of the form  $4\{4\}11$  is 44444444444444411 (not minimal prime, since 444444441 is prime)

\*\*\*\* For the  $4\{4,6\}4\{1,4,6\}1$  family, we can separate this family to  $4\{4,6\}41$ ,  $4\{4,6\}411$ ,  $4\{4,6\}461$

\*\*\*\* For the  $4\{4,6\}41$  family, since 661 and 6441 are primes, we only need to consider the families  $4\{4\}41$  and  $4\{4\}641$  (since any digits combo 64 or 66 between (4,41) will produce smaller primes)

\*\*\*\*\* The smallest prime of the form  $4\{4\}41$  is **444444441**

\*\*\*\*\* The smallest prime of the form  $4\{4\}641$  is **444641**

\*\*\*\* For the  $4\{4,6\}411$  family, since 661 and 6441 are primes, we only need to consider the families  $4\{4\}411$  and  $4\{4\}6411$  (since any digits combo 64 or 66 between (4,411) will produce smaller primes)

\*\*\*\*\* The smallest prime of the form  $4\{4\}411$  is **444444441**

\*\*\*\*\* The smallest prime of the form  $4\{4\}6411$  is 44444444444444446411 (not minimal prime, since 444444441 and 444641 are primes)

\*\*\*\* For the  $4\{4,6\}461$  family, since 661 is prime, we only need to consider the family  $4\{4\}461$

\*\*\*\*\* The smallest prime of the form  $4\{4\}461$  is 44444444461 (not minimal prime, since 444444441 is prime)

\*\*\*\* For the  $4\{4\}6\{4\}1$  family, since 6441 is prime, we only need to consider the families  $4\{4\}61$  and  $4\{4\}641$  (since any digits combo 44 between (4{4}6,1) will produce smaller primes)

\*\*\*\*\* The smallest prime of the form  $4\{4\}61$  is 44444444461 (not minimal prime, since 444444441 is prime)

\*\*\*\*\* The smallest prime of the form  $4\{4\}641$  is **444641**

\* Case (4,3):

\*\* Since 45, 13, 23, 53, 73, **433**, **463** are primes, we only need to consider the family  $4\{0,4\}3$  (since any digits 1, 2, 3, 5, 6, 7 between them will produce smaller primes)

\*\*\* Since **4043** and **4443** are primes, we only need to consider the families  $4\{0\}3$  and  $44\{0\}3$  (since any digits combo 04, 44 between them will produce smaller primes)

\*\*\*\* All numbers of the form  $4\{0\}3$  are divisible by 7, thus cannot be prime.

\*\*\*\* All numbers of the form  $44\{0\}3$  are divisible by 3, thus cannot be prime.

\* Case (4,5):

\*\* **45** is prime, and thus the only minimal prime in this family.

\* Case (4,7):

\*\* Since 45, 27, 37, 57, **407**, **417**, **467** are primes, we only need to consider the family  $4\{4,7\}7$  (since any digits 0, 1, 2, 3, 5, 6 between them will produce smaller primes)

\*\*\* Since 747 is prime, we only need to consider the families  $4\{4\}7$ ,  $4\{4\}77$ ,  $4\{7\}7$ ,  $44\{7\}7$  (since any digits combo 74 between (4,7) will produce smaller primes)





\*\*\*\*\* For the  $6\{0,7\}1\{0,7\}1$  family, since 711 and **6101** are primes, we only need to consider the family  $6\{0\}1\{7\}1$

\*\*\*\*\* Since **60171** is prime, we only need to consider the families  $6\{0\}11$  and  $61\{7\}1$

\*\*\*\*\* All numbers of the form  $6\{0\}11$  are divisible by 3, thus cannot be prime.

\*\*\*\*\* The smallest prime of the form  $61\{7\}1$  is 617771 (not minimal prime, since 6777 is prime)

\* Case (6,3):

\*\* Since 65, 13, 23, 53, 73, **643** are primes, we only need to consider the family  $6\{0,3,6\}3$  (since any digits 1, 2, 4, 5, 7 between them will produce smaller primes)

\*\*\* All numbers of the form  $6\{0,3,6\}3$  are divisible by 3, thus cannot be prime.

\* Case (6,5):

\*\* **65** is prime, and thus the only minimal prime in this family.

\* Case (6,7):

\*\* Since 65, 27, 37, 57, **667** are primes, we only need to consider the family  $6\{0,1,4,7\}7$  (since any digits 2, 3, 5, 6 between them will produce smaller primes)

\*\*\* Since 107, 117, 147, 177, 407, 417, 717, 747, **6007**, **6477**, **6707**, **6777** are primes, we only need to consider the families  $60\{1,4,7\}7$ ,  $6\{0\}17$ ,  $6\{0,4\}4\{4\}7$ ,  $6\{0\}77$  (since any digits combo 00, 10, 11, 14, 17, 40, 41, 47, 70, 71, 74, 77 between them will produce smaller primes)

\*\*\*\* All numbers of the form  $6\{0\}17$  or  $6\{0\}77$  are divisible by 3, thus cannot be prime.

\*\*\*\* For the  $60\{1,4,7\}7$  family, since 117, 147, 177, 417, 6477, 717, 747, 6777 are primes, we only need to consider the numbers 6017, 6047, 6077 and the family  $60\{4\}7$  (since any digit combo 11, 14, 17, 41, 47, 71, 74, 77 between (60,7) will produce smaller primes)

\*\*\*\*\* None of 6017, 6047, 6077 are primes.

\*\*\*\* All numbers of the form  $60\{4\}7$  are divisible by 21, thus cannot be prime.

\*\*\*\* For the  $6\{0,4\}4\{4\}7$  family, since 6007 and 407 are primes, we only need to consider the families  $6\{4\}7$  and  $60\{4\}7$  (since any digit combo 00, 40 between (6,4{4}7) will produce smaller primes)

\*\*\*\*\* All numbers of the form  $6\{4\}7$  are divisible by 3, 5, or 15, thus cannot be prime.

\*\*\*\*\* All numbers of the form  $60\{4\}7$  are divisible by 21, thus cannot be prime.

\* Case (7,1):

\*\* Since 73, 75, 21, 51, **701**, **711** are primes, we only need to consider the family  $7\{4,6,7\}1$  (since any digits 0, 1, 2, 3, 5 between them will produce smaller primes)





\* Case (7,7):

\*\* Since 73, 75, 27, 37, 57, **717**, **747**, **767** are primes, we only need to consider the family  $7\{0,7\}7$  (since any digits 1, 2, 3, 4, 5, 6 between them will produce smaller primes)

\*\*\* All numbers of the form  $7\{0,7\}7$  are divisible by 7, thus cannot be prime.

## base 10

The possible (first digit,last digit) for an element with  $\geq 3$  digits in the minimal set of the strings for primes with at least two digits are:

(1,1), (1,3), (1,7), (1,9), (2,1), (2,3), (2,7), (2,9), (3,1), (3,3), (3,7), (3,9), (4,1), (4,3), (4,7), (4,9), (5,1), (5,3), (5,7), (5,9), (6,1), (6,3), (6,7), (6,9), (7,1), (7,3), (7,7), (7,9), (8,1), (8,3), (8,7), (8,9), (9,1), (9,3), (9,7), (9,9)

\* Case (1,1):

\*\* **11** is prime, and thus the only minimal prime in this family.

\* Case (1,3):

\*\* **13** is prime, and thus the only minimal prime in this family.

\* Case (1,7):

\*\* **17** is prime, and thus the only minimal prime in this family.

\* Case (1,9):

\*\* **19** is prime, and thus the only minimal prime in this family.

\* Case (2,1):

\*\* Since 23, 29, 11, 31, 41, 61, 71, **251**, **281** are primes, we only need to consider the family  $2\{0,2\}1$  (since any digits 1, 3, 4, 5, 6, 7, 8, 9 between them will produce smaller primes)

\*\*\* Since **2221** and **20201** are primes, we only need to consider the families  $2\{0\}1$ ,  $2\{0\}21$ ,  $22\{0\}1$  (since any digits combo 22 or 020 between them will produce smaller primes)

\*\*\*\* All numbers of the form  $2\{0\}1$  are divisible by 3, thus cannot be prime.

\*\*\*\* The smallest prime of the form  $2\{0\}21$  is **20021**

\*\*\*\* The smallest prime of the form  $22\{0\}1$  is **22000001**

\* Case (2,3):

\*\* **23** is prime, and thus the only minimal prime in this family.

\* Case (2,7):

\*\* Since 23, 29, 17, 37, 47, 67, 97, **227**, **257**, **277** are primes, we only need to consider the family  $2\{0,8\}7$  (since any digits 1, 2, 3, 4, 5, 6, 7, 9 between them will produce smaller primes)

\*\*\* Since 887 and **2087** are primes, we only need to consider the families  $2\{0\}7$  and  $28\{0\}7$  (since any digit combo 08 or 88 between them will produce smaller primes)

\*\*\*\* All numbers of the form  $2\{0\}7$  are divisible by 3, thus cannot be prime.

\*\*\*\* All numbers of the form  $28\{0\}7$  are divisible by 7, thus cannot be prime.

\* Case (2,9):

\*\* **29** is prime, and thus the only minimal prime in this family.

\* Case (3,1):

\*\* **31** is prime, and thus the only minimal prime in this family.

\* Case (3,3):

\*\* Since 31, 37, 13, 23, 43, 53, 73, 83 are primes, we only need to consider the family  $3\{0,3,6,9\}3$  (since any digits 1, 2, 4, 5, 7, 8 between them will produce smaller primes)

\*\*\* All numbers of the form  $3\{0,3,6,9\}3$  are divisible by 3, thus cannot be prime.

\* Case (3,7):

\*\* **37** is prime, and thus the only minimal prime in this family.

\* Case (3,9):

\*\* Since 31, 37, 19, 29, 59, 79, 89, **349** are primes, we only need to consider the family  $3\{0,3,6,9\}9$  (since any digits 1, 2, 4, 5, 7, 8 between them will produce smaller primes)

\*\*\* All numbers of the form  $3\{0,3,6,9\}9$  are divisible by 3, thus cannot be prime.

\* Case (4,1):

\*\* **41** is prime, and thus the only minimal prime in this family.

\* Case (4,3):

\*\* **43** is prime, and thus the only minimal prime in this family.

\* Case (4,7):

\*\* **47** is prime, and thus the only minimal prime in this family.

\* Case (4,9):

\*\* Since 41, 43, 47, 19, 29, 59, 79, 89, **409, 449, 499** are primes, we only need to consider the family  $4\{6\}9$  (since any digits 0, 1, 2, 3, 4, 5, 7, 8, 9 between them will produce smaller primes)

\*\*\* All numbers of the form  $4\{6\}9$  are divisible by 7, thus cannot be prime.

\* Case (5,1):

\*\* Since 53, 59, 11, 31, 41, 61, 71, **521** are primes, we only need to consider the family  $5\{0,5,8\}1$  (since any digits 1, 2, 3, 4, 6, 7, 9 between them will produce smaller primes)

\*\*\* Since 881 is prime, we only need to consider the families  $5\{0,5\}1$  and  $5\{0,5\}8\{0,5\}1$  (since any digit combo 88 between them will produce smaller primes)

\*\*\*\* For the  $5\{0,5\}1$  family, since **5051** and **5501** are primes, we only need to consider the families  $5\{0\}1$  and  $5\{5\}1$  (since any digit combo 05 or 50 between them will produce smaller primes)

\*\*\*\*\* All numbers of the form  $5\{0\}1$  are divisible by 3, thus cannot be prime.

\*\*\*\*\* The smallest prime of the form  $5\{5\}1$  is **555555555551**

\*\*\*\* For the  $5\{0,5\}8\{0,5\}1$  family, since **5081, 5581, 5801, 5851** are primes, we only need to consider the number 581

\*\*\*\*\* 581 is not prime.

\* Case (5,3):

\*\* **53** is prime, and thus the only minimal prime in this family.

\* Case (5,7):

\*\* Since 53, 59, 17, 37, 47, 67, 97, **557, 577, 587** are primes, we only need to consider the family  $5\{0,2\}7$  (since any digits 1, 3, 4, 5, 6, 7, 8, 9 between them will produce smaller primes)

\*\*\* Since 227 and **50207** are primes, we only need to consider the families  $5\{0\}7$ ,  $5\{0\}27$ ,  $52\{0\}7$  (since any digits combo 22 or 020 between them will produce smaller primes)

\*\*\*\* All numbers of the form  $5\{0\}7$  are divisible by 3, thus cannot be prime.

\*\*\*\* The smallest prime of the form  $5\{0\}27$  is **5000000000000000000000000000027**

\*\*\*\* The smallest prime of the form  $52\{0\}7$  is **5200007**

\* Case (5,9):

\*\* **59** is prime, and thus the only minimal prime in this family.

\* Case (6,1):

\*\* **61** is prime, and thus the only minimal prime in this family.

\* Case (6,3):

\*\* Since 61, 67, 13, 23, 43, 53, 73, 83 are primes, we only need to consider the family  $6\{0,3,6,9\}3$  (since any digits 1, 2, 4, 5, 7, 8 between them will produce smaller primes)

\*\*\* All numbers of the form  $6\{0,3,6,9\}3$  are divisible by 3, thus cannot be prime.

\* Case (6,7):

\*\* **67** is prime, and thus the only minimal prime in this family.

\* Case (6,9):

\*\* Since 61, 67, 19, 29, 59, 79, 89 are primes, we only need to consider the family  $6\{0,3,4,6,9\}9$  (since any digits 1, 2, 5, 7, 8 between them will produce smaller primes)

\*\*\* Since 449 is prime, we only need to consider the families  $6\{0,3,6,9\}9$  and  $6\{0,3,6,9\}4\{0,3,6,9\}9$  (since any digit combo 44 between them will produce smaller primes)

\*\*\*\* All numbers of the form  $6\{0,3,6,9\}9$  are divisible by 3, thus cannot be prime.

\*\*\*\* For the  $6\{0,3,6,9\}4\{0,3,6,9\}9$  family, since 409, 43, **6469**, 499 are primes, we only need to consider the family  $6\{0,3,6,9\}49$

\*\*\*\*\* Since 349, **6949** are primes, we only need to consider the family  $6\{0,6\}49$

\*\*\*\*\* Since **60649** is prime, we only need to consider the family  $6\{6\}\{0\}49$  (since any digits combo 06 between  $\{6,49\}$  will produce smaller primes)

\*\*\*\*\* The smallest prime of the form  $6\{6\}49$  is **666649**

\*\*\*\*\* Since this prime has just 4 6's, we only need to consider the families with  $\leq 3$  6's

\*\*\*\*\* The smallest prime of the form  $6\{0\}49$  is **6000049**

\*\*\*\*\* The smallest prime of the form  $66\{0\}49$  is **6600049**

\*\*\*\*\* The smallest prime of the form  $666\{0\}49$  is **6660049**

\* Case (7,1):

\*\* **71** is prime, and thus the only minimal prime in this family.

\* Case (7,3):

\*\* **73** is prime, and thus the only minimal prime in this family.

\* Case (7,7):

\*\* Since 71, 73, 79, 17, 37, 47, 67, 97, **727**, **757**, **787** are primes, we only need to consider the family  $7\{0,7\}7$  (since any digits 1, 2, 3, 4, 5, 6, 8, 9 between them will produce smaller primes)

\*\*\* All numbers of the form  $7\{0,7\}7$  are divisible by 7, thus cannot be prime.



\*\*\*\*\* 95801 is the only prime among 901, 9021, 9051, 9081, 9201, 9501, 9801, 90581, 95081, 95801, but it is not minimal prime since 5801 is prime.

\*\*\*\* For the  $9\{0\}2\{0\}1$  family, since 9001 is prime, we only need to consider the numbers 921, 9201, 9021

\*\*\*\*\* None of 921, 9201, 9021 are primes.

\*\*\*\* For the  $9\{0\}5\{0,8\}1$  family, since 9001 and 881 are primes, we only need to consider the numbers 951, 9051, 9501, 9581, 90581, 95081, 95801

\*\*\*\*\* 95801 is the only prime among 951, 9051, 9501, 9581, 90581, 95081, 95801, but it is not minimal prime since 5801 is prime.

\*\*\*\* For the  $9\{0,5\}8\{0\}1$  family, since 9001 and 5581 are primes, we only need to consider the numbers 981, 9081, 9581, 9801, 90581, 95081, 95801

\*\*\*\*\* 95801 is the only prime among 981, 9081, 9581, 9801, 90581, 95081, 95801, but it is not minimal prime since 5801 is prime.

\* Case (9,3):

\*\* Since 97, 13, 23, 43, 53, 73, 83 are primes, we only need to consider the family  $9\{0,3,6,9\}3$  (since any digits 1, 2, 4, 5, 7, 8 between them will produce smaller primes)

\*\*\* All numbers of the form  $9\{0,3,6,9\}3$  are divisible by 3, thus cannot be prime.

\* Case (9,7):

\*\* **97** is prime, and thus the only minimal prime in this family.

\* Case (9,9):

\*\* Since 97, 19, 29, 59, 79, 89 are primes, we only need to consider the family  $9\{0,3,4,6,9\}9$  (since any digits 1, 2, 5, 7, 8 between them will produce smaller primes)

\*\*\* Since 449 is prime, we only need to consider the families  $9\{0,3,6,9\}9$  and  $9\{0,3,6,9\}4\{0,3,6,9\}9$  (since any digit combo 44 between them will produce smaller primes)

\*\*\*\* All numbers of the form  $9\{0,3,6,9\}9$  are divisible by 3, thus cannot be prime.

\*\*\*\* For the  $9\{0,3,6,9\}4\{0,3,6,9\}9$  family, since **9049**, 349, **9649**, **9949** are primes, we only need to consider the family  $94\{0,3,6,9\}9$

\*\*\*\*\* Since 409, 43, 499 are primes, we only need to consider the family  $94\{6\}9$  (since any digits 0, 3, 9 between (94,9) will produce smaller primes)

\*\*\*\*\* The smallest prime of the form  $94\{6\}9$  is **946669**

## base 12

The possible (first digit,last digit) for an element with  $\geq 3$  digits in the minimal set of the strings for primes with at least two digits are:

(1,1), (1,5), (1,7), (1,B), (2,1), (2,5), (2,7), (2,B), (3,1), (3,5), (3,7), (3,B), (4,1), (4,5), (4,7), (4,B), (5,1), (5,5), (5,7), (5,B), (6,1), (6,5), (6,7), (6,B), (7,1), (7,5), (7,7), (7,B), (8,1), (8,5), (8,7), (8,B), (9,1), (9,5), (9,7), (9,B), (A,1), (A,5), (A,7), (A,B), (B,1), (B,5), (B,7), (B,B)

\* Case (1,1):

\*\* **11** is prime, and thus the only minimal prime in this family.

\* Case (1,5):

\*\* **15** is prime, and thus the only minimal prime in this family.

\* Case (1,7):

\*\* **17** is prime, and thus the only minimal prime in this family.

\* Case (1,B):

\*\* **1B** is prime, and thus the only minimal prime in this family.

\* Case (2,1):

\*\* Since 25, 27, 11, 31, 51, 61, 81, 91, **221**, **241**, **2A1**, **2B1** are primes, we only need to consider the family  $2\{0\}1$  (since any digits 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B between them will produce smaller primes)

\*\*\* The smallest prime of the form  $2\{0\}1$  is **2001**

\* Case (2,5):

\*\* **25** is prime, and thus the only minimal prime in this family.

\* Case (2,7):

\*\* **27** is prime, and thus the only minimal prime in this family.

\* Case (2,B):

\*\* Since 25, 27, 1B, 3B, 4B, 5B, 6B, 8B, AB, **2BB** are primes, we only need to consider the family  $2\{0,2,9\}B$  (since any digits 1, 3, 4, 5, 6, 7, 8, A, B between them will produce smaller primes)

\*\*\* Since 90B, **200B**, **202B**, **222B**, **229B**, **292B**, **299B** are primes, we only need to consider the numbers 20B, 22B, 29B, 209B, 220B (since any digits combo 00, 02, 22, 29, 90, 92, 99 between them will produce smaller primes)

\*\*\*\* None of 20B, 22B, 29B, 209B, 220B are primes.

\* Case (3,1):





\*\* Since 51, 57, 5B, 15, 25, 35, 45, 75, 85, 95, B5, **565** are primes, we only need to consider the family  $5\{0,5,A\}5$  (since any digits 1, 2, 3, 4, 6, 7, 8, 9, B between them will produce smaller primes)

\*\*\* All numbers of the form  $5\{0,5,A\}5$  are divisible by 5, thus cannot be prime.

\* Case (5,7):

\*\* **57** is prime, and thus the only minimal prime in this family.

\* Case (5,B):

\*\* **5B** is prime, and thus the only minimal prime in this family.

\* Case (6,1):

\*\* **61** is prime, and thus the only minimal prime in this family.

\* Case (6,5):

\*\* Since 61, 67, 6B, 15, 25, 35, 45, 75, 85, 95, B5, **655**, **665** are primes, we only need to consider the family  $6\{0,A\}5$  (since any digits 1, 2, 3, 4, 5, 6, 7, 8, 9, B between them will produce smaller primes)

\*\*\* Since **6A05** and **6AA5** are primes, we only need to consider the families  $6\{0\}5$  and  $6\{0\}A5$  (since any digit combo A0, AA between them will produce smaller primes)

\*\*\*\* All numbers of the form  $6\{0\}5$  are divisible by B, thus cannot be prime.

\*\*\*\* The smallest prime of the form  $6\{0\}A5$  is **600A5**

\* Case (6,7):

\*\* **67** is prime, and thus the only minimal prime in this family.

\* Case (6,B):

\*\* **6B** is prime, and thus the only minimal prime in this family.

\* Case (7,1):

\*\* Since 75, 11, 31, 51, 61, 81, 91, **701**, **721**, **771**, **7A1** are primes, we only need to consider the family  $7\{4,B\}1$  (since any digits 0, 1, 2, 3, 5, 6, 7, 8, 9, A between them will produce smaller primes)

\*\*\* Since **7BB**, **7441** and **7B41** are primes, we only need to consider the numbers 741, 7B1, 74B1

\*\*\*\* None of 741, 7B1, 74B1 are primes.

\* Case (7,5):

\*\* **75** is prime, and thus the only minimal prime in this family.

\* Case (7,7):

\*\* Since 75, 17, 27, 37, 57, 67, 87, A7, B7, **747**, **797** are primes, we only need to consider the family  $7\{0,7\}7$  (since any digits 1, 2, 3, 4, 5, 6, 8, 9, A, B between them will produce smaller primes)

\*\*\* All numbers of the form  $7\{0,7\}7$  are divisible by 7, thus cannot be prime.

\* Case (7,B):

\*\* Since 75, 1B, 3B, 4B, 5B, 6B, 8B, AB, **70B**, **77B**, **7BB** are primes, we only need to consider the family  $7\{2,9\}B$  (since any digits 0, 1, 3, 4, 5, 6, 7, 8, A, B between them will produce smaller primes)

\*\*\* Since 222B, 729B is prime, we only need to consider the families  $7\{9\}B$ ,  $7\{9\}2B$ ,  $7\{9\}22B$  (since any digits combo 222, 29 between them will produce smaller primes)

\*\*\*\* The smallest prime of the form  $7\{9\}B$  is **7999B**

\*\*\*\* The smallest prime of the form  $7\{9\}2B$  is 79992B (not minimal prime, since 992B and 7999B are primes)

\*\*\*\* The smallest prime of the form  $7\{9\}22B$  is 79922B (not minimal prime, since 992B is prime)

\* Case (8,1):

\*\* **81** is prime, and thus the only minimal prime in this family.

\* Case (8,5):

\*\* **85** is prime, and thus the only minimal prime in this family.

\* Case (8,7):

\*\* **87** is prime, and thus the only minimal prime in this family.

\* Case (8,B):

\*\* **8B** is prime, and thus the only minimal prime in this family.

\* Case (9,1):

\*\* **91** is prime, and thus the only minimal prime in this family.

\* Case (9,5):

\*\* **95** is prime, and thus the only minimal prime in this family.

\* Case (9,7):

\*\* Since 91, 95, 17, 27, 37, 57, 67, 87, A7, B7, **907** are primes, we only need to consider the family  $9\{4,7,9\}7$  (since any digit 0, 1, 2, 3, 5, 6, 8, A, B between them will produce smaller primes)

\*\*\* Since 447, 497, 747, 797, **9777**, **9947**, **9997** are primes, we only need to consider the numbers 947, 977, 997, 9477, 9977 (since any digits combo 44, 49, 74, 77, 79, 94, 99 between them will produce smaller primes)

\*\*\*\* None of 947, 977, 997, 9477, 9977 are primes.

\* Case (9,B):

\*\* Since 91, 95, 1B, 3B, 4B, 5B, 6B, 8B, AB, **90B**, **9BB** are primes, we only need to consider the family  $9\{2,7,9\}B$  (since any digit 0, 1, 3, 4, 5, 6, 8, A, B between them will produce smaller primes)

\*\*\* Since 27, 77B, **929B**, **992B**, **997B** are primes, we only need to consider the families  $9\{2,7\}2\{2\}B$ ,  $97\{2,9\}B$ ,  $9\{7,9\}9\{9\}B$  (since any digits combo 27, 29, 77, 92, 97 between them will produce smaller primes)

\*\*\*\* For the  $9\{2,7\}2\{2\}B$  family, since 27 and 77B are primes, we only need to consider the families  $9\{2\}2\{2\}B$  and  $97\{2\}2\{2\}B$  (since any digits combo 27, 77 between (9,2{2}B) will produce smaller primes)

\*\*\*\*\* The smallest prime of the form  $9\{2\}2\{2\}B$  is 9222B (not minimal prime, since 222B is prime)

\*\*\*\*\* The smallest prime of the form  $97\{2\}2\{2\}B$  is 972222222222B (not minimal prime, since 222B is prime)

\*\*\*\* For the  $97\{2,9\}B$  family, since 729B and 929B are primes, we only need to consider the family  $97\{9\}\{2\}B$  (since any digits combo 29 between (97,B) will produce smaller primes)

\*\*\*\*\* Since 222B is prime, we only need to consider the families  $97\{9\}B$ ,  $97\{9\}2B$ ,  $97\{9\}22B$  (since any digit combo 222 between (97,B) will produce smaller primes)

\*\*\*\*\* All numbers of the form  $97\{9\}B$  are divisible by 11, thus cannot be prime.

\*\*\*\*\* The smallest prime of the form  $97\{9\}2B$  is 979999992B (not minimal prime, since 9999B is prime)

\*\*\*\*\* All numbers of the form  $97\{9\}22B$  are divisible by 11, thus cannot be prime.

\*\*\*\* For the  $9\{7,9\}9\{9\}B$  family, since 77B and 9999B are primes, we only need to consider the numbers 99B, 999B, 979B, 9799B, 9979B

\*\*\*\*\* None of 99B, 999B, 979B, 9799B, 9979B are primes.

\* Case (A,1):

\*\* Since A7, AB, 11, 31, 51, 61, 81, 91, **A41** are primes, we only need to consider the family  $A\{0,2,A\}1$  (since any digits 1, 3, 4, 5, 6, 7, 8, 9, B between them will produce smaller primes)

\*\*\* Since 221, 2A1, **A0A1**, **A201** are primes, we only need to consider the families  $A\{A\}\{0\}1$  and  $A\{A\}\{0\}21$  (since any digits combo 0A, 20, 22, 2A between them will produce smaller primes)

\*\*\*\* For the  $A\{A\}\{0\}1$  family:

\*\*\*\*\* All numbers of the form  $A\{0\}1$  are divisible by B, thus cannot be prime.

\*\*\*\*\* The smallest prime of the form  $AA\{0\}1$  is **AA000001**

\*\*\*\*\* The smallest prime of the form  $AAA\{0\}1$  is **AAA0001**

\*\*\*\*\* The smallest prime of the form  $AAAA\{0\}1$  is **AAAA1**

\*\*\*\*\* Since this prime has no 0's, we do not need to consider the families  $\{A\}1$ ,  $\{A\}01$ ,  $\{A\}001$ , etc.

\*\*\*\* All numbers of the form  $A\{A\}\{0\}21$  are divisible by 5, thus cannot be prime.

\* Case (A,5):

\*\* Since  $A7$ ,  $AB$ ,  $15$ ,  $25$ ,  $35$ ,  $45$ ,  $75$ ,  $85$ ,  $95$ ,  $B5$  are primes, we only need to consider the family  $A\{0,5,6,A\}5$  (since any digits  $1, 2, 3, 4, 7, 8, 9, B$  between them will produce smaller primes)

\*\*\* Since  $565$ ,  $655$ ,  $665$ , **A605**, **A6A5**, **AA65** are primes, we only need to consider the families  $A\{0,5,A\}5$  and  $A\{0\}65$  (since any digits combo  $56, 60, 65, 66, 6A, A6$  between them will produce smaller primes)

\*\*\*\* All numbers of the form  $A\{0,5,A\}5$  are divisible by 5, thus cannot be prime.

\*\*\*\* The smallest prime of the form  $A\{0\}65$  is **A00065**

\* Case (A,7):

\*\* **A7** is prime, and thus the only minimal prime in this family.

\* Case (A,B):

\*\* **AB** is prime, and thus the only minimal prime in this family.

\* Case (B,1):

\*\* Since  $B5$ ,  $B7$ ,  $11$ ,  $31$ ,  $51$ ,  $61$ ,  $81$ ,  $91$ , **B21** are primes, we only need to consider the family  $B\{0,4,A,B\}1$  (since any digits  $1, 2, 3, 5, 6, 7, 8, 9$  between them will produce smaller primes)

\*\*\* Since  $4B$ ,  $AB$ ,  $401$ ,  $A41$ , **B001**, **B0B1**, **BB01**, **BB41** are primes, we only need to consider the families  $B\{A\}0\{4,A\}1$ ,  $B\{0,4\}4\{4,A\}1$ ,  $B\{0,4,A,B\}A\{0,A\}1$ ,  $B\{B\}B\{A,B\}1$  (since any digits combo  $00, 0B, 40, 4B, A4, AB, B0, B4$  between them will produce smaller primes)

\*\*\*\* For the  $B\{A\}0\{4,A\}1$  family, since  $A41$  is prime, we only need consider the families  $B0\{4\}\{A\}1$  and  $B\{A\}0\{A\}1$

\*\*\*\*\* For the  $B0\{4\}\{A\}1$  family, since **B04A1** is prime, we only need to consider the families  $B0\{4\}1$  and  $B0\{A\}1$

\*\*\*\*\* The smallest prime of the form  $B0\{4\}1$  is  $B04441$  (not minimal prime, since  $4441$  is prime)

\*\*\*\*\* The smallest prime of the form  $B0\{A\}1$  is  $B0AAAAA1$  (not minimal prime, since  $AAAA1$  is prime)

\*\*\*\* For the  $B\{A\}0\{A\}1$  family, since  $A0A1$  is prime, we only need to consider the families  $B\{A\}01$  and  $B0\{A\}1$

\*\*\*\*\* The smallest prime of the form  $B\{A\}01$  is **BAA01**

\*\*\*\*\* The smallest prime of the form  $B0\{A\}1$  is  $B0AAAAA1$  (not minimal prime, since  $AAAA1$  is prime)

\*\*\*\* For the  $B\{0,4\}4\{4,A\}1$  family, since  $4441$  is prime, we only need to consider the families  $B\{0\}4\{4,A\}1$  and  $B\{0,4\}4\{A\}1$

\*\*\*\*\* For the  $B\{0\}4\{4,A\}1$  family, since  $B001$  is prime, we only need to consider the families  $B4\{4,A\}1$  and  $B04\{4,A\}1$

\*\*\*\*\* For the  $B4\{4,A\}1$  family, since  $A41$  is prime, we only need to consider the family  $B4\{4\}\{A\}1$

\*\*\*\*\* Since  $4441$  and  $BAAA1$  are primes, we only need to consider the numbers  $B41$ ,  $B441$ ,  $B4A1$ ,  $B44A1$ ,  $B4AA1$ ,  $B44AA1$

\*\*\*\*\* None of  $B41$ ,  $B441$ ,  $B4A1$ ,  $B44A1$ ,  $B4AA1$ ,  $B44AA1$  are primes.

\*\*\*\*\* For the  $B04\{4,A\}1$  family, since **B04A1** is prime, we only need to consider the family  $B04\{4\}1$

\*\*\*\*\* The smallest prime of the form  $B04\{4\}1$  is  $B04441$  (not minimal prime, since  $4441$  is prime)

\*\*\*\* For the  $B\{0,4\}4\{A\}1$  family, since  $401$ ,  $4441$ ,  $B001$  are primes, we only need to consider the families  $B4\{A\}1$ ,  $B04\{A\}1$ ,  $B44\{A\}1$ ,  $B044\{A\}1$  (since any digits combo  $00$ ,  $40$ ,  $44$  between  $(B,4\{A\}1)$  will produce smaller primes)

\*\*\*\*\* The smallest prime of the form  $B4\{A\}1$  is  $B4AAA1$  (not minimal prime, since  $BAAA1$  is prime)

\*\*\*\*\* The smallest prime of the form  $B04\{A\}1$  is **B04A1**

\*\*\*\*\* The smallest prime of the form  $B44\{A\}1$  is  $B44AAAAAAA1$  (not minimal prime, since  $BAAA1$  is prime)

\*\*\*\*\* The smallest prime of the form  $B044\{A\}1$  is  $B044A1$  (not minimal prime, since  $B04A1$  is prime)

\*\*\*\* For the  $B\{0,4,A,B\}A\{0,A\}1$  family, since all numbers in this family with  $0$  between  $(B,1)$  are in the  $B\{A\}0\{4,A\}1$  family, and all numbers in this family with  $4$  between  $(B,1)$  are in the  $B\{0,4\}4\{4,A\}1$  family, we only need to consider the family  $B\{A,B\}A\{A\}1$

\*\*\*\* Since **BAAA1** is prime, we only need to consider the families  $B\{A,B\}A1$  and  $B\{A,B\}AA1$

\*\*\*\*\* For the  $B\{A,B\}A1$  family, since  $AB$  and **BAAA1** are primes, we only need to consider the families  $B\{B\}A1$  and  $B\{B\}AA1$

\*\*\*\*\* All numbers of the form  $B\{B\}A1$  are divisible by  $B$ , thus cannot be prime.

\*\*\*\*\* The smallest prime of the form  $B\{B\}AA1$  is **BBBAA1**

\*\*\*\*\* For the  $B\{A,B\}AA1$  family, since **BAAA1** is prime, we only need to consider the families  $B\{B\}AA1$

\*\*\*\*\* The smallest prime of the form  $B\{B\}AA1$  is **BBBAA1**

\*\*\*\* For the  $B\{B\}B\{A,B\}1$  family, since  $AB$  and  $BAAA1$  are primes, we only need to consider the families  $B\{B\}B1$ ,  $B\{B\}BA1$ ,  $B\{B\}BAA1$  (since any digits combo  $AB$  or  $AAA$  between  $(B\{B\}B,1)$  will produce smaller primes)

\*\*\*\* The smallest prime of the form  $B\{B\}B1$  is **BBBB1**

\*\*\*\* All numbers of the form  $B\{B\}BA1$  are divisible by  $B$ , thus cannot be prime.

\*\*\*\* The smallest prime of the form  $B\{B\}BAA1$  is **BBBAA1**

\* Case  $(B,5)$ :

\*\* **B5** is prime, and thus the only minimal prime in this family.

\* Case  $(B,7)$ :

\*\* **B7** is prime, and thus the only minimal prime in this family.

\* Case  $(B,B)$ :

\*\* Since  $B5$ ,  $B7$ ,  $1B$ ,  $3B$ ,  $4B$ ,  $5B$ ,  $6B$ ,  $8B$ ,  $AB$ ,  **$B2B$**  are primes, we only need to consider the family  $B\{0,9,B\}B$  (since any digits  $1, 2, 3, 4, 5, 6, 7, 8, A$  between them will produce smaller primes)

\*\*\* Since  $90B$  and  $9BB$  are primes, we only need to consider the families  $B\{0,B\}\{9\}B$

\*\*\*\* Since  $9999B$  is prime, we only need to consider the families  $B\{0,B\}B$ ,  $B\{0,B\}9B$ ,  $B\{0,B\}99B$ ,  $B\{0,B\}999B$

\*\*\*\* All numbers of the form  $B\{0,B\}B$  are divisible by  $B$ , thus cannot be prime.

\*\*\*\* For the  $B\{0,B\}9B$  family:

\*\*\*\*\* Since  **$B0B9B$**  and  **$BB09B$**  are primes, we only need to consider the families  $B\{0\}9B$  and  $B\{B\}9B$  (since any digits combo  $0B$ ,  $B0$  between  $(B,9B)$  will produce smaller primes)

\*\*\*\*\* The smallest prime of the form  $B\{0\}9B$  is  **$B000000000000000000000000000009B$**

\*\*\*\*\* All numbers of the form  $B\{B\}9B$  is either divisible by  $11$  (if totally number of  $B$ 's is even) or factored as  $10^{(2*n)}-21 = (10^{n-5}) * (10^{n+5})$  (if totally number of  $B$ 's is odd number  $2*n-1$ ), thus cannot be prime.

\*\*\*\* For the  $B\{0,B\}99B$  family:

\*\*\*\*\* Since  $B0B9B$  and  $BB09B$  are primes, we only need to consider the families  $B\{0\}99B$  and  $B\{B\}99B$  (since any digits combo  $0B$ ,  $B0$  between  $(B,99B)$  will produce smaller primes)

\*\*\*\*\* The smallest prime of the form  $B\{0\}99B$  is  **$B00099B$**

\*\*\*\*\* The smallest prime of the form  $B\{B\}99B$  is  **$BBBBBB99B$**





[2] [https://en.wikipedia.org/wiki/Minimal\\_prime\\_\(recreational\\_mathematics\)](https://en.wikipedia.org/wiki/Minimal_prime_(recreational_mathematics)) (article “minimal prime” in Wikipedia)

[3] [https://www.primepuzzles.net/puzzles/puzz\\_178.htm](https://www.primepuzzles.net/puzzles/puzz_178.htm) (the puzzle of minimal primes (when the restriction of prime>base is not required) in The Prime Puzzles & Problems Connection)

[4] [https://www.primepuzzles.net/problems/prob\\_083.htm](https://www.primepuzzles.net/problems/prob_083.htm) (the problem of minimal primes in The Prime Puzzles & Problems Connection)

[5] <https://github.com/xayahrainie4793/non-single-digit-primes> (my data for these  $M(L_b)$  sets for  $2 \leq b \leq 16$ )

[6] <http://recursed.blogspot.com/2006/12/prime-game.html> (Shallit's The Prime Game page)

[7] <http://www.cs.uwaterloo.ca/~shallit/Papers/minimal5.pdf> (Shallit's proof of base 10 minimal primes, when the restriction of prime>base is not required)

[8] <https://archive.ph/IGZE1> (proofs of minimal primes in bases  $b \leq 10$ , when the restriction of prime>base is not required, warning: the sets of  $M(L_b)$  have errors for  $b = 8$  and  $b = 10$ ,  $b = 8$  misses the prime 6101 and  $b = 10$  missing the primes 9001 and 9049 and instead wrongly including the primes 90001, 90469, and 9000049, thus the correct values of  $|M(L_b)|$  for  $b = 8$  and  $b = 10$  are 15 and 26 (instead of 14 and 27), respectively)

[9] <https://cs.uwaterloo.ca/~cbright/reports/mepn.pdf> (the article for this minimal prime problem in bases  $b \leq 30$ , when the restriction of prime>base is not required)

[10] <https://cs.uwaterloo.ca/~cbright/talks/minimal-slides.pdf> (the article for this minimal prime problem in bases  $b \leq 30$ , when the restriction of prime>base is not required)

[11] <https://archive.ph/ci2yM> (the article for this minimal prime problem in bases  $b \leq 30$ , when the restriction of prime>base is not required)

[12] <https://github.com/curtisbright/mepn-data> (data for these  $M(L_b)$  sets and unsolved families for  $2 \leq b \leq 30$ , when the restriction of prime>base is not required, file “minimal. $b$ .txt” is the data of all known minimal primes or PRPs in base  $b$ , and file “unsolved. $b$ .txt” is the list of all unsolved families (families for which not even a probable prime is known nor can be ruled out as only contain composites) in base  $b$ , the format of the unsolved families is  $xy^*z$  for  $xyyy\dots yyyz$ , for bases  $2 \leq b \leq 16$  and  $b = 18, 20, 22, 23, 24, 30$  are completely solved, except the largest element in  $M(L_{13})$  and largest 9 elements in  $M(L_{23})$  (except the second-largest element in  $M(L_{23})$ , it can be proven prime using  $N-1$  primality test, since  $n-1$  can be trivially fully factored for this number  $n$ ) are only probable primes, i.e. not proven primes, thus we cannot definitely say that the corresponding families can be

removed from the list of unsolved families, and we cannot definitely compute this part of the sets  $M(L_b)$ , search limits of lengths for the unsolved families (families for which not even a probable prime is known nor can be ruled out as only contain composites) in base  $b$ : 1000000 for  $b = 17$ , 707000 for  $b = 19$ , 506000 for  $b = 21$ , 292000 for  $b = 25$ , 486000 for  $b = 26$ , 368000 for  $b = 27$ , 543000 for  $b = 28$ , 233000 for  $b = 29$ )

[13] <https://github.com/RaymondDevillers/primes> (data for these  $M(L_b)$  sets and unsolved families for  $28 \leq b \leq 50$ , when the restriction of prime>base is not required, using lowercase letters a – n to represent digit values 36 to 49 for bases  $b > 36$ , file “kernel  $b$ ” is the data of all known minimal primes or PRPs in base  $b$ , and file “left  $b$ ” is the list of all unsolved families (families for which not even a probable prime is known nor can be ruled out as only contain composites) in base  $b$ , the format of the unsolved families is  $x\{y\}z$  for  $xyyy\dots yyyz$ , only bases  $b = 30$  and  $b = 42$  are completely solved, search limits of lengths for the unsolved families (families for which not even a probable prime is known nor can be ruled out as only contain composites) in base  $b$ : 10000 for all  $b$ )

[14] <http://www.bitman.name/math/article/730> (article for minimal primes, when the restriction of prime>base is not required)

[15] <http://www.bitman.name/math/table/497> (data for minimal primes in bases  $2 \leq b \leq 16$ , when the restriction of prime>base is not required)

[16] <https://oeis.org/A071071/a071071.pdf> (research of minimal sets of powers of 2, when the restriction of >base is not required)

[17] <http://nntdm.net/papers/nntdm-25/NNTDM-25-1-036-047.pdf> (research of minimal set of totients+ $n$  for  $0 \leq n \leq 5$ , when the restriction of >base is not required)

[18] <http://www.prothsearch.com/sierp.html> (the Sierpinski problem)

[19] <http://www.prothsearch.com/rieselprob.html> (the Riesel problem)

[20] <http://www.primegrid.com/> (with projects for the Sierpinski problem, the Riesel problem, the Prime Sierpinski problem, the Extended Sierpinski problem, Sierpinski/Riesel base 5 problem, generalized Fermat prime search)

[21] <http://www.prothsearch.com/> (lists for primes of the form  $k \cdot 2^n + 1$  for odd  $k < 1200$ , also factoring status of generalized Fermat numbers of the form  $a^{2^n} + b^{2^n}$  for  $1 \leq b < a \leq 12$ )

[22] <http://www.15k.org/> (lists for primes of the form  $k \cdot 2^n - 1$  for odd  $k < 10000$ )

[23] <https://www.rieselprime.de/default.htm> (lists for primes of the form  $k \cdot 2^n \pm 1$ )

[24] <http://www.noprimeleftbehind.net/crus/Sierp-conjectures.htm> (generalized Sierpinski conjectures in bases  $b \leq 1030$ , some primes found in these conjectures are minimal primes in base  $b$ , especially, all primes for  $k < b$  (if exist for a  $(k, b)$  combo) are always minimal primes in the base  $b$ ) (also some examples for simple families contain no primes  $> b$ )

[25] <http://www.noprimeleftbehind.net/crus/Riesel-conjectures.htm> (generalized Riesel conjectures in bases  $b \leq 1030$ , some primes found in these conjectures are minimal primes in base  $b$ , especially, all primes for  $k < b$  (if exist for a  $(k, b)$  combo) are always minimal primes in the base  $b$ ) (also some examples for simple families contain no primes  $> b$ )

[26] [http://www.noprimeleftbehind.net/crus/tab/CRUS\\_tab.htm](http://www.noprimeleftbehind.net/crus/tab/CRUS_tab.htm) (list for the status of the generalized Sierpinski conjectures and the generalized Riesel conjectures in bases  $b \leq 1030$ )

[27] <https://www.utm.edu/staff/caldwell/preprints/2to100.pdf> (article for generalized Sierpinski conjectures in bases  $b \leq 100$ )

[28] <https://oeis.org/A076336/a076336c.html> (the dual Sierpinski problem)

[29] <https://mersenneforum.org/showthread.php?t=10761> (list of large (probable) primes for the dual Sierpinski problem)

[30] <http://www.kurims.kyoto-u.ac.jp/EMIS/journals/INTEGERS/papers/i61/i61.pdf> (article for the mixed (original+dual) Sierpinski problem)

[31] <https://mersenneforum.org/showthread.php?t=6545> (research for the mixed (original+dual) Riesel problem)

[32] <https://mersenneforum.org/showthread.php?t=26328> (research for the mixed (original+dual) Sierpinski base 5 problem)

[33] <http://www.fermatquotient.com/> (generalized repunit primes (primes of the form  $(b^n - 1)/(b - 1)$ ) in bases  $b \leq 160$ , the smallest such prime for base  $b$  (if exists) is always minimal prime in base  $b$ ) and (generalized half Fermat primes (primes of the form  $(b^{2^n} + 1)/2$ ) sorted by  $n$ , the smallest such prime for base  $b$  (if exists) is always minimal prime in base  $b$ )

[34] <https://archive.ph/7f7jx> (generalized repunit primes (primes of the form  $(b^n - 1)/(b - 1)$ ) in bases  $b \leq 1000$ , the smallest such prime for base  $b$  (if exists) is always minimal prime in base  $b$ )

[35] <http://jeppesn.dk/generalized-fermat.html> (generalized Fermat primes (primes of the form  $b^{2^n} + 1$ ) in even bases  $b \leq 1000$ , the smallest such prime for base  $b$  (if exists) is always minimal prime in base  $b$ )

[36] <http://www.noprimeleftbehind.net/crus/GFN-primes.htm> (generalized Fermat primes (primes of the form  $b^{2^n} + 1$ ) in even bases  $b \leq 1030$ , the smallest such prime for base  $b$  (if exists) is always minimal prime in base  $b$ )

[37] <https://harvey563.tripod.com/wills.txt> (primes of the form  $(b-1) * b^n - 1$  for bases  $b \leq 2049$ , the smallest such prime for base  $b$  (if exists) is always minimal prime in base  $b$ )

[38] [https://www.rieselprime.de/ziki/Williams\\_prime\\_MM\\_least](https://www.rieselprime.de/ziki/Williams_prime_MM_least) (the smallest primes of the form  $(b-1) * b^n - 1$  for bases  $b \leq 2049$ , these primes (if exists) is always minimal prime in base  $b$ )

[39] [https://www.rieselprime.de/ziki/Williams\\_prime\\_MP\\_least](https://www.rieselprime.de/ziki/Williams_prime_MP_least) (the smallest primes of the form  $(b-1) * b^n + 1$  for bases  $b \leq 1024$ , these primes (if exists) is always minimal prime in base  $b$ )

[40] [https://www.rieselprime.de/ziki/Riesel\\_prime\\_small\\_bases\\_least\\_n](https://www.rieselprime.de/ziki/Riesel_prime_small_bases_least_n) (the smallest primes of the form  $k * b^n - 1$  for  $k \leq 12$  and bases  $b \leq 1024$ , these primes (if exists) is always minimal prime in base  $b$  if  $b > k$ )

[41] [https://www.rieselprime.de/ziki/Proth\\_prime\\_small\\_bases\\_least\\_n](https://www.rieselprime.de/ziki/Proth_prime_small_bases_least_n) (the smallest primes of the form  $k * b^n + 1$  for  $k \leq 12$  and bases  $b \leq 1024$ , these primes (if exists) is always minimal prime in base  $b$  if  $b > k$ )

[42] <https://docs.google.com/spreadsheets/d/e/2PACX-1vTKkSNKGVQkUINlp1B3cXe90FWPwiegdA07EE7-U7sqXntKAEQryno1sbFvvKriieda3LfkqRwmKME/pubhtml> (my list for the smallest primes in given simple family in bases  $b \leq 1024$ )

[43] <https://www.rose-hulman.edu/~rickert/Compositeseq/> (a problem related to this project)

[44] <http://www.worldofnumbers.com/Appending%201s%20to%20n.txt> (a problem related to this project)

[45] <http://www.worldofnumbers.com/deplat.htm> (list of plateau and depression primes)

[46] <http://www.worldofnumbers.com/wing.htm> (list of palindromic wing primes)

[47] <https://stdkmd.net/nrr/prime/primecount.txt> (near- and quasi- repdigit (probable) primes sorted by count)

- [48] <https://stdkmd.net/nrr/prime/primedifficulty.txt> (near- and quasi- repdigit (probable) primes sorted by difficulty)
- [49] <https://stdkmd.net/nrr/coveringset.htm> (covering set of near-repdigit-related sequences)
- [50] [http://www.rieselprime.de/dl/CRUS\\_pack.zip](http://www.rieselprime.de/dl/CRUS_pack.zip) (*srsieve*, *sr1sieve*, *sr2sieve*, *pfgw*, and *llr* softwares)
- [51] <https://www.bc-team.org/app.php/dlxt/?cat=3> (*srsieve*, *sr1sieve*, *sr2sieve*, *sr5sieve* software)
- [52] <https://sourceforge.net/projects/openpfgw/> (*pfgw* software)
- [53] <http://jpenne.free.fr/index2.html> (*llr* software)
- [54] <http://www.ellipsa.eu/public/primos/primos.html> (*PRIMO* software)
- [55] <https://primes.utm.edu/prove/index.html> (website for primality proving)
- [56] [https://www.rieselprime.de/ziki/Primality\\_test](https://www.rieselprime.de/ziki/Primality_test) (list of known primality tests and probable primality tests)
- [57] [https://primes.utm.edu/notes/prp\\_prob.html](https://primes.utm.edu/notes/prp_prob.html) (the probability that a random PRP is composite)
- [58] [https://oeis.org/wiki/User:Charles\\_R\\_Greathouse\\_IV/Tables\\_of\\_special\\_primes](https://oeis.org/wiki/User:Charles_R_Greathouse_IV/Tables_of_special_primes) (expected number of primes in first  $n$  terms of a given sequence)
- [59] [https://primes.utm.edu/curios/page.php?number\\_id=22380](https://primes.utm.edu/curios/page.php?number_id=22380) (the largest base 10 minimal prime in Prime Curios!)
- [60] <https://oeis.org/A347819> (OEIS sequence for base 10 minimal primes)
- [61] <https://oeis.org/A326609> (OEIS sequence for the largest base  $b$  minimal prime, when the restriction of prime  $>$  base is not required)
- [62] <https://primes.utm.edu/primes/lists/all.txt> (top proven primes)
- [63] <http://www.primenumbers.net/prptop/prptop.php> (top PRPs)
- [64] <http://factordb.com> (online factor database, including many primes which are minimal primes in a small base)

For list of more references, see

<https://mersenneforum.org/showpost.php?p=571731&postcount=140> and  
<https://mersenneforum.org/showpost.php?p=582061&postcount=154>

Also see <https://primes.utm.edu/curios/includes/primetest.php?file=primetest.html> and <https://www.numberempire.com/primenumbers.php> and <https://www.bigprimes.net/primaliditytest> and <http://www.proftnj.com/calcprem.htm> and <https://www.archimedes-lab.org/primOmatic.html> and <http://www.sonic.net/~undoc/java/PrimeCalc.html> for links of prime checkers.

Also see <https://www.numberempire.com/numberfactorizer.php> and <https://www.alpertron.com.ar/ECM.HTM> and <http://www.javascripter.net/math/calculators/primefactorscalculator.htm> and <https://primefan.tripod.com/Factorer.html> and <http://www.se16.info/js/factor.htm> and <http://math.fau.edu/Richman/mla/factor-f.htm> for links of integer factorizers.

Also see <https://primes.utm.edu/lists/small/1000.txt> and <https://primes.utm.edu/lists/small/millions/> and [https://oeis.org/A000040/b000040\\_1.txt](https://oeis.org/A000040/b000040_1.txt) and [https://oeis.org/A000040/a000040\\_1B.7z](https://oeis.org/A000040/a000040_1B.7z) and <http://www.primos.mat.br/indexen.html> and [https://www.walter-fendt.de/html5/men/primenumbers\\_en.htm](https://www.walter-fendt.de/html5/men/primenumbers_en.htm) and <http://www.rsok.com/~jrm/printprimes.html> for links of lists of small primes.

Also see <https://baseconvert.com/> and <https://www.calculand.com/unit-converter/zahlen.php> for links of base converters.

(In fact, you can use [Wolfram Alpha](#) for prime checker, integer factorizer, and base converter, besides, many [mathematical softwares](#) also already have prime checkers, integer factorizers, and base converters, including [Maple](#), [wolfram Mathematica](#), [PARI/GP](#), [Python](#), [GMP](#), [Magma](#), [SageMath](#), see the table below, you can download these softwares by clicking the links)

software	<a href="#">Maple</a>	<a href="#">Wolfram Mathematica</a>	<a href="#">PARI/GP</a>	<a href="#">Python</a>	<a href="#">GMP</a>	<a href="#">Magma</a>	<a href="#">SageMath</a>
check if a number is <a href="#">probable prime</a>		<a href="#">PrimeQ[<i>number</i>]</a>	ispseudo prime( <i>number</i> )				
check if a number is <a href="#">proven prime</a>		<a href="#">ProvablePrimeQ[<i>number</i>]</a>	isprime( <i>number</i> )				
<a href="#">factor</a> a number		<a href="#">FactorInteger[<i>number</i>]</a>	factor( <i>number</i> )				
convert a number		<a href="#">BaseForm[<i>number</i></a>	digits( <i>number</i> , <i>base</i> )	<a href="#">int(<i>number</i>, <i>base</i>)</a>			

to <a href="#">base</a> $b$		$r, base]$	$base)$				
		<a href="#">IntegerDigits</a> [ $number, base]$					

Finally, there is a [C code](#) for the problem in this article: (need run with [GMP](#)), see [this forum post](#).