MATHEMATICS OF COMPUTATION, VOLUME 26, NUMBER 120, OCTOBER 1972

Some Prime Numbers of the Forms $2A3^n + 1$ and $2A3^n - 1$

By H. C. Williams and C. R. Zarnke

Abstract. All primes of the form $2A3^n + 1$ and of the form $2A3^n - 1$, where $1 \leqq A \leqq 50$ and $1 \leqq n \leqq 325$, are found. Some large twin primes are also determined.

1. Introduction. Robinson [2] has given a table of primes of the form $k2^n + 1$ and Williams and Zarnke [3] and Riesel [1] have given tables of primes of the form $k2^n - 1$. By comparing these tables, it is possible to determine some large twin primes, namely $9 \cdot 2^{211} \pm 1$ and $45 \cdot 2^{189} \pm 1$. The purpose of this paper is to present a table of all primes of the form $2A3^n + 1$ and a table of all primes of the form $2A3^n - 1$ for $1 \leqq A \leqq 50$ and $1 \leqq n \leqq 325$. By comparing these two tables, we also find some large twin primes.

2. The Algorithm. The following theorem was used to test the primality of integers of the form $2A3^n \pm 1$.

THEOREM. *Let $N = 2A3^n \pm 1$, where $(A, 3) = 1$, $1 \leqq A \leqq 3^n$. Let q be a prime ($\equiv 1 \pmod 3$) such that $N^{(q-1)/3} \not\equiv 1 \pmod q$ and let $4q = r^2 + 27s^2$, where $r \equiv 1 \pmod 3$. If $(qs, N) = 1$, N is a prime if and only if*

$$P_n \equiv \pm 1 \pmod N,$$

where

$$P_1 \equiv K^A V_{2A} \pmod N$$

and

$$P_{k+1} \equiv P_k(P_k^2 - 3) \pmod N.$$

Here K is an integer such that $Kq \equiv 1 \pmod N$ and $V_{2A} = \alpha^{2A} + \beta^{2A}$, where α, β are the zeros of $x^2 + rx + q$.

A proof of this theorem for integers of the form $2A3^n - 1$ is given in Williams [4]. By using methods similar to those in [4], it is not difficult to demonstrate that the theorem is also true for integers of the form $2A3^n + 1$. This theorem can also be generalized for integers of the form $2Ap^n \pm 1$, where p is any odd prime (see Williams [5]).

The above theorem was used to construct an algorithm (see [4]) which was programmed for an IBM/360-65 computer. The results of running this program are given in Table 1 and Table 2. The computer required about five hours of CPU time to complete all the calculations.

Received July 1971.
AMS 1970 *subject classifications.* Primary 10A25.
Key words and phrases. Primes, twin primes, algorithm.

TABLE 1. *Table of Primes of the Form* $2A3^n + 1$

2A	n $(1 \leq n \leq 325)$
2	1,2,4,5,6,9,16,17,30,54,57,60,65,132,180,320
4	1,2,3,6,14,15,39,201,249
8	2,7,8,10,22,52,58,76,130,143
10	1,3,4,7,9,12,18,22,102,112,157,162,289
14	1,2,3,18,22,26,27,33,39,57,62,94,145,246
16	3,4,5,12,24,36,77,195,296,297
20	1,2,3,4,5,8,16,19,28,50,134,280
22	1,2,4,5,10,12,14,24,34,37,52,56,65,68,96,106,128,156,169,236,254
26	1,12,15,17,20,29,31,32,35,37,77,95,193,203,224,296
28	3,4,8,11,14,15,18
32	1,4,8,9,32,36,48,74,112,186,204
34	1,2,3,5,11,15,19,25,46,65,83,85,145,211
38	4,19,115
40	5,7,15,17,18,27,29,30,33,35,53,54,59,60,150,203,229
44	2,6,9,10,90,194
46	1,4,12,39,220
50	1,4,6,8,20,38,46,49,59,60,95,148,168,308
52	1,5,17,22,37,73,96,113,205,245
56	9,20,21,45,53,59,68,113,135,168,255,299,308
58	2,3,6,15,18,32,35,36,52,172,224,255,296,303
62	4,9,40,184,297
64	1,2,7,11,13,31,41,61,121,127,157,167,181,203,229,278
68	2,24,26,30,31,32,42,54,72,119
70	1,2,5,6,8,9,12,15,19,20,21,25,39,44,49,52,55,69,85,94,115,162,195, 222,225,271
74	1,3,7,50,70,115,202
76	1,3,11,19,52,59,88,103,121,139,189,268
80	1,3,4,5,6,10,21,35,54,71,90,202,306
82	2,5,6,9,17,21,26,29,32,90,138,180,278,290
86	4,5,17,27,39,48,57,60,65,68,116,128,132,165,208
88	3,4,6,16,34,43,67
92	1,2,8,12,14,36,46,54,58,62,74,85,94,118,169,182,186
94	1,3,53,55,83,99,113,114,154,186,223
98	2,3,6,11,16,19,22,66,103,111,123,151,239
100	4,6,9,10,11,14,22,24,28,64,69,70,105,117,161,236,323

TABLE 2. *Table of Primes of the Form* $2A3^n - 1$

2A	n ($1 \leq n \leq 325$)
2	1,2.3,7,8,12,20,23,27,35,56,62,68,131,222
4	1,3,5,7,15,45,95,235
8	1,2,4,10,17,50,170,184,194,209
10	1,2,3,4,8,10,14,20,22,26,30,38,39,49,54,58,70,81,84,87,102,111,140, 159,207,224
14	1,11,16,80,83,88,136,187
16	1,3,9,13,31,43,81,121,235
20	1,2,4,10,11,17,19,24,32,35,37,60,80,114,140,314
22	2,3,8,14,23,32,167
26	2,3,5,9,15,17,39,45,50,53,93,122,165
28	1,2,4,5,6,8,12,18,24,49,64,76,110,125,138,168,237
32	3,4,6,46,59,84,94,124,239,267
34	1,4,7,24,107,168,248
38	1,6,8,9,14,16,25,28,30,56,64,105,156,168,169,325
40	2,5,10,14,16,40,56,70,95,242
44	1,3,5,8,9,11,81,108,188,308,313
46	1,5,6,10,13,46,54,58,65,71,78,93,127,151,161,187,193,246
50	1,2,4,5,9,13,15,17,23,58,65,119,244,292,323
52	2,4,6,7,8,10,19,20,30,46,60,74,98,122,138,142,158
56	1,2,3,6,7,18,19,22,37,38,54,89,98,106,151,177,229,234,241
58	1,2,6,12,17,41,48,56,96,116,140,312
62	2,4,6,7,11,24,43,46,52,92,103,215,224
64	1,5,7,67,295,325
68	4,9,10,12,30,46,102,108,153,177,297
70	3,4,7,12,24,25,27,35,45,57,144,160,179,180,183,212,223
74	3,5,9,15,21,63,119
76	1,2,10,66,91,127,139,222
80	1,2,7,12,13,15,20,45,72,75,82,102,126,216,277,282,321
82	3,8,11,16,18,20,26,35,59,170,179
86	1,2,5,9,11,21,30,35,45,66,86,95,105,125,194
88	1,4,5,6,10,12,13,16,33,40,41,46,53,54,65,102,121,125,162,210,294
92	2,4,7,10,32,64,79,119
94	1,24,36,55,73,111,139,157,192,205
98	1,2,4,5,8,22,30,34,45,61,90,126,129,154,292
100	3,113,231

3. Remarks. Several pairs of twin primes were found by comparing Tables 1 and 2. The largest ones are

$$10 \cdot 3^{102} \pm 1, \quad 68 \cdot 3^{30} \pm 1, \quad 70 \cdot 3^{25} \pm 1, \quad 76 \cdot 3^{139} \pm 1, \quad 82 \cdot 3^{26} \pm 1, \quad 94 \cdot 3^{55} \pm 1.$$

The largest pair here, $76 \cdot 3^{139} \pm 1$, seems to be the largest pair of twin primes currently known.

Of the numbers analogous to the Cullen numbers, i.e., integers of the form $n3^n + 1$, only $2 \cdot 3^2 + 1$, $8 \cdot 3^8 + 1$ and $32 \cdot 3^{32} + 1$ are primes for $n \leqq 100$. Unfortunately, $128 \cdot 3^{128} + 1$ is composite.

Department of Computer Science
University of Manitoba
Winnipeg 19, Manitoba, Canada

1. H. Riesel, "Lucasian criteria for the primality of $N = h \cdot 2^n - 1$," *Math. Comp.*, v. 23, 1969, pp. 869–875. MR **41** #6773.

2. R. M. Robinson, "A report on primes of the form $k \cdot 2^n + 1$ and on factors of Fermat numbers," *Proc. Amer. Math. Soc.*, v. 9, 1958, pp. 673–681. MR **20** #3097.

3. H. C. Williams & C. R. Zarnke, "A report on prime numbers of the forms $M = (6a + 1)2^{2m-1} - 1$ and $M' = (6a - 1)2^{2m} - 1$," *Math. Comp.*, v. 22, 1968, pp. 420–422. MR **37** #2680.

4. H. C. Williams, "The primality of $2A3^n - 1$," *Canad. Math. Bull.*, (To appear.)

5. H. C. Williams, "An algorithm for determining certain large primes," *Proc. Second Louisiana Conference on Combinatorics, Graph Theory and Computing*, Baton Rouge, 1971, pp. 533–556.