MATHEMATICS OF COMPUTATION
Volume 71, Number 238, Pages 833–835
S 0025-5718(01)01319-9
Article electronically published on March 30, 2001

REPUNIT R49081 IS A PROBABLE PRIME

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ABSTRACT. The Repunit R49081 = $(10^{49081} - 1)/9$ is a probable prime. In order to prove primality R49080 must be approximately 33.3% factored. The status of this factorization is included.

Repunits are numbers of the form R$n = (10^n - 1)/9$. Considerable computer time has been spent factoring repunits, creating interesting primes based on the characteristics of repunits, and searching for repunit primes. One intriguing feature of repunit primes is their apparent scarcity with the only known primes being R2, R19, R23, R317, and R1031. R1031 was found to be a probable prime in 1978 [5] by Williams and Seah, and proved prime in 1985 [4] by Williams and Dubner. The history of the search limits is shown in the following table.

n max	year	searchers
2,000	1978	Williams & Seah
10,000	1985	Dubner
20,000	1992	Dubner
30,000	1994	J. Young
45,000	1998	T. Granlund
60,000	2000	Dubner

In September 1999, it was discovered that R49081 is a probable prime. This was verified on several different computers with different software. Although it is virtually certain that R49081 is prime, it is necessary to prove rigorously that it is prime. Because of its size, 49081 digits, the only hope of proving it prime with current theory and technology is using the BLS method [1]. This requires that (R49081 − 1) be about 1/3 factored, that is, the product of the known prime factors of (R49081 − 1) should be about $(\text{R}49081 - 1)^{1/3}$.

Since

$$\frac{10^{49081} - 1}{9} - 1 = \frac{10}{9}(10^{49080} - 1)$$

and

$$10^{49080} - 1 = (10^{24540} + 1)(10^{12270} + 1)(10^{6135} + 1)(10^{6135} - 1)$$

in Table 1 we have grouped the known prime factors according to the above equation. We follow the format of the Cunningham project [2] so that each line lists the prime factors of the primitive cofactor of n for base 10.

Received by the editor March 21, 2000 and, in revised form, May 30, 2000.
2000 *Mathematics Subject Classification.* Primary 11A41.
Key words and phrases. Prime numbers, primality proving.

TABLE 1. Factoring status of $10^{49080} - 1$

n	Factors of $10^{6135} - 1$	
1-	3.3	
3-	3.37	
5-	41.271	
15-	31.2906161	
409-	1637.13907.77711.1375877.2777111.5371851809.	
	7061270715258437.	c358
1227-	3334987.22123889761.p800	
2045-	110431.163601.1265039515351.	c1610
6135-	prp3265	
n	Factors of $10^k + 1, k = 6135 * 1$	
1+	11	
3+	7.13	
5+	9091	
15+	211.241.2161	
409+	53171.1358791302758702868906124409.	c377
1227+	1008741241.7833811446444211.	c792
2045+	4091.18601321 .3166190815957718461l.	c1602
6135+	49081.674851.394308721.	c3245
n	Factors of $10^k + 1, k = 6135 * 2$	
2+	101	
6+	9901	
10+	3541.27961	
30+	61.4188901.39526741	
818+	4909.16361.2396741.34876249.2091195610248881.	
	4829616990104344590241.	c757
2454+		c1633
4090+		
L	417181.	c1627
M		c1632
12270+		
L	687121.7572258721.1495049855581.	c3236
M		c3265
n	Factors of $10^k + 1, k = 6135 * 4$	
4+	73.137	
12+	99990001	
20+	1676321.5964848081	
60+	1000099999998999899990000000010001	
1636+	18598049.8890622777.	c1615
4908+	9817.1059411433.prp3251	
8180+	822449921.	c6520
24540+		c13056
	2.54 % factored	

Note that R49080 is only 2.54% factored. There are two large prp factors (3265 and 3251 digits) which conceivably could be proved prime in the not-so-distant future increasing the factored part to 15.8%. However, it is obvious that proving true primality will require significant breakthroughs in hardware and theory.

The equivalent of four Pentium II/400MHz PC computers were used in the search. It took about 130 computer-days to test the range of n from 45000 to 60000. The time for a Fermat test for probable primality of R49081 was about 4.5 hours using software that included FFT multiplication.

Reexamining generalized repunit primes, $(b^n - 1)/(b - 1)$, for various bases near 10 [3], it becomes clear that base 10 repunit primes are not exceptionally scarce. For example, when $b = 18$ there is only one prime for n up to 12000. For $b = 23$ there are only two primes in this range. There are several other bases near 10 which have prime patterns not too different from base 10. Unfortunately, these facts and finding that R49081 is prp removes some of the mystery from repunit primes.

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