Math 124 Final Exam

Joseph Nguyen

TOTAL POINTS

58.5 / 100

QUESTION 1

1 Problem 1(a) 5 / 5

✓ **+ 5 pts** Correct

+ 1 pts chain rule from outer ln

+ 1 pts one part of tan^2 x

+ 1 pts another part of tan^2 x

+ 1 pts chain rule from inner ln

+ 1 pts derivative of 2+sin^2 x

- 1 pts arithmetic error

- 1 pts parentheses issues

- 1 pts fraction error

+ 0 pts no points

QUESTION 2

2 Problem 1(b) 5 / 5

+ 1 pts Product rule

+ 1 pts derivative of x is 1

+ 1 pts derivative of cos is -sine

+ 1 pts Correct derivative of fraction is 7-10/x^3 or quotient rule

+ 1 pts keep x in the second part of the derivative

✓ **+ 5 pts All correct derivative**

QUESTION 3

3 Problem 1(c) 5 / 5

✓ **+ 5 pts** Correct

+ 1 pts ln h = sqrt(t) ln (t+1)

+ 2 pts h^'/h=1/(2sqrt(t)) ln(t+1) +sqrt(t)/(t+1)

+ 2 pts multiply by h to get h^'= (t+1)^{sqrt(t)}(1/(2sqrt(t)) ln(t+1) +sqrt(t)/(t+1))

+ 1 pts or write as e^{sqrt(t) ln t+1}

- 1 pts small algebra mistakes

+ 0 pts nothing correct

- 0.5 pts tiny algebra mistakes

QUESTION 4

4 Problem 2(a) 1 / 5

+ 5 pts Correct limit of -infinity with proper work

+ 1 pts turning it into a quotient

+ 2 pts Using L'Hospital on the quotient

+ 1 pts cleaning up post L'Hospital

✓ **+ 1 pts correct limit of -infinity**

+ 2 pts other attempt toward a solution

+ 2 pts completing other attempt toward solution

+ 0 pts no or irrelevant response

QUESTION 5

5 Problem 2(b) 5 / 5

✓ **+ 5 pts Correct answer of-5/6 with the complete work**

+ 1 pts conjugate multiplication

+ 1 pts cleaned up after multiplication to get to infinity/infinity

+ 1 pts divide top and bottom by x

+ 1 pts clean up after dividing top and bottom by x

+ 1 pts correct answer of -5/6

+ 0 pts no or irrelevant work

QUESTION 6

6 Problem 2(c) 5 / 5

✓ **+ 5 pts** Correct

+ 2 pts Manipulated function correctly (or used L'Hospital) so that a limit can be taken

+ 2 pts Correctly checked both LH and RH limit

+ 1 pts Final answer is correct

+ 0 pts No work or irrelevant work

QUESTION 7

7 Problem 3 0 / 13

+ 2 pts fan shape area theta/2pi * pi (0.8)^2

+ 2 pts triangle area is 0.8cos (theta/2) * 0.8 sin (theta/2)

+ **2 pts** cross section area is 0.32theta -0.32sin (theta)
+ **1 pts** volume V= 12(0.32theta -0.32sin (theta)) (relate V and theta)
+ **3 pts** correct differentiation dV/dt= 12(0.32 dtheta/dt -0.32cos theta dtheta/dt)
+ **2 pts** plug in 0.3=12(0.32 dtheta/dt -0.32(-1/2) dtheta/dt) and solve dtheta/dt
+ **1 pts** correct fraction 5/96
- **0.5 pts** adjust: correct decimal 0.0520833333
- **1 pts** minor algebra erros
- **2 pts** algebra erros

✓ + **0 pts** nothing correct
+ **13 pts** all correct
+ **2 pts** adjust: only realizing the rate dV/dt=0.3 and V and theta and did nothing correct
- **1 pts** too messy to read

QUESTION 8

8 Problem 4 2 / 13

+ **13 pts** Correct answer of 175/12~14.58 with proper work

✓ + **2 pts** working out the distance it swims using the pythagorean theorem
+ **2 pts** writing time as run distance/13 plus swim distance/5
+ **1 pts** correct time function
+ **2 pts** differentiating their (of same level of difficulty as answer) function
+ **2 pts** Solving their T'=0
+ **1 pts** correct critical number
+ **2 pts** Checking endpoint values or using other justification
+ **1 pts** choosing the min from their set
+ **0 pts** No or irrelevant response
- **1 pts** Arithmetic errors

QUESTION 9

9 Problem 5 11 / 12

+ **12 pts** All correct with dy/dx=(x^2-y^2)/(xy), plus or minus fourth root of 56, y=-5x/6+14/3, 2.917

✓ + **3 pts** (a) Correct derivative of dy/dx=(x^2-y^2)/(xy)

✓ + **1 pts** (a) setting their numerator for dy/dx equal to 0
+ **1 pts** (a) correct 4 points listed

✓ + **1 pts** (b) evaluating their dy/dx from (a) at (2,3)

✓ + **1 pts** (b) using their slope for tangent line at (2,3)

✓ + **1 pts** (b) correct answer of -5x/6+14/3

✓ + **1 pts** (c) Using their tangent line from (b)

✓ + **1 pts** (c) evaluating their tangent line from (b) at 2.1

✓ + **1 pts** correct answer of 2.917

✓ + **1 pts** (a)Find x values from setting dy/dx=0 and plugging back into the function.

QUESTION 10

10 Problem 6 9 / 14

+ **14 pts** Both (a), (b) correct

✓ + **7 pts** (a) correct
+ **7 pts** (b) correct
+ **2 pts** (a) Obtained the correct $$t$$ values from the information given
+ **3 pts** (a) Used the formula $$dy/dx=y'(t)/x'(t)$$ to find the slope
+ **2 pts** (a) Obtained the two tangent lines

✓ + **2 pts** (b) Used the formula $$g(t)=dy/dx=y'(t)/x'(t)$$
+ **4 pts** (b) Correctly computed $$g'(t)$$
+ **2 pts** Computed $$\frac{d^2y}{dx^2}$$ instead of $$g'(t)$$.
+ **1 pts** (b) Final answer consistent with work
+ **0 pts** (a) No work/irrelevant work
+ **0 pts** (b) No work/irrelevant work

QUESTION 11

11 Problem 7 10.5 / 18

✓ + **3 pts** (a) vertical asymptotes x=2 and x=-2
+ **1.5 pts** (a) gave only one vertical asymptote

✓ + **1 pts** (b) horizontal asypmtote of y=1
+ **2 pts** (b) checked both limits for the horizontal asymptotes

✓ + **1 pts** checked only one limit in (b)

✓ + **1 pts** (c) Derivative f'(x) using quotient rule
+ **1 pts** (c) critical number from 8x=0

+ **1 pts** (c) answer

✓ + **1 pts** **(d) second derivative f''(x) using quotient rule**

+ **2 pts** (d) concave down (-2,2)

+ **1 pts** (e) x=0 is the only critical number

✓ + **1 pts** **(e) it gives a max**

✓ + **1 pts** **(e) reason for max**

+ **3 pts** (f) graph

✓ + **2 pts** **(f) graph**

+ **1 pts** (f) graph

+ **0 pts** no or irrelevant response

- 0.5 Point adjustment

limit work in part b

Math 124 **Final Examination** **Autumn 2018**

Your Name

Joseph Nguyen

Your Signature

Joseph Nguyen

Student ID #

1	5	2	1	1	7	4

Seat Number

Quiz Section

E	5

Professor's Name

Ebru Bekyel

TA's Name

- Turn off and stow away all cell phones, pagers, radios, mp3 players, and other similar devices.
- This exam is closed book. You may use one 8.5″ × 11″ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.
- You can only use a Texas Instruments TI-30X IIS calculator.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Place $\boxed{\text{a box around your answer}}$ to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 8 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	15	
2	15	
3	13	
4	13	

Question	Points	Score
5	12	
6	14	
7	18	
Total	100	

You may use this page for scratch-work.

All writing on this page will be ignored unless you write "see back of first page" below a problem.

$$\frac{2(-5)(25-4)-(25)(-10)}{(25-4)^2} = \frac{-10(21)+250}{(21)^2}$$

$$= \frac{-210+250}{21^2} = \frac{40}{(21)^2}$$

$$0 = 4x\left(x^2 - \sqrt{\frac{-56-x^4}{-2x^2}}\right)$$

$$4x\left(\sqrt{\frac{-56-x^4}{-2x^2}}\right)$$

$$\frac{x^2 - \left(\frac{-56-x^4}{-2x^2}\right)}{\sqrt{\frac{-56+x^4}{-2x^2}}} =$$

$$x^2 + \frac{56}{2x^2} + \frac{x^4}{-2x^2} \qquad \frac{1}{2}x^4 = 23$$

$$x^2 + \frac{56}{2x^2} - \frac{1}{2}x^2$$

$$\frac{1}{2}x^2 - \frac{23}{x^2} = 0 \qquad \frac{23}{x^2} = \frac{1}{2}x^2$$

1. (15 total points) Calculate the derivatives of the following functions. You do not need to simplify your answers.

   (a) (5 points) $f(x) = \ln\left[\tan^2 x + \ln\left(2 + \sin^2 x\right)\right]$

$$\frac{1}{\tan^2 x + \ln(2+\sin^2 x)}\left(2\tan x \sec^2 x + \frac{1}{2+\sin^2 x}(2\sin x \cos x)\right)$$

$\frac{d}{dx}\ln x = \frac{1}{x}$ $\quad \frac{d}{dx}\sin x = \cos x$

$\frac{d}{dx}\tan x = \sec^2 x$ $\quad \frac{d}{dx}(\tan x)^2 = 2\tan x \sec^2 x$

$2\tan x\left(\frac{d}{dx}\tan x\right)$ $\quad \frac{d}{dx}(\sin x)^2 = 2\sin x\left(\frac{d}{dx}\sin x\right)$

   (b) (5 points) $g(x) = x \cdot \cos\left(\frac{7x^3+5}{x^2}\right)$.

$$\cos\left(\frac{7x^3+5}{x^2}\right) - x\sin\left(\frac{7x^3+5}{x^2}\right)\left(\frac{21x^2(x^2) - 2x(7x^3+5)}{x^4}\right)$$

$\frac{d}{dx}\cos x = -\sin x$

$\frac{d}{dx}\left[\frac{7x^3+5}{x^2}\right] = $ (use quotient rule) $= \frac{21x^2(x^2) - 2x(7x^3+5)}{(x^2)x^2}$

$2 \cdot 2 = 4$

   (c) (5 points) $h(t) = (t+1)^{\sqrt{t}}$ $= h(t) = (t+1)^{t^{1/2}}$

$\ln y = \ln(t+1)^{\sqrt{t}}$

$\ln y = \sqrt{t}\ln(t+1)$

$\frac{1}{y}y' = \frac{1}{2\sqrt{t}}\ln(t+1) + \sqrt{t}\frac{1}{t+1}$

$$h'(t) = (t+1)^{\sqrt{t}}\left[\frac{1}{2\sqrt{t}}\ln(t+1) + \sqrt{t}\frac{1}{t+1}\right]$$

2. (15 total points) Evaluate the following limits. Your answer should be a number, ∞, $-\infty$ or DNE.

(a) (5 points) $\lim_{x\to 0^+} \frac{1}{\sqrt{x}} - \frac{1}{\sin x}$

$\frac{\sin x - \sqrt{x}}{\sqrt{x}\sin x}$

$\frac{1}{\sqrt{0.0001}} - \frac{1}{\sin(0.0001)} = \frac{1}{0.0001} - \frac{1}{0.0001}$

$\lim_{x\to 0^+} \frac{1}{\sqrt{x}}$ is too insignificant compared to $\lim_{x\to 0^+} \frac{1}{\sin x} - \frac{1}{0.0001}$

$\lim_{x\to 0^+} \sin x = 0$

$\to \infty - (\infty)$

$\boxed{-\infty}$

$\lim_{x\to 0^+} \frac{1}{\sqrt{x}} = \infty$ (smaller infinity) than $\frac{1}{\sin x}$

(b) (5 points) $\lim_{x\to\infty} 3x - \sqrt{9x^2+5x}$ $\left(\frac{3x+\sqrt{9x^2+5x}}{3x+\sqrt{9x^2+5x}}\right)$

$\lim_{x\to\infty} \frac{9x^2-(9x^2+5x)}{3x+\sqrt{9x^2+5x}} = \lim_{x\to\infty} \frac{-5x}{3x+\sqrt{9x^2+5x}}$

$= \lim_{x\to\infty} \frac{-5x/x}{\frac{3x}{3x} + \sqrt{\frac{9x^2}{x^2} + \frac{5}{x}}} = \frac{-5}{3+\sqrt{9+0}} = \boxed{\frac{-5}{6}}$

(c) (5 points) $\lim_{x\to 2} \frac{x^2+x-6}{x^3-4x^2+4x} = \lim_{x\to 2} \frac{(x-2)(x+3)}{x(x-2)(x-2)}$

$\lim_{x\to 2} \frac{x+3}{x(x-2)} = \frac{2.01+3}{2.01(2.01-2)} = \frac{5.01}{(2.01)(0.01)} = \infty$

$\frac{1.99999+3}{1.99999(1.99999-2)} = \frac{4.99999}{1.99999(-0.01)} = \frac{4.9999}{-0.000199} \approx -\infty$ (DNE)

cause $\lim_{x\to 2^-} \frac{x^2+x-6}{x^3-4x^2+4x} \neq \lim_{x\to 2^+} \frac{x^2+x-6}{x^3-4x^2+4x}$ | $\lim_{x\to 2} \frac{x^2+x-6}{x^3-4x^2+4x} =$ D

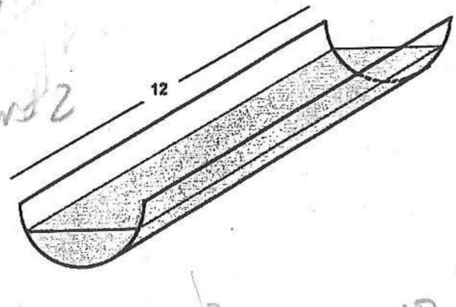
.866 4 or $\left[\frac{4}{3}\pi r^3\right]h$ $\pi(\pi r^2 h)$

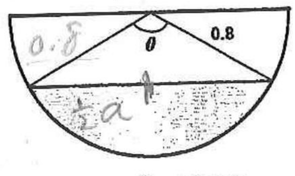
3. (13 points) A water trough is 12 meters long with a semi-circular cross section of radius 0.8 meters. The trough and its cross section are shown below. The pictures are not to scale. The trough is being filled at a rate of 0.3 cubic meters per minute. At what rate is the angle θ shown changing when it is $\frac{2\pi}{3}$ radians? Give an exact answer with units.

(Hint: Start by calculating the shaded area in the cross section.)

$1\cdot 1 = 1$
r=1
rength wd 2

$1.28 - \frac{1.28}{2}$





cross-section of trough

ur of
osines

$$a^2 = (0.8)^2 + (0.8)^2 - 2(0.8)(0.8)(\cos\theta)$$

$$2a = 2(0.8)(0.8)\sin\theta\,\frac{d\theta}{dt}$$

"r" $= \frac{1}{2}(a)$

$$\left(\frac{(\frac{1}{2}a)^2\pi}{2}\right)\left[\frac{4}{3}\right] = V$$

$\sin\frac{2\pi}{3} = \sqrt{3}/2$

$\frac{1}{4}a^2 = \frac{1}{2}a$

$$6\left[2a\left(\tfrac{1}{4}\right)\right]\left[\frac{4}{3}\right](a) \quad \frac{dV}{dt}$$

$\cos\frac{2\pi}{3} = -0.5$

$$\frac{6}{2}\pi\left[\sqrt{(0.8)^2+(0.8)^2-2(0.8)(0.8)(\cos\theta)}\right]\left[\frac{2(0.8)(0.8)\sin\theta\,\frac{d\theta}{dt}}{2}\right]$$

$$\frac{a^2}{2a} = \frac{a}{2}$$

$0.031084 \approx$ $\boxed{\frac{d\theta}{dt} = \frac{0.3}{1.28\sqrt{3}\sqrt{1.92}}}$

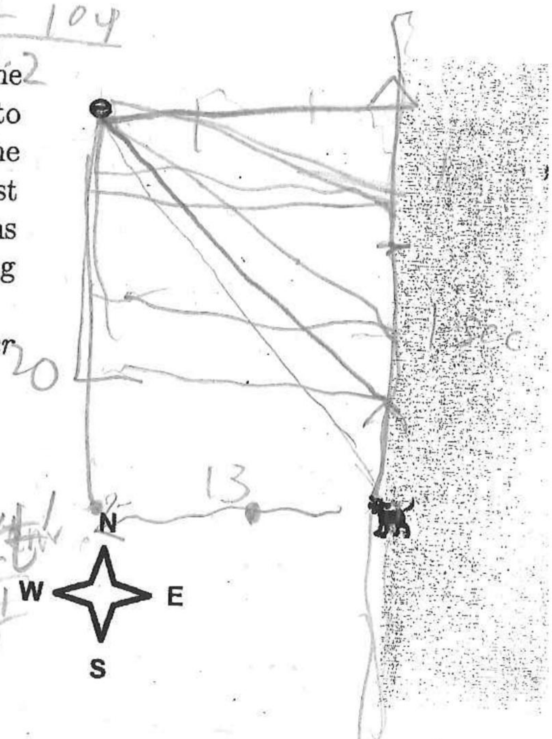
$$\frac{4}{3}(3\pi)\sqrt{1.92}\left[\frac{1.28}{2}\left(\frac{\sqrt{3}}{2}\right)\right]\frac{d\theta}{dt} = 0.3$$

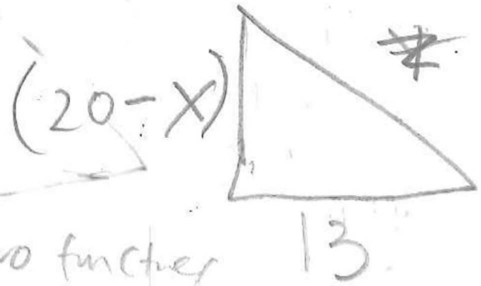
$$\frac{d\theta}{dt} = \frac{0.3(4)}{\pi(\sqrt{3})(1.28)(4)\sqrt{1.92}}$$

10820.16

$60 = \frac{3\ldots}{7}$ $\frac{120}{7} = 17$ $\frac{230 \pm 104}{7.2}$

4. (13 points) Suppose you are at the beach, standing at the edge of the water with your dog. As you look westward into the ocean, you throw a tennis ball out into the water. The ball lands in the water 20 meters North and 13 meters West of where you are standing. The dog swims at 5 m/s but runs at 13 m/s. How far North does the dog run before jumping in the water to minimize his time to get to the ball?
Assume the coastline is straight and ignore both water currents and wind drag.





~~13 + 5 =~~

two functions

$13x + 5z = t$ C

$(20-x)^2 + (13)^2 = (z)^2$ constrain

$z^2 = \sqrt{13^2 + (20-x)}$

$2(20-x)(-1) + 0 = \frac{dz}{dt} 2z$

$\left(\frac{-520+26x}{10}\right)^2$

$\boxed{17.56 \text{ meters North}}$

$\left(-52 + \frac{26}{10}x\right)^2$

$-2(20-x)\frac{dx}{dt} + \cancel{0\frac{dy}{dt}} = \frac{dz}{dt} 2z$

$-2935 + (-1352x)(2) + 40x + 3.6x$

$-2(20-x)(13) = 5(2z)$

$2935 - 230.4x + 3.6x^2$

$-26(20-x) = 10\sqrt{13^2 - (20-x)^2}$

$-520 + 26x = 10\sqrt{169 - (400 + 40x + x^2)}$

$\left(\frac{-520+26x}{10}\right)^2 = 169 - 400 - 40x - x^2$

$x = \frac{230.4 \pm \sqrt{230.4^2 - 4(3.6)(29\ldots)}}{2(3.6)}$

$y^2 = \frac{-56 - x^4}{-2x^2}$ $y = \sqrt{\frac{-56 - x^4}{-2x^2}}$

5. (12 total points) Consider the curve defined by the equation $x^4 - 2x^2y^2 = -56$.

(a) (6 points) Find all points on the curve with a horizontal tangent line. (See back of first page for more work hand)

$4x^3 - 4xy^2 - 2x^2(2y)\frac{dy}{dx} = 0$

$4x^3 - 4xy^2$

$\frac{4x^3 - 4xy^2}{2x^2\,2y} = \frac{dy}{dx}$

$\frac{4x^3 - 4xy^2}{2x^2\,2y} = 0$

$\frac{x(x^2 - y^2)}{y^2 y}$

$x^2 - \frac{-56 - x^4}{-2x^2}$

$\frac{4x(x^2 - y^2)}{4x^2y} = 0$

$\boxed{(-\sqrt[4]{56},\ 0)}$

$x^4 = 56$

$\boxed{\left(\sqrt[4]{56}, \sqrt{\frac{-112}{-2\sqrt{56}}}\right)}$

(b) (3 points) Find the equation of the tangent line at the point $(2, 3)$.

$\frac{dy}{dx} = \frac{4(2)^3 - 4(2)(3)^2}{2(2)^2\,2(3)} = \frac{32 - 72}{48} = \frac{-40}{48} = -\frac{20}{24}$

$= -\frac{5}{6}$ $\boxed{y - 3 = -\frac{5}{6}(x - 2)}$

(c) (3 points) Let P be the point on the curve near $(2, 3)$ with x-coordinate 2.1. Find an approximate value of the y-coordinate of P. Round your answer to three digits after the decimal.

$L_x = 3 + -\frac{5}{6}(x - 2)$

$L_x = 3 + -\frac{5}{6}(2.1 - 2) = \boxed{2.917}$

6. (14 total points) Consider the curve given by the following parametric equations:

$$x(t) = t^3 - 4t, \quad y(t) = t^2; \quad -\infty < t < \infty$$

(a) (7 points) Find the equations of the <u>two</u> tangent lines to the curve at the point $(0, 4)$.

$x'(t) = 3t^2 - 4$ $y'(t) = 2t$ $y = t^2$

$0 = t^3 - 4t$ $4 = t^2$ $\sqrt{y} = t$

$0 = t(t^2 - 4)$ $\pm 2 = t$ $\frac{dy}{dx} = \frac{2t}{3t^2-4} = \frac{2(2)}{3(2)^2}$

$0 = t(t-2)(t+2)$

$\cancel{x} = (\sqrt{y})^3 - 4(\sqrt{y})$ $\frac{dy}{dx} = \frac{1}{\frac{3y}{2\sqrt{y}} - \frac{4}{2\sqrt{y}}}$

$1 = \frac{3(\sqrt{y})^2}{2\sqrt{y}} y' - \frac{4}{2\sqrt{y}} y'$ $\frac{dy}{dx} = \frac{2\sqrt{y}}{3y-4} = \frac{2\sqrt{4}}{3(4)}$

$\frac{dy}{dx}(-2) = \frac{-4}{12-4} = \frac{-4}{8} = \frac{-1}{2}$ $= \frac{4}{8}$

$$\boxed{\begin{array}{l} y - 4 = \frac{1}{2}(x - 0) \\ y - 4 = \frac{-1}{2}(x - 0) \end{array}}$$

(b) (7 points) Let $g(t)$ be the slope of the curve at time t. Compute the instantaneous rate of change of $g(t)$ at time $t = 1$.

$\frac{dy}{dx} = \frac{2\sqrt{y}}{3y-4}$ $\frac{dy}{dx} = \frac{2t}{3t^2-4} = \frac{2(1)}{3(1)-4}$

$\frac{dy}{dx} = \frac{2}{-1} = \boxed{-2}$

7

7. (18 total points)Consider the function $f(x) = \frac{x^2}{x^2-4}$.

(a) (3 points) Find the vertical asymptotes of $y = f(x)$, if there are any.

$f(x) = \frac{x^2}{(x-2)(x+2)}$ $\boxed{x=2,\ x=-2}$

(b) (3 points) Find the horizontal asymptotes of $y = f(x)$, if there are any.

$\lim_{x\to\infty} f(x) = \frac{x^2}{x^2-4} = \frac{1}{1} = \boxed{1}$ or $\boxed{y=1}$

(c) (3 points) Find the intervals where the function is increasing and the intervals where the function is decreasing.

$f'(x) = \frac{2x(x^2-4) - x^2(2x)}{(x^2-4)^2}$ $(-\infty, -2)\ (-2,0)\ (0,2)\ ($

$f'(-100) = +$

$0 = \frac{2x(x-2)(x+2) - x^2(2x)}{[(x-2)(x+2)]^2}$ $f'(-3) = \frac{-6(5) - 9(}{25}$

$=30$ $= +$

$f'(-1) = \frac{-2(-3) - (1)(-1)}{(-3)^2}$

$f'(1) = \frac{2(-3) - (1\cdot 2)}{9}$ $= -$ $= +$

$f'(3) = \frac{6(5) - 54}{25} = \frac{30-54}{25} = -$

Increasing $(-\infty, -2)$ and $(-2, 0)$ | Decreasing $(0, 2)$ and $(2, \infty$

$(-\infty, 0)\ (0, \frac{3}{8})\ (\frac{3}{8}, \infty)$

$-8x \pm 3$

$x = \frac{3}{8}$

Recall that the function is $f(x) = \dfrac{x^2}{x^2-4}$.

$\dfrac{2x^3 - 8x - 2x^3}{x^4 - 8x^2 + 16} = -8$

(d) (3 points) Find the intervals where the function is concave up and the intervals where the function is concave down.

$\dfrac{2x(x^2-4) - x^2(2x)}{(x^2-4)^2}$ $f''(x) = \dfrac{-8(x^4 - 8x^2 + 16) + 8x(4x^3 \ldots}{((x^2-4)^2)^2}$

$128 = x^3(-8x$

$0 = \dfrac{-8x^4 + 64x^2 - 128 + 32x^3 - 128x^2}{(x-2)(x-2)(x+2)(x+2)}$ $f''(-1) = -$

$0 = -8x^4 - 64x^2 + 3x^3 - 128$ $128 = -8x^4 + 3x^3$ $f(\frac{1}{100}) =$

$\boxed{\text{concave up: } (0, \frac{3}{8})}$ $\boxed{\text{concave down } (-\infty, 0)(\frac{3}{8}, \infty)}$ $f(1) = -$

(e) (3 points) Find all critical numbers of the function and characterize them as the x-coordinates of local minima, local maxima, or neither.

$\boxed{0, \frac{3}{8}, 2, -2}$

$x = -2$ neither

$x = 0$ = local maxi

$x = \frac{3}{8}$ = neither

$x = 2$ = neither

$f'(-3)$ = positive derivative

$f'(-1)$ = positive derivative

$f'(1)$ = negative derivative

$f'(3)$ = negative derivative

(f) (3 points) Sketch the graph of $y = f(x)$ on the axis provides below. Be sure to include asymptotes in your picture. Also mark the coordinates of all local maxima, local minima, and inflection points (if any exist).

$\frac{1}{3}$

$\dfrac{1/100^2}{100^2 - 4}$

$\dfrac{4.001}{4.001 - 4} = \dfrac{4.2}{101}$

horizontal asymptote $(y = 1)$

$\dfrac{1}{-4} =$