Math 125 C

Webassign office hours
11:00 AM - 3:00 PM
this Thursday in MSC

MSC M-T
 11-5

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Exam Dates:

Exam 1, Thursday, July 18
Exam 2, Thursday, August 8
Final Friday August 23

Make name tag

Entry task:
Find (or look up) the following:
① $\frac{d}{dx} x = 1$
② $\frac{d}{dx} x^2 = 2x$
③ $\frac{d}{dx} \ln x = \frac{1}{x}$
④ $\frac{d}{dx} \sin x = \cos x$
⑤ $\frac{d}{dx} \cos x = -\sin x$

First Homework Due Wed, July 3

Classroom guidelines

① Entry task
② Lots of Questions / Comments (small class, easy to make individual)
   a) No cold calling
③ Missing class matters more in the summer
④ Formal feedback after first midterm

Goal: Somebody hands you the derivative of a function. Can you tell them the original function? How? Why? Has many applications.

Math 124
$a(t)$ = acceleration
$v(t)$ = velocity
$s(t)$ = Position
Math 125

Definition: A function F is called an antiderivative of f if $F'(x) = f(x)$

$$|a_n x^n| \leq |a_n| |x|^n$$
$$\leq |a_n| |a^n|$$

Ex: We can turn derivative info into antiderivative info

function	derivative
x	1
x^2	$2x$
$\ln x$	$\frac{1}{x}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$

ASK How can we turn this into an antiderivative table?

function	antiderivative
1	x
$2x$	x^2
$\frac{1}{x}$	$\ln x$
$\cos x$	$\sin x$
$-\sin x$	$\cos x$
x	?
$\sin x$	?
$\ln x$	?

other examples:

1. If $f(x) = x^6$ an antiderivative is $F(x) = \frac{1}{7} x^7$
2. $f(x) = \cos(x) + \frac{1}{x} + e^x$, , , $F(x) = \sin x + \ln x + e^x$

Note: These are not the only antiderivatives

For $f(x) = x$, have $F(x) = \frac{x^2}{2}$, $F(x) = \frac{x^2}{2} + 5$,

$F(x) = \frac{x^2}{2} - e^{\pi}$,

all have similar form.

Theorem: If $F(x)$ is an antiderivative of $f(x)$, then ALL other antiderivatives have the form.

$f(x) + C$ (THE GENERAL ANTIDERIVATIVE)

"Proof": If $F'(x) = G'(x) = f(x)$, then

$$0 = G'(x) - F'(x) = (G-F)'(x).$$

So $F-G$ has zero rate of change, i.e., it's constant.

$G(x) - F(x) = C$, so $G(x) = F(x) + C$

$\square$

Ex: The general antiderivative of

$$f(x) = x^2 + 4$$

$$F(x) = \frac{x^3}{3} + 4x + C$$

Warning: ① Don't forget the "$+C$"

② Antiderivatives are "harder" to compute than derivatives

Project Loveless's list

Ex: Find general antiderivative of

$$f(x) = \sqrt{x} + \sin x$$

$$= x^{1/2} + \sin x$$

$$F(x) = \frac{1}{\frac{1}{2}+1} x^{\frac{1}{2}+1} - \cos x + C$$

$$= \frac{2}{3} x^{3/2} - \cos x + C$$

no quotient rule

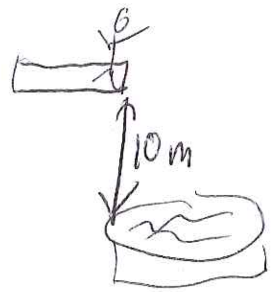
Ex: $f(u) = \frac{u - \sqrt{u}}{u^2} + \sec^2(u)$

$$= \frac{u}{u^2} - \frac{\sqrt{u}}{u^2} + \sec^2(u)$$

$$= \frac{1}{u} - u^{-3/2} + \sec^2(u)$$

$F(u) = \ln|u| - \frac{1}{(-\frac{1}{2})} u^{-1/2} + \tan(u) +$

$$= \boxed{\ln|u| + 2u^{-1/2} + \tan(u) + C}$$



Motivation: (Like HW)

Suppose Ron steps off a diving board 10 meters above a pool. How much time will pass before he hits the water? Assume gravity is the only relevant force.

Solution:

~~From Physics~~ Let t = time passed since Ron steps off

Physics says $a(t) = -9.8 \ m/sec^2$

acceleration at time t.

orient up means move meters

~~Since~~ Then $v(t) = -9.8t + C$ m/sec

velocity

Since Ron steps off, we have $v(0) = 0$, so

$0 = -9.8(0) + C = C$. so $v(t) = -9.8t$.

Then $s(t) = -\frac{9.8}{2} t^2 + D = -4.9t^2 + D$.

Position

constant.

By assumption, we have $s(0) = 10$, so

$10 = -4.9(0)^2 + D = D$, so

$s(t) = -4.9t^2 + 10$.

He hits water when $s(t) = 0$, so we solve

$0 = -4.9t^2 + 10 \iff t^2 = \frac{10}{4.9} \iff t = \pm\sqrt{\frac{10}{4.9}} \approx \boxed{\pm 1.43 \text{ sec}}$

Choose [illegible]