Math 125 - Summer 2019
Exam 1
July 18, 2019

Name: ______________________________

Section: ______________________________

Student ID Number: ______________________________

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- There are 5 questions spanning 5 pages. Make sure your exam contains all these questions.
- You are allowed to use the Ti-30x IIS scientific calculator (**no graphing calculators**) and one **hand-written** 8.5 by 11 inch page (front and back) of notes.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.** Give exact answers wherever possible.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 60 minutes to complete the exam. Budget your time wisely.
SPEND NO MORE THAN 10-15 MINUTES PER PAGE!

GOOD LUCK!

1. (12 points) Evaluate the integrals:

(a) $\int \frac{x^4+2x}{x^2} + e^x + 5 \; dx$

Solution:

$$\int \frac{x^4+2x}{x^2} + e^x + 5 \; dx = \int x^2 + \frac{2}{x} + e^x + 5 \; dx = \frac{x^3}{3} + 2\ln|x| + e^x + 5x + C$$

(b) $\int_0^{\pi/4} \frac{\sin(x)}{\cos^3(x)} \; dx$

Solution: Let $u = \cos x$, so $du = -\sin x dx$. Then the integral becomes

$$\int_0^{\pi/4} \frac{\sin(x)}{\cos^3(x)} \; dx = -\int_{\cos 0}^{\cos(\pi/4)} \frac{du}{u^3} = -\int_1^{1/\sqrt{2}} u^{-3} du = -\frac{u^{-2}}{-2}\Big|_1^{1/\sqrt{2}} = \frac{1}{2u^2}\Big|_1^{1/\sqrt{2}} = \frac{1}{2\cdot\frac{1}{2}} - \frac{1}{2} = \frac{1}{2}$$

(c) $\int x\sqrt{1+2x} \; dx$

Solution: Let $u = 1+2x$, so $x = (u-1)/2$, and $du = 2dx$, so $dx = du/2$. Then the integral becomes

$$\int x\sqrt{1+2x} \; dx = \int \frac{x\sqrt{u}}{2} \; du = \frac{1}{2}\int \left(\frac{u-1}{2}\right)\sqrt{u} \; du = \frac{1}{4}\int (u^{3/2} - u^{1/2}) \; du = \frac{1}{4}\left(\frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2}\right) + C.$$

Plugging in $u = 1+2x$ yields

$$\boxed{\frac{(1+2x)^{5/2}}{10} - \frac{(1+2x)^{3/2}}{6} + C}$$

2. (10 points) Kobe Bryant is preparing to come out of retirement to compete for one more championship with the Los Angeles Lakers. In preparation for this Kobe runs along a straight track. A trainer finds that Kobe's acceleration fits to the function $a(t) = 6t - 12$ measured in miles per hour. Assuming after half an hour his velocity is 15/4 miles per hour (i.e. $v(1/2) = 15/4$), how much did Kobe run after 5 hours?

   Solution: Kobe's total distance after 5 hours is given by the integral $\int_0^5 |v(t)|\, dt$. Consequently, we first must find the velocity function.

   If $a(t) = 6t - 12$, then $v(t) = 3t^2 - 12t + C$. Given the initial condition on the velocity of $v(1/2) = 15/4$, we deduce that $C = 9$ and so

$$v(t) = 3t^2 - 12t + 9.$$

   The absolute value in total distance makes us consider the roots of $v(t)$:

$$v(t) = 0 \implies 3t^2 - 12t + 9 = 0 \implies 3(t-1)(t-3) = 0 \implies t = 1, 3.$$

   Notice that

$$\int v(t)dt = t^3 - 6t + 9t.$$

   Plugging in the intervals of interest yields

   (*i*)

$$\int_0^1 v(t)dt = t^3 - 6t + 9t\Big|_0^1 = (1 - 6 + 9) - (0 - 0 + 0) = 4.$$

   (*ii*)

$$\int_1^3 v(t)dt = t^3 - 6t + 9t\Big|_1^3 = 27 - 54 + 27 - (1 - 6 + 9) = -4.$$

   (*iii*)

$$\int_3^5 v(t)dt = t^3 - 6t + 9t\Big|_3^5 = (125 - 150 + 45) - (27 - 54 + 27) = 20.$$

   In total we obtain

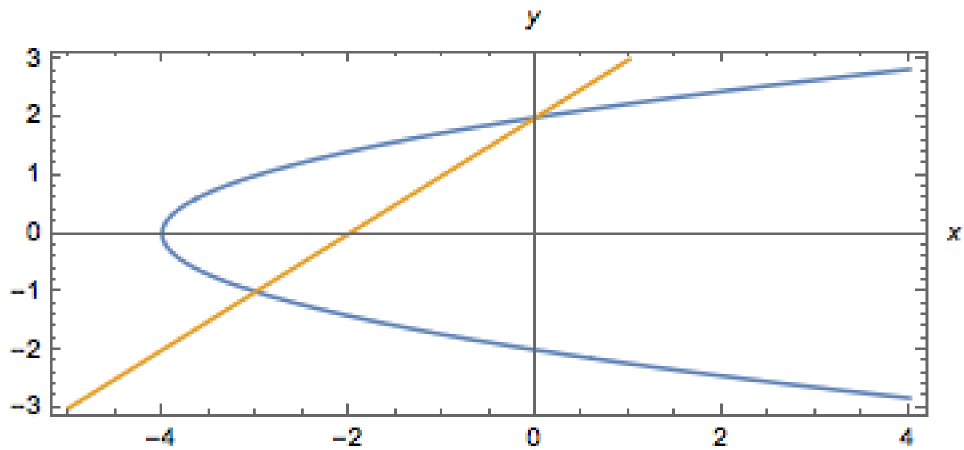
$$\int_0^5 |v(t)|\, dt = 4 - (-4) + 20 = 28 \text{ miles}.$$

3. (8 points) Find the area of the region between the curves

$$x = y^2 - 4, \text{ and } x = y - 2$$

Include a picture of the region.

Solution: Here is a picture of the region:



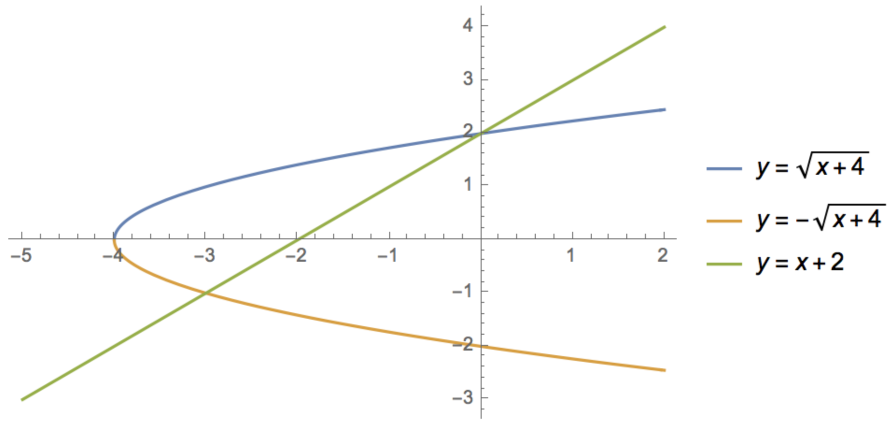
Motivated by this picture, we slice this region horizontally, using dy. First we find the intersection points:

$$y^2 - 4 = y - 2 \implies y^2 - y - 2 = 0 \implies (y-2)(y+1) = 0 \implies y = 2, -1.$$

Since $y - 2$ is on the right over our region, the desired area is

$$\begin{aligned}
\int_{-1}^{2} y - 2 - (y^2 - 4)\ dy &= \int_{-1}^{3} -y^2 - y + 2\ dy \\
&= \frac{-y^3}{3} + \frac{y^2}{2} + 2y \Big|_{-1}^{2} \\
&= \left(\frac{-8}{3} + 2 + 4\right) - \left(\frac{-(-1)^3}{3} + \frac{1}{2} - 2\right) \\
&= \frac{-8}{3} + 6 - \frac{1}{3} - \frac{1}{2} + 2 \\
&= \frac{-9}{3} - \frac{1}{2} + 8 \\
&= \boxed{5 - \frac{1}{2} = 4.5}
\end{aligned}$$

If we were to slice the region vertically, we'd have the following picture:

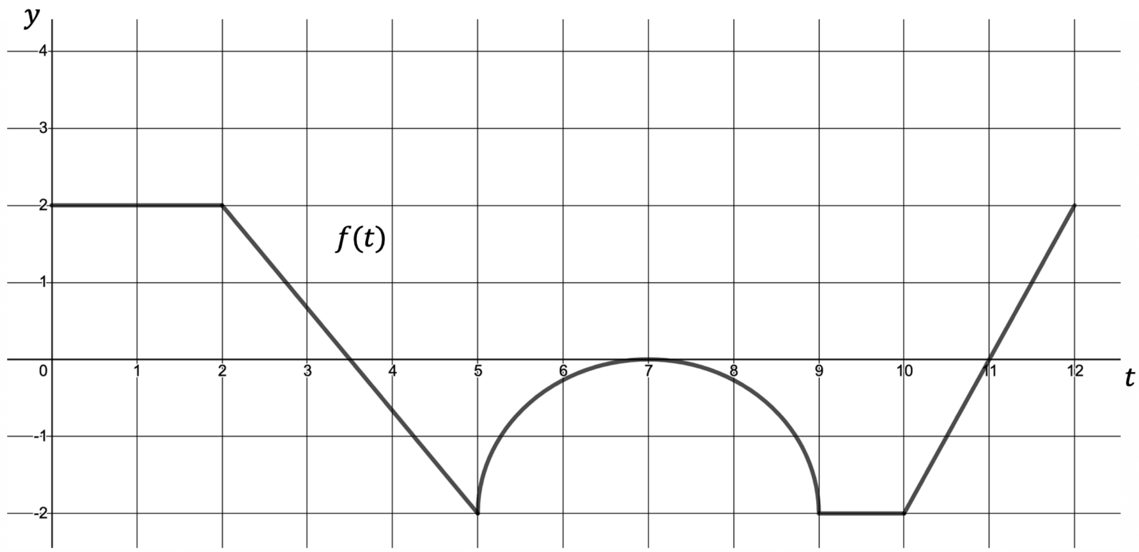


where we switch between going from blue to orange, to blue to green. You can get the formulas by solving for y as a function of x in the original equations for the curves. The x coordinate where we switch is $x = -3$ (we know it's $y = -1$ from before, just plug that in to $x = y - 2$). This gives the area as

$$\int_{-4}^{-3} \sqrt{x+4} - (-\sqrt{x+4})\ dx + \int_{-3}^{0} \sqrt{x+4} - (x+2)\ dx.$$

The substitution $u = x + 4$ simplifies this computation.

4. (10 points) Let $f(t)$ be the piece-wise defined function consisting of the line segments and circular segment shown below.



(a) $\int_0^7 f(t)\ dt =$

Solution: $\int_0^7 f(t)\ dt = \int_0^2 f(t)\ dt + \int_2^5 f(t)\ dt + \int_5^7 f(t)\ dt = 4 + 0 - (4 - \pi) = \pi$ where $\int_2^5 f(t) = 0$ by symmetry.

(b) $\int_2^{10} |f(t)|\ dt =$

Solution: $\int_2^{10} |f(t)|\ dt = \int_2^5 |f(t)|\ dt + \int_5^9 |f(t)|\ dt + \int_9^{10} |f(t)|\ dt = 2(1.5) + (8 - 2\pi) + 2$
$= 13 - 2\pi$

(c) If $g(x) = \int_{x-2}^{x^2} (f(t) - \sqrt{t})\ dt$, evaluate $g'(3)$.

Solution: By the chain rule and FTOC, $g'(x) = (f(x^2) - \sqrt{x^2})2x - (f(x-2) - \sqrt{x-2})(1)$ and so $g'(3) = (f(9) - 3)(6) - (f(1) - 1) = (-2 - 3)(6) - (2 - 1) = -31$.

5. (10 points) Consider the region A enclosed by $y = x^2$, $y = 4$ and $x = 0$.
(i) Compute the volume of the solid of revolution obtained by rotating the region A about the axis $y = -1$ using the disk/washer method. (6 points)

Solution: We find the two endpoints as $x = 0, x = 2$ and each cross-section at fixed $0 \leq x \leq 2$ is a ring with outer radius $R = 5$ and inner radius $r = x^2 + 1$. Therefore the volume is

$$\begin{aligned}\int_0^2 \pi(25 - (x^2+1)^2)dx &= \pi \int_0^2 (-x^4 - 2x^2 + 24)dx \\ &= \frac{544\pi}{15} \approx 113.94\end{aligned}$$

(ii) An oracle says that the volume of the above solid can also be computed using the limit of the following Riemann sum:

$$V = \lim_{n\to\infty} \sum_{i=1}^{n} \left(2\pi \cdot \left(\frac{4i}{n} + 1\right) \cdot \sqrt{\frac{4i}{n}}\right) \cdot \frac{4}{n}.$$

Solve the volume by solving this limit (Hint: don't try to evaluate it directly. Instead, write it as a definite integral and then solve the integral). For this problem, you get no points for copying over the number above without justification. (4 points)

Solution: Using the interval $[0, 4]$ as integration interval and right endpoints, the limit of above Riemann sum is the definite integral

$$\int_0^4 2\pi(x+1)\sqrt{x}dx.$$

Solve the integral yields:

$$\begin{aligned}\int_0^4 2\pi(x+1)\sqrt{x}dx &= 2\pi \left(\int_0^4 x\sqrt{x}dx + \int_0^4 \sqrt{x}dx\right) \\ &= \frac{544\pi}{15} \approx 113.94\end{aligned}$$