7.7 Disjunctive and Hypothetical Syllogisms

Propositions are *categorical* when they affirm or deny the inclusion or exclusion of categories or classes. Syllogisms, arguments consisting of two premises and a conclusion, are called categorical when the propositions they contain are categorical. Up to this point our analysis has been of categorical syllogisms only. However, a syllogism may contain propositions that are not categorical. Such cases are not called categorical syllogisms but are instead named on the basis of the kind of propositions they contain. Here we look briefly at some other kinds of propositions and the syllogisms to which they give rise.

The categorical propositions with which we are familiar are *simple* in the sense that they have a single component, which affirms or denies some class relation. In contrast, some propositions are *compound*, in that they contain more than one component, each of which is itself some other proposition.

Consider first the *disjunctive* (or alternative) proposition. An example is "She was driven either by stupidity or by arrogance." Its two components are "she was driven by stupidity" and "she was driven by arrogance." The disjunctive proposition contains those two component propositions, which are called its *disjuncts*. The disjunctive proposition does not categorically affirm the truth of either one of its disjuncts, but says that at least one of them is true, allowing for the possibility that both may be true.

If we have a disjunction as one premise, and as another premise the denial or contradictory of one its two disjuncts, then we can validly infer that the other disjunct in that disjunction is true. Any argument of this form is a valid **disjunctive syllogism.** A letter writer, critical of a woman nominated for high office by President George W. Bush, wrote:

> In trying to cover up her own illegal peccadillo or stonewall her way out of it, she was driven either by stupidity or arrogance. She's obviously not stupid; her plight must result, then, from her arrogance.[6]

As we use the term in this section, not every disjunctive syllogism is valid. The argument

> She was either arrogant or stupid.
>
> She was arrogant.
>
> Therefore she was not stupid.

is an example of what may be called an invalid disjunctive syllogism. We readily see that, even if the premise were true, she may have been arrogant *and* stupid. The truth of one disjunct of a disjunction does not imply the falsehood of the other disjunct, because both disjuncts of a disjunction can be true. We have

a valid disjunctive syllogism, therefore, only where the categorical premise contradicts one disjunct of the disjunctive premise and the conclusion affirms the other disjunct of the disjunctive premise.

An objection might be raised at this point, based on such an argument as the following:

Either Smith is in New York or Smith is in Paris.

Smith is in New York.

Therefore Smith is not in Paris.

Here the categorical premise affirms one disjunct of the stated disjunction, and the conclusion contradicts the other disjunct, yet the conclusion seems to follow validly. Closer analysis shows, however, that the stated disjunction plays no role in the argument. The conclusion follows enthymematically from the second, categorical premise, with the unexpressed additional premise being the obviously true proposition that "Smith cannot be both in New York and in Paris," which can be stated in disjunctive form as

Either Smith is not in New York or Smith is not in Paris.

When this tacit premise is supplied and the superfluous original disjunction is discarded, the resulting argument is easily seen to be a valid disjunctive syllogism. The apparent exception is not really an exception, and the objection is groundless.

The second kind of compound proposition we consider is the *conditional* (or *hypothetical*) proposition, an example of which is "If the first native is a politician, then the first native lies." A conditional proposition contains two component propositions: The one following the "if" is the *antecedent*, and the one following the "then" is the *consequent*. A syllogism that contains conditional propositions exclusively is called a **pure hypothetical syllogism;** for example,

If the first native is a politician, then he lies.

If he lies, then he denies being a politician.

Therefore if the first native is a politician, then he denies being a politician.

In this argument it can be observed that the first premise and the conclusion have the same antecedent, that the second premise and the conclusion have the same consequent, and that the consequent of the first premise is the same as the antecedent of the second premise. It should be clear that any pure hypothetical syllogism whose premises and conclusion have their component parts so related is a valid argument.

A syllogism that has one conditional premise and one categorical premise is called a **mixed hypothetical syllogism.** Two valid forms of the mixed

hypothetical syllogism have been given special names. The first is illustrated by

If the second native told the truth, then only one native is a politician.

The second native told the truth.

Therefore only one native is a politician.

Here the categorical premise affirms the antecedent of the conditional premise, and the conclusion affirms its consequent. Any argument of this form is valid and is said to be in the *affirmative mood* or ***modus ponens*** (from the Latin *ponere*, meaning "to affirm"). One must not confuse the valid form *modus ponens* with the clearly invalid form displayed by the following argument:

If Bacon wrote *Hamlet*, then Bacon was a great writer.

Bacon was a great writer.

Therefore Bacon wrote *Hamlet*.

This argument differs from *modus ponens* in that its categorical premise affirms the consequent, rather than the antecedent, of the conditional premise. Any argument of this form is said to commit the **fallacy of affirming the consequent.**

The other valid form of mixed hypothetical syllogism is illustrated by:

If the one-eyed prisoner saw two red hats, then he could tell the color of the hat on his own head.

The one-eyed prisoner could not tell the color of the hat on his own head.

Therefore the one-eyed prisoner did not see two red hats.

Here the categorical premise denies the consequent of the conditional premise, and the conclusion denies its antecedent. Any argument of this form is valid and is said to be in the form ***modus tollens*** (from the Latin *tollere*, meaning "to deny"). One must not confuse the valid form *modus tollens* with the clearly invalid form displayed by the following argument:

If Carl embezzled the college funds, then Carl is guilty of a felony.

Carl did not embezzle the college funds.

Therefore Carl is not guilty of a felony.

This argument differs from *modus tollens* in that its categorical premise denies the antecedent, rather than the consequent, of the conditional premise. Any argument of this form is said to commit the **fallacy of denying the antecedent.**

OVERVIEW

Principal Kinds of Syllogisms

1. **Categorical syllogisms,** which contain only categorical propositions affirming or denying the inclusion or exclusion of categories. Example:

   All *M* is *P*.

   All *S* is *M*.

   Therefore all *S* is *P*.

2. **Disjunctive syllogisms,** which contain a compound, disjunctive (or alternative) premise asserting the truth of at least one of two alternatives, and a premise that asserts the falsity of one of those alternatives. Example:

   Either *P* is true or *Q* is true.

   P is not true.

   Therefore *Q* is true.

3. **Hypothetical syllogisms,** which contain one or more compound, hypothetical (or conditional) propositions, affirming that if one of its components (the antecedent) is true then the other of its components (the consequent) is true. Two subtypes are distinguished:

   A. Pure hypothetical syllogisms contain conditional propositions only. Example:

   If *P* is true, then *Q* is true.

   If *Q* is true, then *R* is true.

   Therefore if *P* is true, then *R* is true.

   B. Mixed hypothetical syllogisms contain both a conditional premise and a categorical premise.

   If the categorical premise affirms the truth of the antecedent of the conditional premise, and the consequent of that conditional premise is the conclusion of the argument, the form is valid and is called *modus ponens*. Example:

   If *P* is true, then *Q* is true.

   P is true.

   Therefore *Q* is true.

   If the categorical premise affirms the falsity of the consequent of the conditional premise, and the falsity of the antecedent of that

(Continued)

conditional premise is the conclusion of the argument, the form is valid and is called *modus tollens*. Example:

If *P* is true, then *Q* is true.

Q is false.

Therefore *P* is false.

EXERCISES

Identify the form and discuss the validity or invalidity of each of the following arguments.

EXAMPLE

1. If a man could not have done otherwise than he in fact did, then he is not responsible for his action. But if determinism is true, it is true of every action that the agent could not have done otherwise. Therefore, if determinism is true, no one is ever responsible for what he does.

—Winston Nesbit and Stewart Candlish, "Determinism and the Ability to Do Otherwise," *Mind*, July 1978

SOLUTION

This is a pure hypothetical syllogism. Valid.

2. Men, it is assumed, act in economic matters only in response to pecuniary compensation or to force. Force in the modern society is largely, although by no means completely, obsolete. So only pecuniary compensation remains of importance.

—John Kenneth Galbraith, *The New Industrial State*, (Boston: Houghton Mifflin, 1967)

3. If each man had a definite set of rules of conduct by which he regulated his life he would be no better than a machine. But there are no such rules, so men cannot be machines.

—A. M. Turing, "Computing Machinery and Intelligence," *Mind*, vol. 59, 1950

4. If the second native told the truth, then the first native denied being a politician. If the third native told the truth, then the first native denied

being a politician. Therefore if the second native told the truth, then the third native told the truth.

*5. If the one-eyed prisoner does not know the color of the hat on his own head, then the blind prisoner cannot have on a red hat. The one-eyed prisoner does not know the color of the hat on his own head. Therefore the blind prisoner cannot have on a red hat.

6. If all three prisoners have on white hats, then the one-eyed prisoner does not know the color of the hat on his own head. The one-eyed prisoner does not know the color of the hat on his own head. Therefore all three prisoners have on white hats.

7. The stranger is either a knave or a fool. The stranger is a knave. Therefore the stranger is no fool.

8. If the first native is a politician, then the third native tells the truth. If the third native tells the truth, then the third native is not a politician. Therefore if the first native is a politician, then the third native is not a politician.

9. Mankind, he said, judging by their neglect of him, have never, as I think, at all understood the power of Love. For if they had understood him they would surely have built noble temples and altars, and offered solemn sacrifices in his honor; but this is not done.

—Plato, *Symposium*

*10. I have already said that he must have gone to King's Pyland or to Capleton. He is not at King's Pyland, therefore he is at Capleton.

—Arthur Conan Doyle, *The Adventure of Silver Blaze*

11. If then, it is agreed that things are either the result of coincidence or for an end, and that these cannot be the result of coincidence or spontaneity, it follows that they must be for an end.

—Aristotle, *Physics*

12. There is no case known (neither is it, indeed, possible) in which a thing is found to be the efficient cause of itself; for in such a case it would be prior to itself, which is impossible.

—Thomas Aquinas, *Summa Theologiae*, I, question 2, art. 3

13. Either wealth is an evil or wealth is a good; but wealth is not an evil; therefore wealth is a good.

—Sextus Empiricus, *Against the Logicians*, second century A.D.

14. I *do* know that this pencil exists; but I could not know this, if Hume's principles were true; *therefore*, Hume's principles, one or both of them, are false.

—G. E. Moore, *Some Main Problems of Philosophy* (New York: Allen & Unwin, 1953)

***15.** It is clear that we mean something, and something different in each case, by such words [as *substance*, *cause*, *change*, etc.]. If we did not we could not use them consistently, and it is obvious that on the whole we do consistently apply and withhold such names.

—C. D. Broad, *Scientific Thought*, 1923

16. If number were an idea, then arithmetic would be psychology. But arithmetic is no more psychology than, say, astronomy is. Astronomy is concerned, not with ideas of the planets, but with the planets themselves, and by the same token the objects of arithmetic are not ideas either.

—Gottlob Frege, *The Foundations of Arithmetic*, 1893

17. . . . If a mental state is to be identical with a physical state, the two must share all properties in common. But there is one property, spatial localizability, that is not so shared; that is, physical states and events are located in space, whereas mental events and states are not. Hence, mental events and states are different from physical ones.

—Jaegwon Kim, "On the Psycho-Physical Identity Theory," *American Philosophical Quarterly*, 1966

18. When we regard a man as morally responsible for an act, we regard him as a legitimate object of moral praise or blame in respect of it. But it seems plain that a man cannot be a legitimate object of moral praise or blame for an act unless in willing the act he is in some important sense a "free" agent. Evidently free will in some sense, therefore, is a precondition of moral responsibility.

—C. Arthur Campbell, *In Defence of Free Will*, 1938

19. In spite of the popularity of the finite-world picture, however, it is open to a devastating objection. In being finite the world must have a limiting boundary, such as Aristotle's outermost sphere. That is impossible, because a boundary can only separate one part of space from another. This objection was put forward by the Greeks, reappeared in the scientific skepticism of the early Renaissance and probably occurs to any schoolchild who thinks about it today. If one accepts the objection, one must conclude that the universe is infinite.

—J. J. Callahan, "The Curvature of Space in a Finite Universe," *Scientific American*, August 1976

***20.** Total pacifism might be a good principle if everyone were to follow it. But not everyone does, so it isn't.

—Gilbert Harman, *The Nature of Morality*, 1977

7.8 The Dilemma

The **dilemma** is a common form of argument in ordinary language. It is, in essence, an argumentative device in which syllogisms on the same topic are combined, sometimes with devastating effect. Each of the constituent syllogisms may be quite ordinary, and therefore the dilemma is not of special importance from a strictly logical point of view. But the premises of the syllogisms so combined are formulated disjunctively, and devised in a way designed to trap the opponent by forcing him to accept one or the other of the disjuncts. Thus the opponent is forced to accept the truth of the conclusion of one or the other of the syllogisms combined. When this is done successfully, the dilemma can prove to be a powerful instrument of persuasion.

We say somewhat loosely that a person is "in" a dilemma (or "impaled on the horns of a dilemma") when that person must choose between two alternatives, both of which are bad or unpleasant. The dilemma is a form of argument intended to put one's opponent in just that kind of position. In debate, one uses a dilemma to offer alternative positions to one's adversary, from which a choice must be made, and then to prove that no matter which choice is made, the adversary is committed to an unacceptable conclusion.

The distinguished physicist Richard Feynman, recounting his experiences in the 1986 investigation of the catastrophic explosion of the *Challenger* space shuttle, was caustic in his criticism of mismanagement by administrators in the National Aeronautics and Space Administration (NASA). He said:

> Every time we talked to higher level managers, they kept saying they didn't know anything about the problems below them. . . . Either the group at the top didn't know, in which case they should have known, or they did know, in which case they were lying to us.[7]

An attack of this kind is designed to push the adversaries (in this case the NASA administrators) into a corner and there annihilate them. The only explicitly stated premise of the argument is a disjunction, but one of the disjuncts must obviously be true; Either they knew or they didn't know about the problems below them. And whichever disjunct is chosen, the result for the adversary is very bad. The conclusion of a dilemma can itself be a disjunction (for example, "Either the NASA administrators did not know what they should have known, or they lied") in which case we call the dilemma a