# Case Study 2

## The Origins of Maxwell’s Equations

# Case Study 2

## The Origins of Maxwell's Equations

800 BC: Greek writers describe magnetic materials, the name being derived from the material **magnetite**, which was known to attract iron in its natural state – it was mined in the Greek province of Magnesia in Thessaly.

They also knew about static electricity which arises when amber rods are rubbed with fur – the Greek word for amber is **elektron**.

The first systematic study of magnetic and electric phenomena was published in 1600 by *William Gilbert* in *De Magnete, Magnetisque Corporibus, et de Magno Magnete Tellure*.

# Early History

- *Benjamin Franklin* systematised the laws of electrostatics and defined the concepts of positive and negative electric charges.
- In 1767, *Joseph Priestly* showed that there are no electric forces inside a conducting sphere, the forerunner of the Williams, Faller and Hall experiment of 1971 which demonstrated the the remarkable precision of the inverse square law of electrostatics.
- In the period 1779-1780, Charles-Augustin Coulomb derived experimentally his *Laws of Electrostatics and Magnetostatics*:

$$\boxed{\boldsymbol{f} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\,\boldsymbol{i}_r \quad \text{and} \quad \boldsymbol{f} = \frac{\mu_0 p_1 p_2}{4\pi r^2}\,\boldsymbol{i}_r,}$$

  where $\boldsymbol{i}_r$ is the unit vector directed radially *away* from either charge in the direction of the other.

# Mathematics of Electrostatics and Magnetostatics

In 1812, Siméon-Denis Poisson published his famous *Mémoire sur la Distribution de l'Électricité à la Surface des Corps Conducteurs* and wrote down Poisson's equation for the electrostatic potential

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho_{\mathrm{e}}}{\epsilon_0},$$

where $\rho_e$ is the electric charge density distribution. The electric field strength $\boldsymbol{E}$ is given by

$$\boldsymbol{E} = -\mathrm{grad}\, V.$$

# Mathematics of Electrostatics and Magnetostatics

In 1812, Siméon-Denis Poisson published his famous *Mémoire sur la Distribution de l'Électricité à la Surface des Corps Conducteurs* and wrote down Poisson's equation for the electrostatic potential $V$

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho_\mathrm{e}}{\epsilon_0},$$

where $\rho_e$ is the electric charge density distribution. The electric field strength $\boldsymbol{E}$ is given by

$$\boldsymbol{E} = -\mathrm{grad}\, V.$$

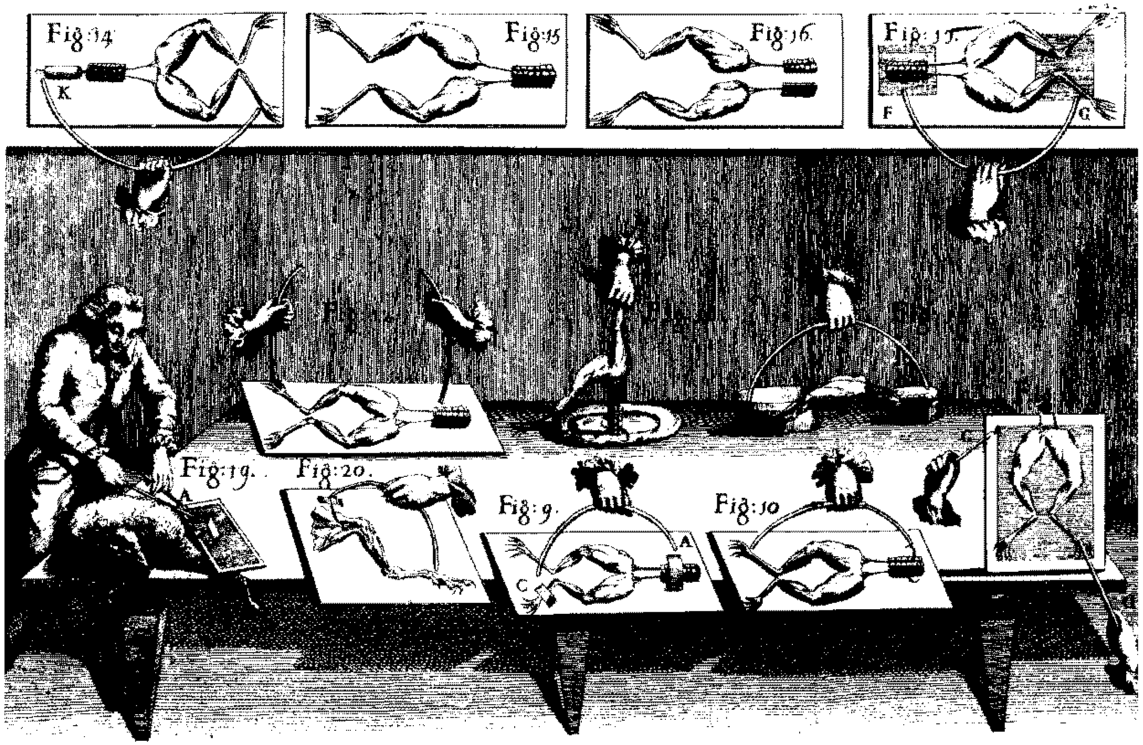
In 1826, the corresponding expressions for the magnetic flux density $\boldsymbol{B}$ in terms of the magnetostatic potential $V_\mathrm{mag}$ were presented:

$$\frac{\partial^2 V_\mathrm{mag}}{\partial x^2} + \frac{\partial^2 V_\mathrm{mag}}{\partial y^2} + \frac{\partial^2 V_\mathrm{mag}}{\partial z^2} = 0,$$

where the magnetic flux density $\boldsymbol{B}$ is given by

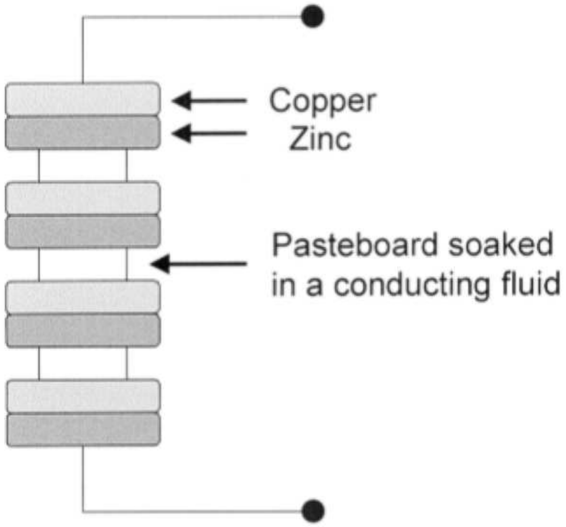
$$\boldsymbol{B} = -\mu_0\, \mathrm{grad}\, V_\mathrm{mag}.$$

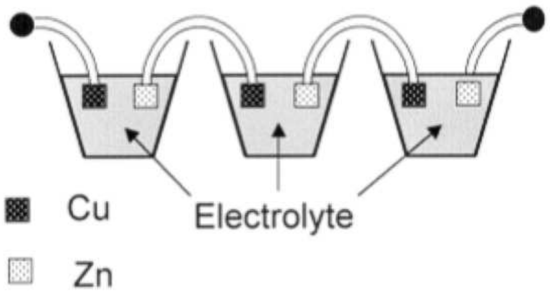
# Luigi Galvani




Luigi Galvani discovered that electrical effects could stimulate the muscular contraction of frogs' legs. In 1791, he showed that, when two dissimilar metals were used to make the connection between nerve and muscle, the same muscular contraction was observed. This was the discovery of *animal electricity*.

# Alessandro Volta



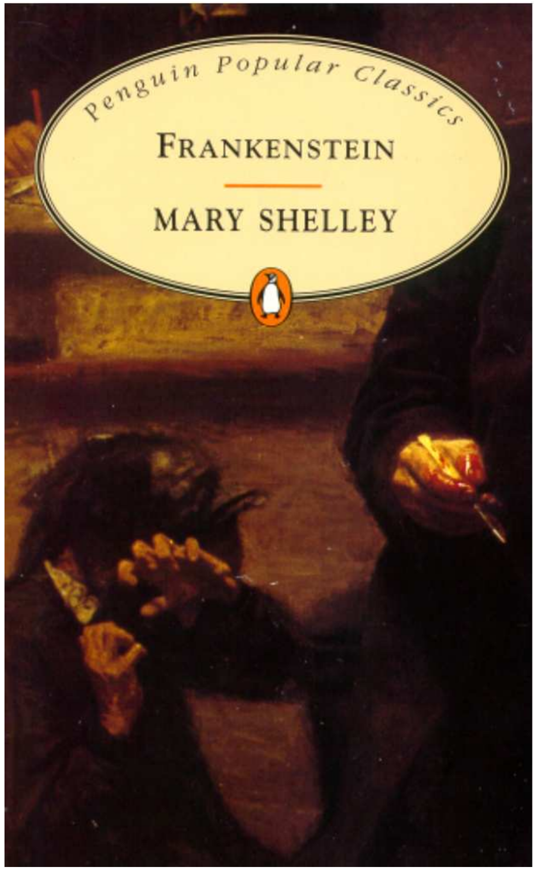

Alessandro Volta suspected that the electric current was associated with the presence of different metals in contact with a moist body. In 1800, he built what became known as a *voltaic pile*, consisting of interleaved layers of copper and zinc separated by layers of pasteboard soaked in a conducting liquid. This led to his construction of his *crown of cups*, which resembles a modern car battery.

The most important outcome was the discovery of a controllable source of electric current.

# Cultural Resonances

Mary Shelley's *Frankenstein*

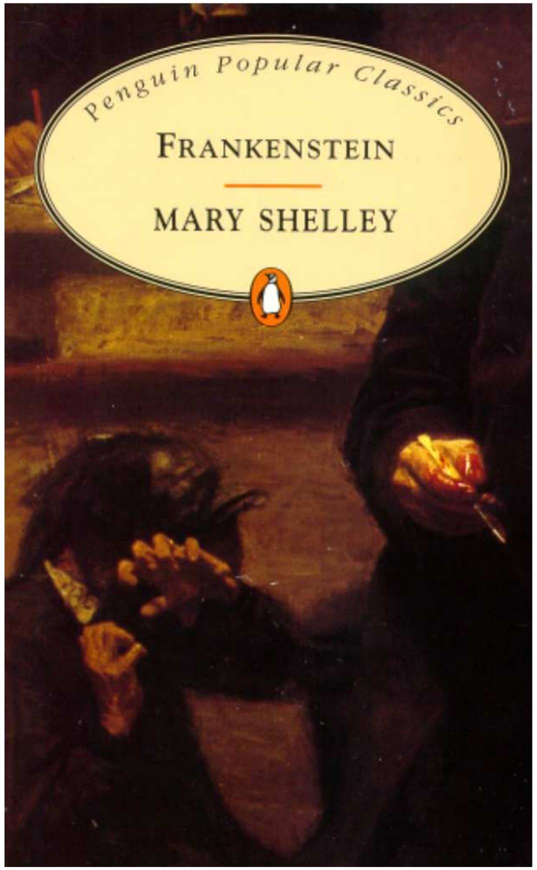
This book was written in 1816. In the preface, the monster was said to be revived by galvanism.



# Cultural Resonances

Mary Shelley's *Frankenstein*

This book was written in 1816. In the preface, the monster was said to be revived by galvanism.




Mozart's *Cosi fan Tutti*

This opera was first performed in 1791. The heavily disguised Ferrando and Giuglielmo are revived by magnetism and galvanism.

# Currents and Magnetism

1820 Hans-Christian Øersted: there is always a magnetic field associated with an electric current – the beginning of the science of electromagnetism.

1820 Jean-Baptiste Biot (1774–1862) and Félix Savart (1791–1841) discovered the dependence of the strength of the magnetic field at distance $\boldsymbol{r}$ from a current element of length $\mathrm{d}\boldsymbol{l}$ in which a current $I$ is flowing, the *Biot-Savart law*

$$\mathrm{d}\boldsymbol{B} = \frac{\mu_0 I \,\mathrm{d}\boldsymbol{l} \times \boldsymbol{r}}{4\pi r^3}.$$

André-Marie Ampère (1775–1836) extended the Biot-Savart law to relate the current flowing though a closed loop to the integral of the component of the magnetic flux density around the loop, *Ampère's law*.

$$\int_C \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{s} = \mu_0 I_{\mathrm{enclosed}}.$$

# Currents and Magnetism

In 1825, Ampère

- represented the magnetic field of a current loop by an equivalent *magnetic shell*.
- formulated the equation for the force between two current elements, $\mathrm{d}\boldsymbol{l}_1$ and $\mathrm{d}\boldsymbol{l}_2$ carrying currents $I_1$ and $I_2$;

$$\boxed{\mathrm{d}\boldsymbol{F}_2 = \frac{\mu_0 I_1 I_2\, \mathrm{d}\boldsymbol{l}_1 \times (\mathrm{d}\boldsymbol{l}_2 \times \boldsymbol{r})}{4\pi r^3}.}$$

1827, Georg Simon Ohm (1787–1854) formulated the relation between potential difference $V$ and the current $I$, *Ohm's law*, $V = RI$.

All these results were known by 1830 and comprise the whole of *static electricity*, the forces between *stationary* charges, magnets and currents.

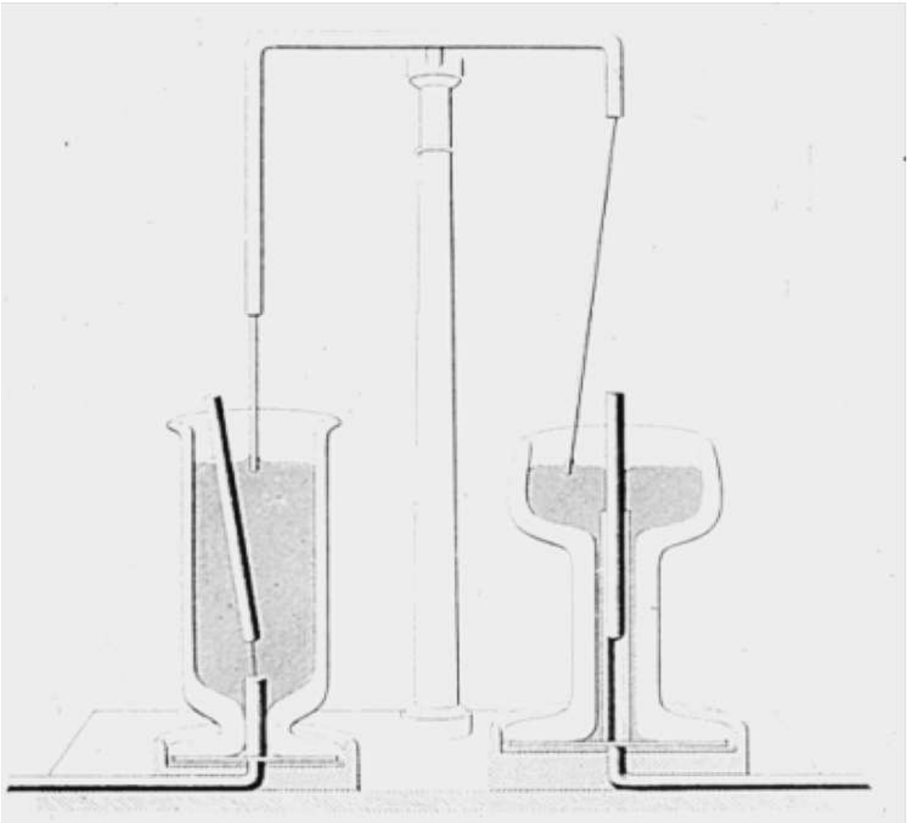
# Michael Faraday
# Mathematics without Mathematics

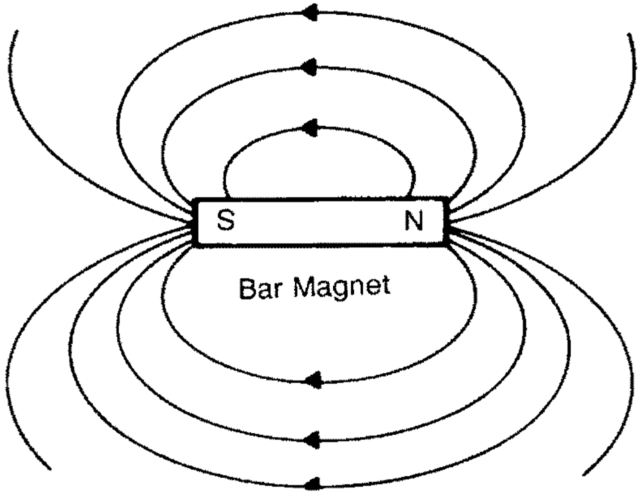
Maxwell's equations deal with *time-varying* phenomena. Over the succeeding 20 years, all the basic experimental features of time-varying electric and magnetic fields were established. The hero of this story is Michael Faraday (1791–1867), an experimenter of genius.

In 1820, at the suggestion of Humphry Davy, Faraday surveyed the many articles submitted to the scientific journals describing electromagnetic effects and began his systematic study of electromagnetic phenomena.

# Faraday's Electric Motors

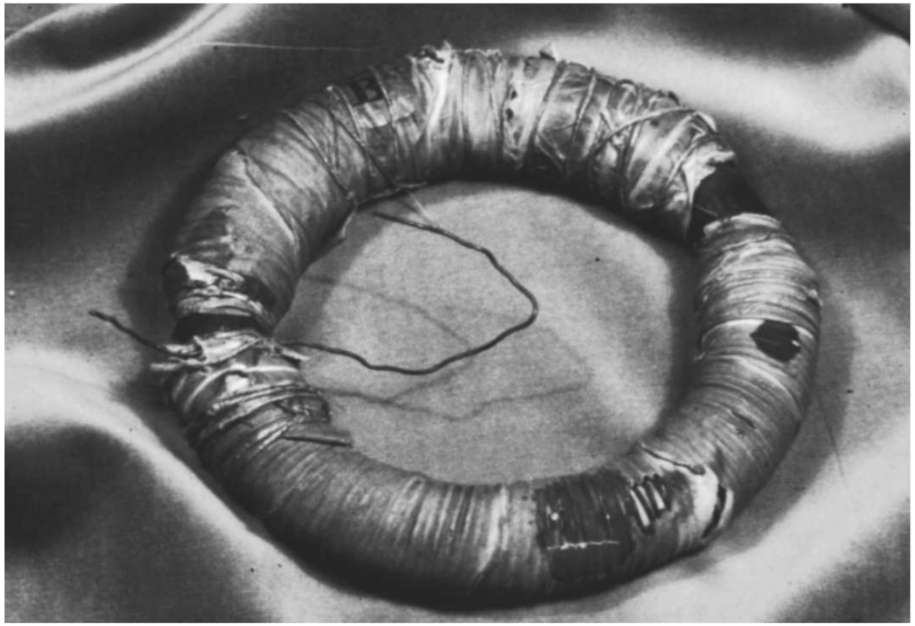
In the course of these experiments, he built the first electric motors.





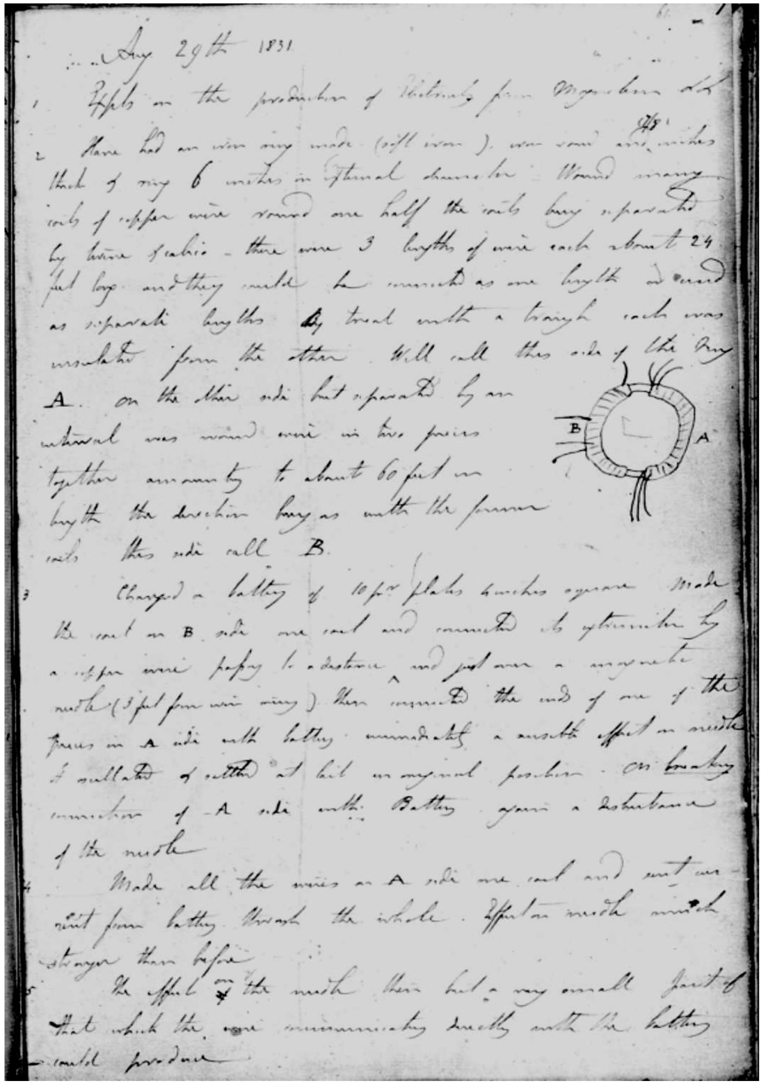

These experiments led Faraday to the concept of *magnetic lines of force*, which he used as a means of visualising the effects of stationary and time-varying magnetic fields.

# The Discovery of Electromagnetic Induction



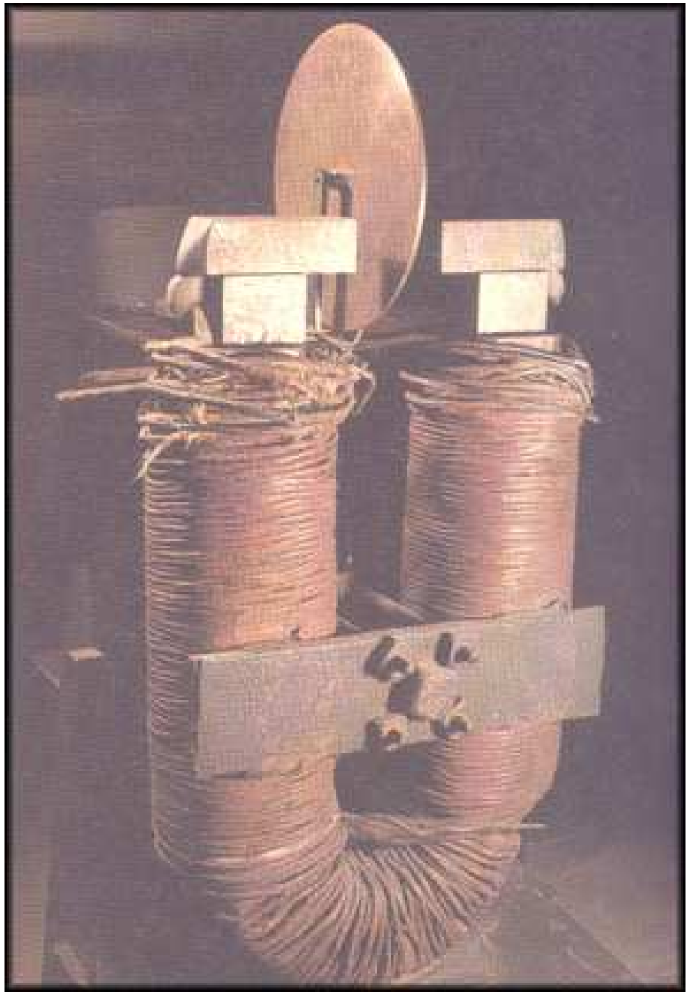
In 1831, Faraday heard about Joseph Henry's experiments in Albany, New York with very powerful electromagnets. He had the idea of observing the strain in the material of the electromagnet caused by the lines of force. He built a strong electromagnet by winding an insulating wire, through which a current could be passed, onto a thick iron ring. The effects of the strain were to be detected with another winding on the ring attached to a galvanometer.

# The Discovery of Electromagnetic Induction



The experiment was conducted on 29 August 1931 and is recorded in Faraday's laboratory notebooks. When the primary circuit was closed, there was a displacement of the galvanometer needle in the secondary winding. Deflections of the galvanometer were only observed when the current in the electromagnet was switched on and off. This was the discovery of *electromagnetic induction*.

# The Invention of the Dynamo



There followed a magnificent series of experiments

- He tried coils of different shapes and sizes and discovered that the iron bar was not needed to create the effect.
- On 28 October 1831, he showed how a continuous electric current could be generated by rotating a copper disc between the poles of the ‘great horse-shoe magnet’ belonging to the Royal Society. This was the invention of the dynamo.

# The Laws of Electromagnetic Induction

As early as 1831, Faraday established the qualitative form his law of induction in terms of the concept of lines of force – *the electromotive force induced in a current loop is directly related to the rate at which magnetic field lines are cut*.

It took Faraday many years to complete all the necessary experimental work to demonstrate the general validity of the law. In 1834, Lenz's law cleared up the problem of the direction of the induced electromotive force in the circuit – *the electromotive force acts in such a direction as to oppose the change in magnetic flux*.

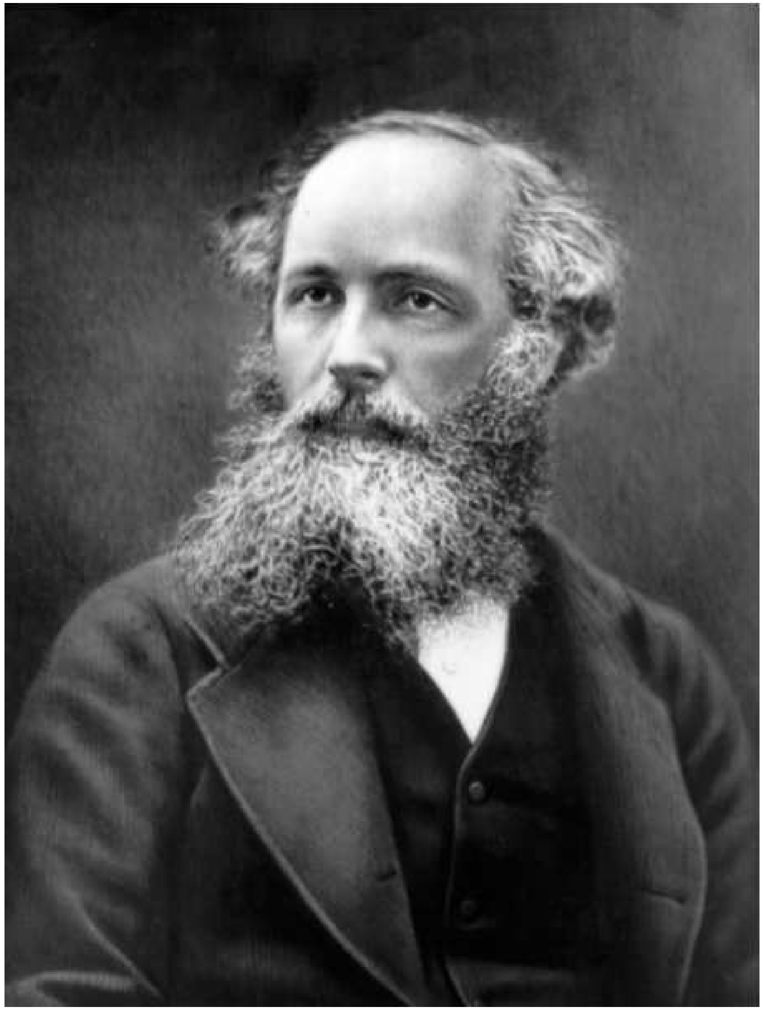
Faraday could not formulate his theoretical ideas mathematically, but he was convinced that the concept of lines of force provided the key to understanding electromagnetic phenomena. In 1846, he speculated in a discourse to the Royal Institution that light might be some form of disturbance propagating along the field lines. He published these ideas in a paper entitled *Thoughts on Ray Vibrations*, but they were received with considerable scepticism.

# Maxwell on Faraday in 1864

‘The electromagnetic theory of light as proposed by (Faraday) is the same in substance as that which I have begun to develop in this paper, except that in 1846 there was no data to calculate the velocity of propagation.’

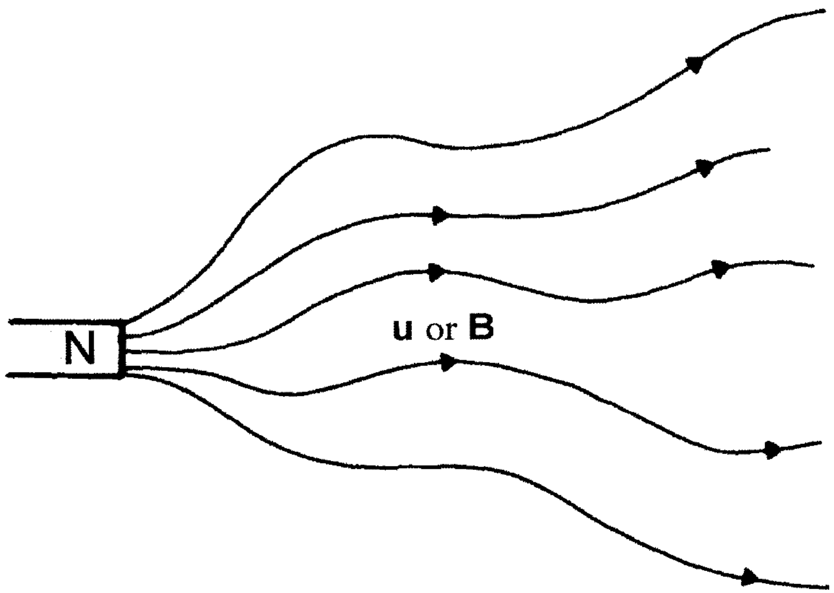
‘As I proceeded with the study of Faraday, I perceived that his method of conceiving of phenomena was also a mathematical one, though not exhibited in the conventional form of mathematical symbols... I found, also, that several of the most fertile methods of research discovered by the mathematicians could be expressed much better in terms of ideas derived from Faraday than in their original form.’

# James Clerk Maxwell



Maxwell was born and educated in Edinburgh. He had a physical imagination which could appreciate the empirical models of Faraday and give them mathematical substance. In 1856, he published the essence of his approach in an essay entitled *Analogies in Nature*. The technique consists of recognising mathematical similarities between quite distinct physical problems and seeing how far one can go in applying the successes of one theory to different circumstances.

# Maxwell and Analogy




For example, the analogy between *incompressible fluid flow* and *magnetic lines of force*. For fluid flow, the equation of continuity is

$$\text{div}\,\rho\boldsymbol{u} = -\frac{\partial\rho}{\partial t}.$$

If the fluid is incompressible, $\rho$ = constant and hence

$$\text{div}\,\boldsymbol{u} = 0$$

# Building up Maxwell's Equations (1)

Maxwell drew an immediate analogy between the behaviour of magnetic field lines and the streamlines of incompressible fluid flow. The velocity $\boldsymbol{u}$ is analogous to the magnetic flux density $\boldsymbol{B}$. For example, if the tubes of force, or steamlines, diverge, the strength of the field decreases, as does the fluid velocity. This suggests

$$\boxed{\mathrm{div}\,\boldsymbol{B} = 0}$$

Faraday's law of electromagnetic induction was first put into mathematical form by Neumann in 1845 – the induced electromotive force $\mathcal{E}$ is proportional to the rate of change of magnetic flux, $\Phi$,

$$\boxed{\mathcal{E} = -\frac{\mathrm{d}\Phi}{\mathrm{d}t},}$$

where $\Phi$ is the total magnetic flux through the circuit.

Note: We have used modern vector notation rather than the more cumbersome set of equations used by Maxwell.

# Building up Maxwell's Equations (2)

Since $\mathcal{E} = \int \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{s}$ and $\Phi = \int \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{S}$, Stokes' theorem leads directly to

$$\boxed{\mathrm{curl}\, \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}.}$$

Since $I_{\mathrm{enclosed}} = \int \boldsymbol{J} \cdot \mathrm{d}\boldsymbol{S}$, Stokes' theorem converts Ampère's law

$$\int_C \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{s} = I_{\mathrm{enclosed}},$$

into

$$\boxed{\mathrm{curl}\, \boldsymbol{H} = \boldsymbol{J}.}$$

# Building up Maxwell's Equations (2)

Since $\mathcal{E} = \int \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{s}$ and $\Phi = \int \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{S}$, Stokes' theorem leads directly to

$$\boxed{\mathrm{curl}\,\boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}.}$$

Since $I_{\mathrm{enclosed}} = \int \boldsymbol{J} \cdot \mathrm{d}\boldsymbol{S}$, Stokes' theorem converts Ampère's law

$$\int_C \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{s} = I_{\mathrm{enclosed}},$$

into

$$\boxed{\mathrm{curl}\,\boldsymbol{H} = \boldsymbol{J}.}$$

Finally, from Poisson's equation, he knew that

$$\boxed{\mathrm{div}\,\boldsymbol{E} = \frac{\rho_{\mathrm{e}}}{\epsilon_0}.}$$

in free space.

# The Primitive Form of Maxwell's Equations

These results form the *primitive* and *incomplete* set of Maxwell's equations.

$$
\begin{aligned}
\text{curl}\, \boldsymbol{E} &= -\frac{\partial \boldsymbol{B}}{\partial t} \\
\text{curl}\, \boldsymbol{H} &= \boldsymbol{J} \\
\text{div}\, \epsilon_0 \boldsymbol{E} &= \rho_\text{e} \\
\text{div}\, \boldsymbol{B} &= 0
\end{aligned}
$$

Maxwell still lacked a physical model for the phenomena of electromagnetism. He developed his solution in 1861-2 in a remarkable series of papers entitled *On physical lines of force*.

Since his earlier work on the analogy between $\boldsymbol{u}$ and $\boldsymbol{B}$, he had become convinced that magnetism was essentially rotational in nature. He began with the model of a rotating vortex tube as an analogue for a tube of magnetic flux.

# Maxwellian Vortices

A rotating vortex tube was taken as an analogue for a tube of magnetic flux.

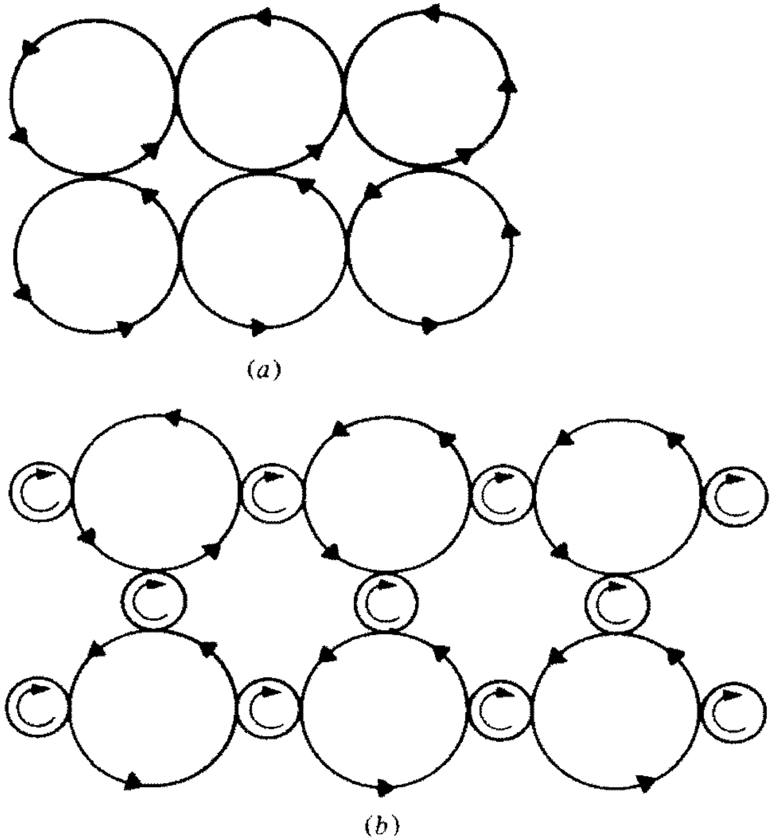
If left on their own, magnetic field lines expand apart, exactly as occurs in the case of a vortex tube if the rotational centrifugal forces are not balanced. The rotational kinetic energy of the vortices as

$$\int_v \rho \boldsymbol{u}^2 \, \mathrm{d}v,$$

where $\rho$ is the density of the fluid and $\boldsymbol{u}$ its rotational velocity. This is similar to the expression for the energy of a magnetic field, $\int_v (\boldsymbol{B}^2/2\mu_0) \, \mathrm{d}v$. Thus, again $\boldsymbol{u}$ is analogous to $\boldsymbol{B}$.

Maxwell postulated that everywhere the local magnetic field strength should be proportional to the angular velocity of the vortex, so that the angular momentum vector is parallel to the axis of the vortex, and so parallel to the magnetic field direction.

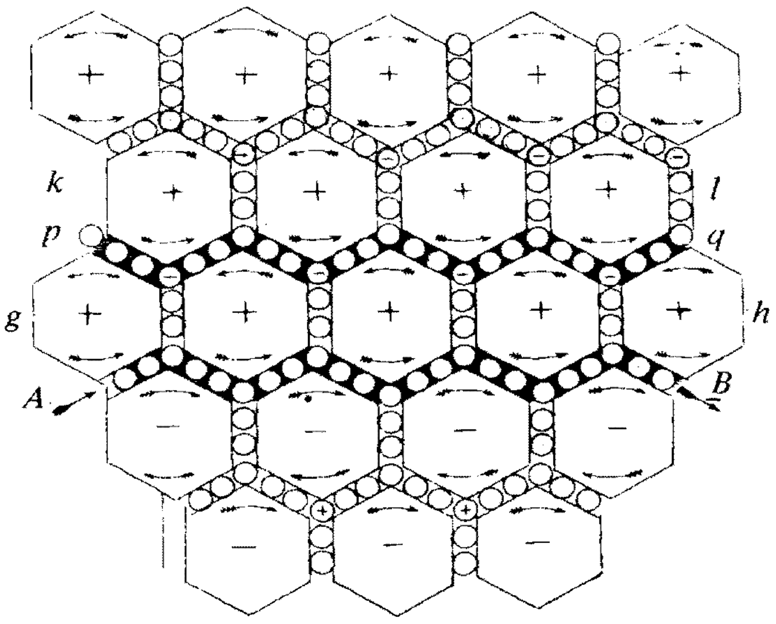
# Maxwellian Vortex Tubes




Maxwell began with a model in which the whole of space was filled with vortex tubes. However, friction between neighbouring vortices would cause them to dissipate.

Maxwell therefore inserted 'idler-wheels', or 'ball-bearings', between the vortices so that they could all rotate in the same direction without friction.

# Maxwellian Vortex Tubes

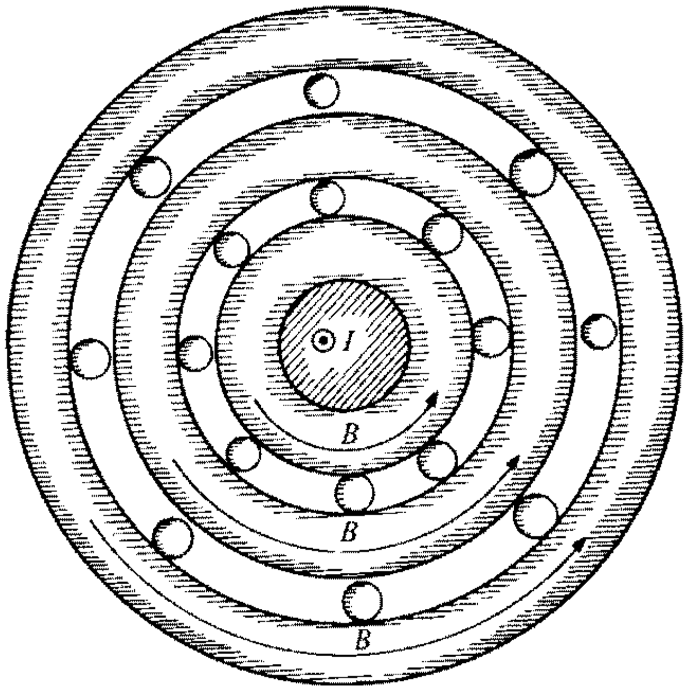



Maxwell's original picture of the vortices represented by rotating hexagons. The diagram shows a current flowing through vortices.

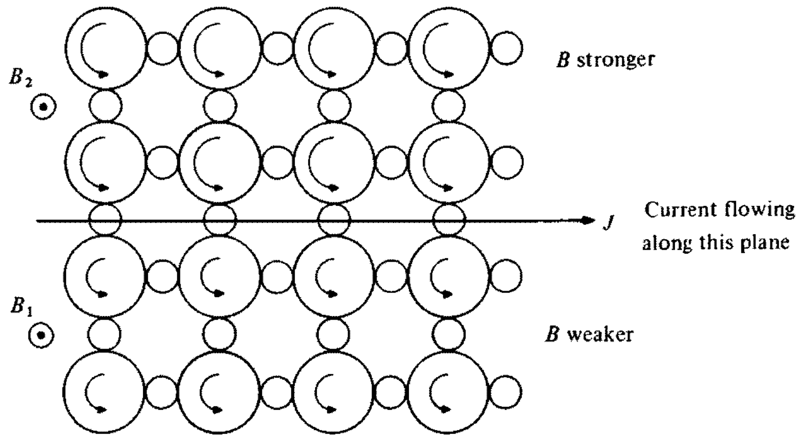
He then identified the 'idler-wheels' with electric particles which, if they were free to move, would carry an electric current. In conductors, these electric particles are free to move, whereas in insulators, *including free space*, they cannot move and so cannot carry an electric current.

Remarkably, this model could explain all known phenomena of electromagnetism.

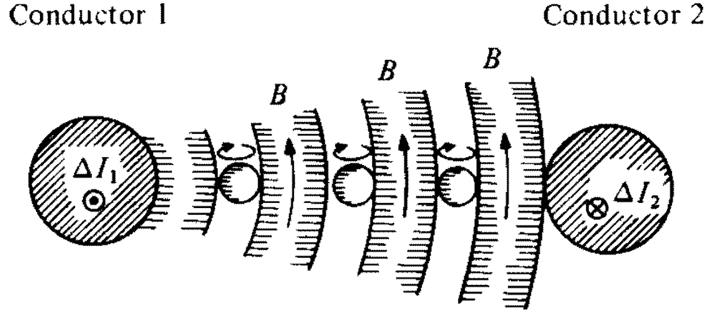
# Electromagnetism and Vortices




The magnetic field of a current carrying wire.




The magnetic field distribution in the presence of a current sheet.




Electromagnetic induction

# The Displacement Current (1)

Maxwell next considered how insulators store electrical energy. In insulators, the medium is elastic so that the electric particles can be displaced from their equilibrium positions by the action of an electric field. The electrostatic energy in the medium was identified with the elastic potential energy associated with the displacement of the electric particles. This had two key consequences:

- When the electric field applied to a medium is varying, there are small changes in the positions of the electric particles in the insulating medium or vacuum, and so there are small currents associated with this elastic motion. There is a current associated with the *displacement* of the electric particles from their equilibrium positions which Maxwell called the *displacement current*.

# The Displacement Current (2)

- Because the medium is elastic, the speed at which disturbances can be propagated through the insulator, or vacuum, can be calculated. The calculation is straightforward.

The displacement of the electric particles is assumed to be proportional to the electric field strength

$$\boldsymbol{r} = \alpha \boldsymbol{E}.$$

When the strength of the field varies, the charges move causing a *displacement current*. If $N_{\mathrm{q}}$ is the number density of electric particles and $q$ their charge, the *displacement current density* is

$$\boldsymbol{J}_{\mathrm{d}} = q N_{\mathrm{q}} \dot{\boldsymbol{r}} = q N_{\mathrm{q}} \alpha \dot{\boldsymbol{E}} = \beta \dot{\boldsymbol{E}}.$$

# The Displacement Current (3)

This displacement current density should be included in the equation for curl $\boldsymbol{H}$:

$$\text{curl}\,\boldsymbol{H} = \boldsymbol{J} + \boldsymbol{J}_{\text{d}} = \boldsymbol{J} + \beta\dot{\boldsymbol{E}}.$$

$\alpha$ and $\beta$ are unknown constants to be found from the electric properties of the medium. Assuming there are no currents, $\boldsymbol{J} = 0$, the speed of propagation of a disturbance through the medium is the solution of the equations

$$\boxed{\begin{aligned} \text{curl}\,\boldsymbol{H} &= \beta\dot{\boldsymbol{E}}, \\ \text{curl}\,\boldsymbol{E} &= -\dot{\boldsymbol{B}}. \end{aligned}}$$

We seek wave solutions of the form $e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}$ and then, using the standard procedure, we find a dispersion relation

$$\boldsymbol{k} \times (\boldsymbol{k} \times \boldsymbol{H}) = -\omega^2 \beta \mu_0 \boldsymbol{H}.$$

For *transverse waves*, this reduces to

$$c^2 = 1/\beta\mu_0$$

# The Determination of $\beta$

The energy density stored in the dielectric is the work done per unit volume in displacing the electric particles a distance $\boldsymbol{r}$,

$$\text{Work done} = \int \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{r} = \int N_{\mathrm{q}} q \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{r}.$$

But

$$\boldsymbol{r} = \alpha \boldsymbol{E} \quad \text{and hence} \quad \mathrm{d}\boldsymbol{r} = \alpha\, \mathrm{d}\boldsymbol{E}.$$

Therefore, the work done is

$$\int_0^E N_{\mathrm{q}} q \alpha E \,\mathrm{d}E = \tfrac{1}{2} \alpha N_{\mathrm{q}} q E^2 = \tfrac{1}{2} \beta E^2.$$

But the electrostatic energy density in the dielectric is $\frac{1}{2}\boldsymbol{D} \cdot \boldsymbol{E} = \frac{1}{2}\epsilon_0 E^2$. Therefore $\beta = \epsilon_0$. Inserting this value into the expression for the speed of the waves,

$$\boxed{c = (\mu_0 \epsilon_0)^{-1/2}}$$

# Light as Electromagnetic Waves

Maxwell inserted the best available values for the electrostatic and magnetostatic constants into the expression for $c$ and found, to his amazement, that it turned out to be the speed of light, within a few percent. In his own words:

> 'we can scarcely avoid the inference that light consists in the transverse modulations of the same medium which is the cause of electric and magnetic phenomena'.

> 'I do not bring it forward as a mode of connection existing in Nature....
>
> It is however a mode of connection which is mechanically conceivable and it serves to bring out the actual mechanical connections between known electromagnetic phenomena.'

# *A Dynamical Theory of the Electromagnetic Field* (1865)

In 1864, Maxwell developed the whole theory on a much more abstract basis without any special assumptions about the nature of the medium through which electromagnetic phenomena are propagated. To quote Whittaker:

> ‘In this, the architecture of his system was displayed, stripped of the scaffolding by aid of which it had been first erected’.

Maxwell’s own view of the significance of this paper is revealed in what Everitt calls ‘a rare moment of unveiled exuberance’ in a letter to his cousin Charles Cay:

> ‘I have also a paper afloat, containing an electromagnetic theory of light, which, till I am convinced to the contrary, I hold to be great guns’.

# Maxwell's Equations in their Final Form (1)

In Maxwell's great papers, eight equations were involved. The standard form of the equations was introduced by Heaviside and Hertz in the 1880s.

$$
\begin{aligned}
\mathrm{curl}\,\boldsymbol{E} &= -\frac{\partial \boldsymbol{B}}{\partial t}, \\
\mathrm{curl}\,\boldsymbol{H} &= \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}, \\
\mathrm{div}\,\boldsymbol{D} &= \rho_{\mathrm{e}}, \\
\mathrm{div}\,\boldsymbol{B} &= 0.
\end{aligned}
$$

The inclusion of the *displacement current* $\partial \boldsymbol{D}/\partial t$ resolves a problem with the equation of continuity in electromagnetism. Taking the divergence of the second equation,

$$\mathrm{div}(\mathrm{curl}\,\boldsymbol{H}) = \mathrm{div}\,\boldsymbol{J} + \frac{\partial}{\partial t}(\mathrm{div}\,\boldsymbol{D}) = 0$$

# Maxwell's Equations in their Final Form (2)

Since $\text{div}\boldsymbol{D} = \rho_\text{e}$,

$$\text{div}\,\boldsymbol{J} + \frac{\partial \rho_\text{e}}{\partial t} = 0,$$

the continuity equation for conservation of electric charge in electrostatics.

Notes:

1. Maxwell's discovery gave real physical content to the wave theory of light.
2. Poincaré: In his experience, all Frenchmen were oppressed by a 'feeling of discomfort, even of distress' at their first encounter with the works of Maxwell.
3. When Maxwell became the first Cavendish Professor in 1871, he devoted considerable effort to the precise determination of the ratio of the electrostatic and electromagnetic units.

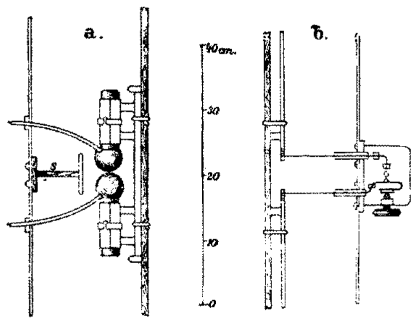
# Hertz's Experiments (1)

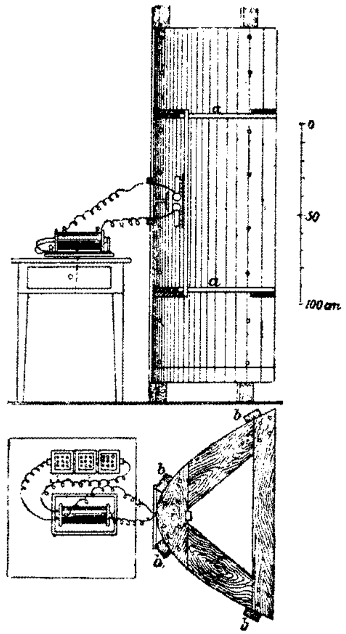
Heinrich Hertz

Maxwell died in 1879 before direct experimental evidence was obtained for electromagnetic waves.

In the period 1887-89, Heinrich Hertz carried out a classic series of experiments which demonstrated that electromagnetic waves have all the properties of light.

# Hertz's Experiments (2)




This apparatus is on show in the Deutsches Museum in Munich.

He found standing waves as the detector was moved along the line between the emitter and the conducting sheet. The frequency of the waves was found from the resonant frequency of the receiving loop, $\omega = 2\pi\nu = (LC)^{-1/2}$, where $L$ and $C$ are the inductance and capacitance of the detector. The wavelength of the waves was twice the distance between the minima of the standing waves and $c = \nu\lambda$. The result was precisely the speed of light.

# Hertz and the Photoelectric Effect

Hertz went on to demonstrate that the electromagnetic waves exhibited all the properties of light waves. In his great book *Electric Waves* (1893), the chapter headings include:

Rectilinear Propagation, Polarisation, Reflection, Refraction.

To demonstrate refraction he constructed a prism weighing 12 cwt out of ‘so-called hard pitch, a material like asphalt’.

Ironically, in the same set of experiments which completely established Maxwell’s theory, he also discovered the *photoelectric effect*. This effect was central to Einstein’s demonstration that light has particle as well as wave properties. It led directly to the development of the quantum theory of matter and radiation.

# Why is Maxwell not Better Known?

Maxwell was very modest about his achievements. In his 1870 Presidential Address to the BAAS reviewing advances in physics and electromagnetism, he reviewed all the other theories but not his own, merely referring to:

> ‘Another theory of electromagnetism which I prefer.’

Freeman Dyson (1999)

> ‘The moral of this story is that modesty is not always a virtue.’

# Freeman Dyson on Maxwell

‘Maxwell’s theory becomes simple and intelligible only when you give up thinking in terms of mechanical models. Instead of thinking of mechanical objects as primary and electromagnetic stresses as secondary consequences, you must think of the electromagnetic field as primary and mechanical forces as secondary. The idea that the primary constituents of the universe are fields did not come easily to the physicists of Maxwell’s generation. Fields are an abstract concept, far removed from the familiar world of things and forces. The field equations of Maxwell are partial differential equations. They cannot be expressed in simple words like Newton’s law of motion, force equals mass times acceleration.

Maxwell’s theory had to wait for the next generation of physicists, Hertz and Lorentz and Einstein, to reveal its power and clarify its concepts. The next generation grew up with Maxwell’s equations and was at home with a Universe built out of fields. The primacy of fields was as natural to Einstein as the primacy of mechanical structures had been for Maxwell.’