Math 120 A - Winter 2018
Midterm Exam Number Two
February 22nd, 2018

Name: _____ Student ID no. : _____

Signature: _____ Section: _____

1	12	
2	12	
3	12	
4	12	
5	12	
Total	60	

- This exam consists of FIVE problems on THREE double-sided pages.
- Show all work for full credit.
- You may use a TI-30X IIS calculator during this exam. Other calculators and electronic devices are not permitted.
- You do not need to simplify your answers.
- If you use a trial-and-error or guess-and-check method when a more rigorous method is available, you will not receive full credit.
- Draw a box around your final answer to each problem.
- **Do not write within 1 centimeter of the edge!** Your exam will be scanned for grading.
- If you run out of room, write on the back of the last page and indicate that you have done so. If you still need more room, ask your TA for an extra page to staple to your exam.
- You may use one hand-written double-sided 8.5" by 11" page of notes.
- You have 50 minutes to complete the exam.

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1. **[12 points]** I have placed some plums in an icebox. Their temperature (in Celsius) is an exponential function of time.

   10 hours from now, the plums will be at a temperature of $11°$ Celsius.

   22 hours from now, they will be at a temperature of $5°$ Celsius.

   (a) Write a function $p(t)$ for the temperature of the plums (in degrees Celsius) t hours from now.

$p(t) = A_0 b^t$

$A_0 b^{10} = 11$

$A_0 b^{22} = 5$

$b^{12} = \frac{5}{11} \rightarrow b = \left(\frac{5}{11}\right)^{\frac{1}{12}}$

$A_0 \left(\frac{5}{11}\right)^{\frac{10}{12}} = 11 \rightarrow A_0 = \frac{11}{\left(\frac{5}{11}\right)^{\frac{10}{12}}}$

$A_0 \approx 21.22$

$$p(t) = 21.22 \left(\frac{5}{11}\right)^{\frac{t}{12}}$$

   (b) I will eat the plums when their temperature reaches $4°$ Celsius. How many hours from now is this?

$$21.22 \left(\frac{5}{11}\right)^{\frac{t}{12}} = 4$$

$$\left(\frac{5}{11}\right)^{\frac{t}{12}} = \frac{4}{21.22}$$

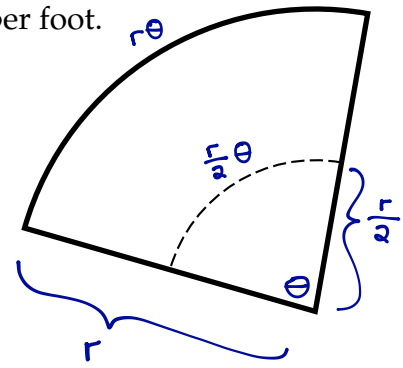
$$\frac{t}{12} \ln\left(\frac{5}{11}\right) = \ln\left(\frac{4}{21.22}\right)$$

$$t = \frac{12 \ln\left(\frac{4}{21.22}\right)}{\ln\left(\frac{5}{11}\right)} \approx 25.396 \text{ hours}$$

2. **[12 points]** I have $3500 with which I'd like to build a fence in the shape of a sector, with a partition in the shape of an arc running between the midpoints of the straight sides, as shown below.

   The outside fence costs $5 per foot, and the partition costs $2 per foot.

   What is the **maximum possible total area** inside the fence?



$A = \frac{1}{2} r^2 \theta$

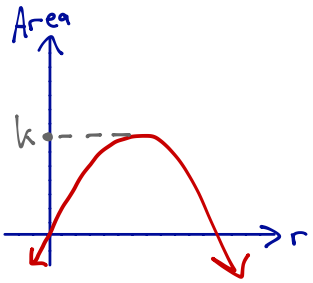
$5\underbrace{(2r + \theta r)}_{\text{outside length}} + 2\underbrace{\left(\frac{1}{2} r\theta\right)}_{\text{partition}} = 3500$

$10r + 6\theta r = 3500$

$\theta = \frac{3500 - 10r}{6r}$

$A = \frac{1}{2} r \left(\frac{3500 - 10r}{6r}\right)$

$= \frac{-5}{6} r^2 + \frac{875}{3} r$



$k = c - \frac{b^2}{4a} = -\frac{\left(\frac{875}{3}\right)^2}{4\left(\frac{-5}{6}\right)} \approx 25520.833$

3. **[12 points]** Shuri is developing a shock-absorbing material. The amount of energy it can absorb is a linear-to-linear rational function of how long Shuri conducts her research.

Right now, the material can absorb 2 megajoules of energy.

If she researches for 2 more days, it will be able to absorb 2.8 megajoules.

As Shuri's research continues, the amount of energy the material is able to absorb will approach (but not reach) 5 megajoules.

(a) Write a linear-to-linear rational function $f(x)$ for the amount of energy the material can absorb, in megajoules, if Shuri researches for x days.

$$f(x) = \frac{ax+b}{x+d}$$

$a = 5$

$2 = \frac{b}{d} \rightarrow b = 2d$

$2.8 = \frac{2a+b}{2+d} \rightarrow 5.6 + 2.8d = 2a + b \rightarrow 5.6 + 2.8d = 10 + 2d$

$0.8d = 4.4$

$d = 5.5$

$b = 11$

$$f(x) = \frac{5x+11}{x+5.5}$$

(b) Find the inverse of the function you found in part (a).

$$y = \frac{5x+11}{x+5.5}$$

$$\downarrow$$

$$xy + 5.5y = 5x + 11$$

$$xy - 5x = 11 - 5.5y$$

$$x(y-5) = 11 - 5.5y$$

$$x = \frac{11-5.5y}{y-5}$$

$$f^{-1}(x) = \frac{11-5.5x}{x-5}$$

4. **[12 points]** Check out these wheels:

   Wheel A has a radius of 3 inches and makes 2 revolutions per minute.

   Wheel B is connected by a belt to wheel A, and by an axle to wheel C.

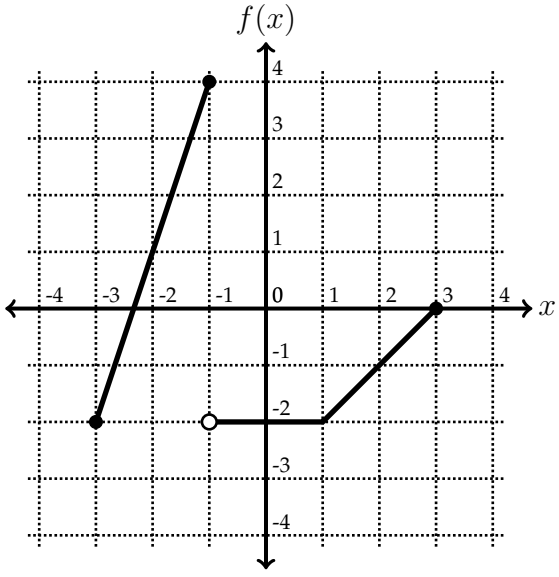
   Wheel C has a radius of 5 inches and rotates at a speed of 10 inches per minute.

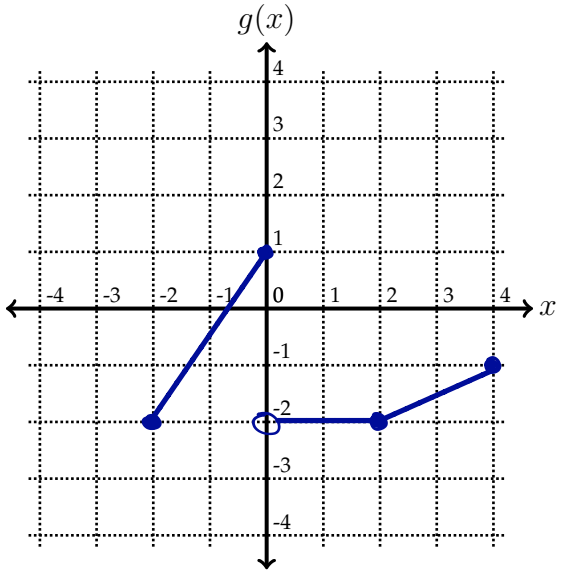
   What's the radius of wheel B?

Wheel	v (in./min)	ω (rad/min)	r (in.)
A	12π	4π	3
B	12π	2	6π
C	10	2	5

→ 6π in.

5. **[12 points]** On the left is the graph of $f(x)$. On the right, please graph $g(x) = \frac{1}{2}f(x-1)-1$.





① Shift right 1
② Scale vert. by $\frac{1}{2}$
③ Shift down 1.

$y = \frac{1}{2}f(x-1)-1 \rightarrow \frac{y+1}{\frac{1}{2}} = f(x-1)$