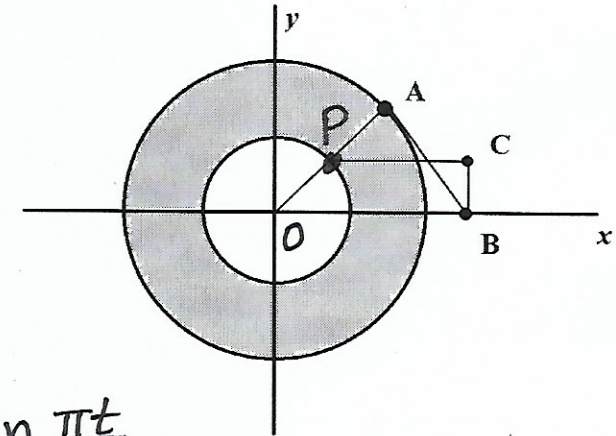
Solutions to Math 124 G Winter '18 MT I

1. A wheel with inner radius 3 and outer radius 7 centered at the origin is turning counterclockwise completing one revolution in 12 seconds. Find parametric equations for the position of the following points using time t measured in seconds as a parameter.



(a) (4 points) Point A.

1 revolution in 12 seconds
$\parallel$
2π radians

$\rightarrow \omega = \frac{2\pi}{12} = \frac{\pi}{6}$ rad/sec.

$x = 7\cos\frac{\pi t}{6} \qquad y = 7\sin\frac{\pi t}{6}$

(b) (4 points) Point B.The line AB is tangent to the circle.

Slope of $OA = \frac{\cancel{7}\sin \pi t/6}{\cancel{7}\cos \pi t/6} \rightarrow$ slope of $AB = \frac{-\cos \pi t/6}{\sin \pi t/6}$

equation of AB

$$y - 7\sin\frac{\pi t}{6} = \frac{-\cos \pi t/6}{\sin \pi t/6}\left(x - 7\cos\frac{\pi t}{6}\right)$$

at B, $\boxed{y=0}$ $\quad -7\sin\frac{\pi t}{6} = \frac{-\cos \pi t/6}{\sin \pi t/6}\left(x - 7\cos\frac{\pi t}{6}\right)$

$$\rightarrow 7\frac{\sin^2 \pi t/6}{\cos \pi t/6} + 7\cos\frac{\pi t}{6} = \boxed{x = 7\sec\frac{\pi t}{6}}$$

(c) (2 points) Point C.

$x = 7\sec\frac{\pi t}{6}$ (same as B)

$y = 3\sin\frac{\pi t}{6}$ (same as P)

2. Evaluate the following limits. Your answer must be justified by your work. Plugging in values in a calculator and guessing the answer will not get credit. Your answer must be one of DNE, ∞, $-\infty$ or a number. If you are answering DNE, briefly explain why.

(a) (3 points) $\lim_{x\to 2} \frac{3x^2-5x-2}{x^2-5x+6} =$ $\lim_{x\to 2} \frac{(3x+1)(x-2)}{(x-2)(x-3)}$

$$= \lim_{x\to 2} \frac{3x+1}{x-3} = \frac{7}{-1} = -7$$

(b) (3 points) $\lim_{x\to\infty} \sqrt{9x^2+7x} - 3x =$

$$= \lim_{x\to\infty} \sqrt{9x^2+7x} - 3x \cdot \frac{\sqrt{9x^2+7x}+3x}{\sqrt{9x^2+7x}+3x}$$

$$= \lim_{x\to\infty} \frac{9x^2+7x-9x^2}{\sqrt{9x^2+7x}+3x} = \lim_{x\to\infty} \frac{7x/x}{\left(\sqrt{9x^2+7x}+3x\right)/x}$$

$$= \lim_{x\to\infty} \frac{7}{\sqrt{9+\frac{7}{x}}+3} = \frac{7}{\sqrt{9}+3} = \frac{7}{6}$$

(c) (3 points) $\lim_{y\to 0} \frac{(3+y)^7-3^7}{y} =$

$$= f'(3)$$

$$f(x) = x^7$$

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^7 - x^7}{h}$$

with $h = y$ + $x = 3$

$$f'(x) = 7x^6$$

$$f'(3) = 7 \cdot 3^6$$

$$= 5103$$

3. (11 points) Let l_1 be the tangent line to

$$y = (x^2 + x + 1)e^x$$

at the point where $x = 0$. Let l_2 be the tangent line to

$$y = -x^2 + \frac{3}{2}x + 7$$

at $x = a$. The tangent lines l_1 and l_2 intersect at right angles at the point C. Find the value of a and the coordinates of the point C.

l_1 : slope $y'(0)$

$y'(x) = (2x+1)e^x + (x^2+x+1)e^x$

$y'(0) = 1 + 1 = 2$

point $(0, y(0)) = (0, 1)$

equation $y - 1 = 2(x-0)$

$\boxed{y = 2x+1}$

l_2 : slope $= \dfrac{-1}{\text{slope of } l_1} = \dfrac{-1}{2}$

point $x = a$ slope from derivative $y'(a)$

where $y' = -2x + \frac{3}{2}$

so $\frac{-1}{2} = -2a + \frac{3}{2}$

$2a = \frac{4}{2}$

$\boxed{a = 1}$

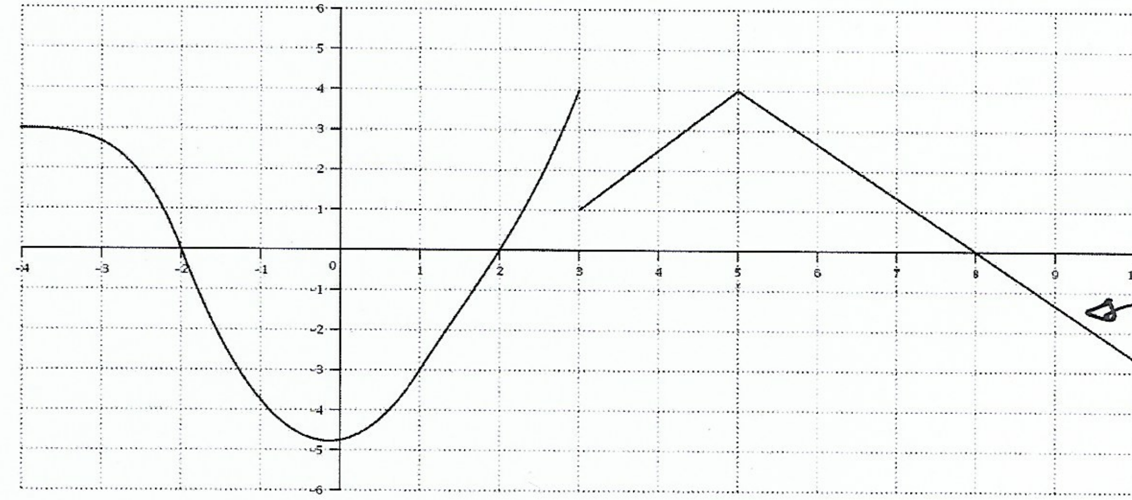
point for l_2 : $(1, -1 + \frac{3}{2} + 7)$

$= (1, \frac{15}{2}) \rightarrow \boxed{y - \frac{15}{2} = \frac{-1}{2}(x-1)}$

OR $y = \frac{-1}{2}x + 8$

$2x + 1 = -\frac{1}{2}x + 8 \rightarrow \frac{5}{2}x = 7$ so $\boxed{x = \frac{14}{5}, \ y = \frac{33}{5}}$

4. (10 points) The following is the graph of $y = f(x)$. You do not have to explain your answers below.



line
slope $= \frac{-4}{3}$
$y = \frac{-4}{3}(x-8)$

(a) $f'(4) =$ slope $= \frac{3}{2}$

(b) $f''(7) =$ 0 (f' is constant around $x=7$)

(c) $\lim_{x \to 3} f(x) =$ DNE

(d) $\lim_{x \to 8} \frac{f(x)}{x-8} =$ $\lim_{x \to 8} \frac{\frac{-4}{3}(x-8)}{x-8} = \frac{-4}{3}$

(e) $\lim_{x \to 3^-} f(x) =$ 4

(f) $\lim_{h \to 0^+} \frac{f(5+h) - f(5)}{h} =$ $\frac{-4}{3}$

(g) List all the intervals where the derivative $f'(x)$ is negative.

$(-4, 0)$ + $(5, 10)$

(h) List all the values of x where the derivative $f'(x)$ doe not exist.

$x=3$, $x=5$ (also $x=-4$, $x=10$)