Solutions to Math 124G Fall 2019 Midterm I

1. (12 points) Evaluate the following limits. Show the algebra work where applicable. Your answer must be one of DNE, ∞, $-\infty$ or a number.

(a) $\lim_{x\to 11}\left(\frac{x^2-x-132}{x^2-x-110}\right) = \lim_{x\to 11} \frac{(x-12)(x+11)}{(x-11)(x+10)}$

$\lim_{x\to 11^+} \frac{(x-12)(x+11)}{(x-11)(x+10)} = -\infty$ $\quad$ $\lim_{x\to 11^-} \frac{(x-12)(x+11)}{(x-11)(x+10)} = +\infty$

so limit DNE

"$\frac{\infty}{\infty}$"

(b) $\lim_{x\to -\infty}\left(\frac{3x+2}{\sqrt{5x^2+1}}\right) = \lim_{x\to -\infty} \frac{\frac{3x+2}{x}}{\frac{\sqrt{5x^2+1}}{x}} = \lim_{x\to -\infty} \frac{3+\frac{2}{x}}{\frac{\sqrt{5x^2+1}}{-\sqrt{x^2}}}$

$x<0$ so $x = -\sqrt{x^2}$

$= \lim_{x\to \infty} \frac{3+\frac{2}{x}}{-\sqrt{\frac{5x^2+1}{5x^2}}} = -\lim_{x\to\infty} \frac{3+\frac{2}{x}}{\sqrt{5+\frac{1}{5x^2}}} = -\frac{3}{\sqrt{5}}$

"$\infty-\infty$"

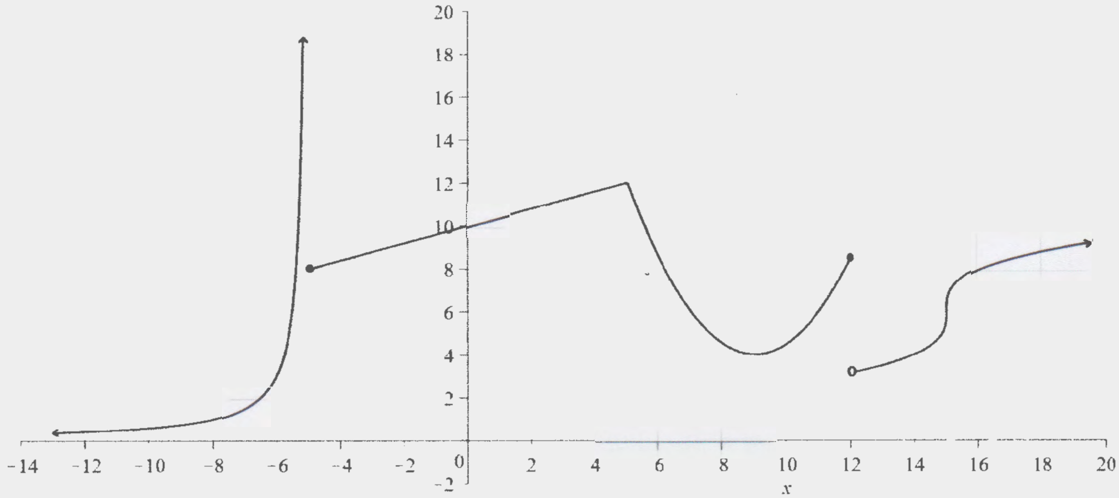
(c) $\lim_{x\to 2^+}\left(\frac{1}{x^2-4}-\frac{1}{x^2-2x}\right) = \lim_{x\to 2^+} \frac{1}{(x-2)(x+2)} - \frac{1}{x(x-2)}$

$= \lim_{x\to 2^+} \frac{x - (x+2)}{x(x+2)(x-2)} = \lim_{x\to 2^+} \frac{-2}{x(x+2)(x-2)} = -\infty$

(d) $\lim_{h\to 0} \frac{e^{h+\ln 3}-3}{h} = f'(\ln 3)$ with $f(x)=e^x$ so $f'(x)=e^x$

$= e^{\ln 3} = 3$

2. (11 points) The following is the graph of $y = f(x)$ with domain all real numbers. You do not have to explain your answers below. For limit questions your answer must be one of DNE, ∞, $-\infty$ or a number. In the graph below $f(-5) = 8$ and $f(12) = 8.5$.



(a) $f'(9) =$ 0

(b) $f''(2) =$ 0

(c) $\lim_{x \to 12^+} f(x) =$ 3

(d) List all the intervals where the derivative $f'(x)$ is negative.

(5, 9) OR $5 < x < 9$

(e) List all the values of x where the derivative $f'(x)$ doe not exist.

$-5, 5, 12, 15$

(f) $f(3.17) =$

Slope $= \frac{12-8}{5-(-5)} = 0.4$ line equation $y = 0.4x + 10$

$f(3.17) = 0.4(3.17) + 10 = 11.268$

(g) $\lim_{h \to 0^-} \frac{f(5+h) - 12}{h} =$ 0.4

(h) $\lim_{h \to 0} \frac{f(-5+h) - f(-5)}{h} =$ DNE

3. Answer the following.

(a) (4 points) Use the limit definition of the derivative to find the derivative of $f(x) = \frac{1}{\sqrt{x+1}}$.

$$\lim_{h\to 0} \frac{\frac{1}{\sqrt{x+h+1}} - \frac{1}{\sqrt{x+1}}}{h} = \lim_{h\to 0} \frac{\sqrt{x+1} - \sqrt{x+h+1}}{h(\sqrt{x+h+1})(\sqrt{x+1})} \cdot \frac{\sqrt{x+1} + \sqrt{x+h+1}}{\sqrt{x+1} + \sqrt{x+h+1}}$$

$$= \lim_{h\to 0} \frac{\cancel{x}\cancel{+1}-\cancel{x}-\cancel{h}-\cancel{1}}{\cancel{h}\sqrt{x+h+1}\sqrt{x+1}(\sqrt{x+1}+\sqrt{x+h+1})} = \lim_{h\to 0} \frac{-1}{\sqrt{x+h+1}\sqrt{x+1}(\sqrt{x+1}+\sqrt{x+h+1})}$$

$$= \frac{-1}{\sqrt{x+1}\sqrt{x+1}(\sqrt{x+1}+\sqrt{x+1})} = \frac{-1}{2(x+1)^{3/2}}$$

(b) (5 points) Find the x- intercept for the equation of the tangent line to $f(x) = xe^x - \frac{3}{x^2}$ at the point where $x = 1$.

$$f'(x) = e^x + xe^x + \frac{6}{x^3} \qquad f'(1) = e + e + 6 = 6 + 2e$$

$$f(1) = e - 3$$

Tangent Line: $y - (e-3) = (6+2e)(x-1)$

x-intercept when $y = 0$

$$-(e-3) = (6+2e)(x-1)$$

$$\frac{3-e}{6+2e} = x - 1$$

$$\frac{3-e}{6+2e} + 1 = x \longrightarrow x = \frac{9+e}{6+2e}$$

4. (8 points) Find the equations of the two tangent lines to $f(x) = \frac{2x+3}{x-4}$ which pass through the point $(1, 13)$.

$$f'(x) = \frac{2(x-4) - (2x+3)\cdot 1}{(x-4)^2} = \frac{-11}{(x-4)^2}$$

point of tangency $\left(a, \frac{2a+3}{a-4}\right)$

Slope of tangent line

$$\frac{-11}{(a-4)^2} = \frac{13 - \frac{2a+3}{a-4}}{1-a}$$

$$\frac{-11}{a-4} = \frac{13(a-4) - 2a - 3}{1-a} = \frac{11a - 55}{1-a} = \frac{11(a-5)}{1-a}$$

$$a - 1 = (a-4)(a-5) = a^2 - 9a + 20$$

$$0 = a^2 - 10a + 21 = (a-3)(a-7)$$

So. $a = 3$ OR $a = 7$

	$a = 3$	$a = 7$
	$f(a) = -9$	$f(a) = \frac{17}{3}$
	$f'(a) = -11$	$f'(a) = \frac{-11}{9}$
tangent line	$y - 13 = -11(x-1)$	$y - 13 = \frac{-11}{9}(x-1)$
	$y = -11x + 24$	$y = \frac{-11}{9}x + \frac{128}{9}$