Midterm 2

Calculus I (Math 124)
Instructor: Jarod Alper
Fall 2019
November 19, 2019 **Name:** ______________________

Section: ______________________

Read all of the following information before starting the exam:

- You may use a Ti-30x IIS calculator.
- You may refer to your hand-written note sheet (8.5"x11", two-sided).
- You may not consult any other outside sources (phone, computer, textbook, other students, ...) to assist in answering the exam problems. All of the work will be your own!
- Simplify your answers as much as possible. Unless otherwise stated, give exact answers to questions. For example, 2ln(3)/pi and 1/3 are exact while 0.699 and 0.333 are approximations for the those numbers.
- Write clearly!! You need to write your solutions carefully and clearly in order to convince me that your solution is correct. Partial credit will be awarded.
- Good luck!

Problem	Points
1 (20 points)	______
2 (20 points)	______
3 (20 points)	______
4 (20 points)	______
5 (20 points)	______
Total (100 points)	

Problem 1.

(a) Find the derivative of $f(x) = \arctan(\sqrt{x})$. (*Recall that* $\arctan(x)$ *is the same function as the inverse tangent function* $\tan^{-1}(x)$.)

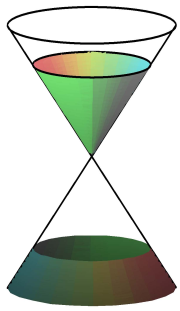
(b) If $y = \ln(x)^{\ln(x)}$, find an expression for $\frac{dy}{dx}$ in terms of x.

Problem 2. Consider the curve defined by the equation $x^4 - 2x^2y^2 = -56$.

(a) Find the equation of the tangent line at the point $(2, 3)$.

(b) Let P be the point on the curve near $(2, 3)$ with x-coordinate 2.1. Find an approximate value of the y-coordinate of P. (*Please round your answer to three digits after the decimal.*)

Problem 3. An hourglass is made up of two glass cones connected at their tips (as in the diagram below). Both cones have radius 1 cm and height 2 cm. When the hourglass is flipped over, sand starts falling to the lower cone.



(a) When the sand remaining in the *upper cone* has height y cm, give a formula for its volume A in terms of y.

(b) When the sand in the *lower cone* has reached a height of h cm, give a formula for its volume B in terms of h.

(c) Assume that the total volume of sand is $2\pi/3$ cm^3 and that the height of the sand in the upper cone is decreasing at a rate of 1 cm/sec. At the instant that the sand in the lower cone is 1 cm high, determine the rate at which the height of the sand in the lower cone is increasing.

Problem 4. Beginning at time $t = 0$ seconds, an ant crawls according to the equations

$$x(t) = t^3 + 45t + 1 \qquad \text{and} \qquad y(t) = -12t^2.$$

(a) At what times t within the first 10 seconds is the ant's direction of travel parallel to the line $x + y = 2$? (*Please round your answer to three digits after the decimal.*)

(b) At what time within the first 10 seconds does the ant attain its maximal speed? (*Please round your answer to three digits after the decimal.*)

Problem 5. Consider the function

$$f(x) = x^4 - 6x^2 + 4.$$

(a) Using interval notation, determine where $f(x)$ is increasing and decreasing.

(b) Determine the critical numbers of $f(x)$.

(c) Using interval notation, determine where $f(x)$ is concave up and down.

(d) Determine the inflection points of $f(x)$.

(e) For each critical number, determine whether it is a local minimum, local maximum or neither.