Answers to Second Midterm

1. $a(t)^2 + 5^2 = c(t)^2$. Taking d/dt of both sides and then substituting $a(t) = 12$ and $c(t) = 13$, we get $2 \cdot 12\dot{a} = 2 \cdot 13\dot{c}$, and so the desired ratio $\dot{c}/\dot{a}$ is equal to $12/13$.

2. The curve has equation $\sqrt{x^2 + (y-10)^2} = 2\sqrt{(x-5)^2 + y^2}$. Next take d/dx of both sides, then substitute $x = 8$, $y = 4$, and solve for y'. It saves a little work to square both sides before taking the derivative, in which case after taking the derivative you have $2x + 2(y-10)y' = 4(2(x-5) + 2yy')$. Substituting $x = 8$, $y = 4$ and solving for y', you get $16 - 12y' = 24 + 32y'$ so that $y' = -8/44 = -2/11$. (If you don't first square both sides, then when you substitute $x = 8$, $y = 4$ into

$$\frac{x + (y-10)y'}{\sqrt{x^2 + (y-10)^2}} = 2\frac{x - 5 + yy'}{\sqrt{(x-5)^2 + y^2}},$$

and multiply through by 20, you get the same thing.) The equation of the tangent line (which passes through $(8, 4)$ with slope $-2/11$) is then $y = -\frac{2}{11}x + \frac{60}{11}$.

3. (a) L is the derivative of $x^{x/2}$ at the point $(4, 16)$.

(b) Set $y = x^{x/2}$, so that $\ln(y) = \frac{1}{2}x\ln(x)$, and, taking d/dx of both sides, $\frac{y'}{y} = \frac{1}{2}(1 + \ln(x))$. At $(4, 16)$ you get $y' = \frac{y}{2}(1 + \ln(x)) = 8(1 + \ln(4))$.

4. Let $f(x) =$Arcsin(x), so that $f'(\frac{\sqrt{2}}{2}) = 1/\sqrt{1 - (\sqrt{2}/2)^2} = 1/\sqrt{1/2} = \sqrt{2}$. Then $f(\frac{\sqrt{2}}{2} + h) \approx$Arcsin$(\sqrt{2}/2) + h\sqrt{2} = \frac{\pi}{4} + h\sqrt{2}$.

5.

(a) $\dfrac{dy}{dx} = \dfrac{\sqrt{3}\sin(\theta)}{2 - \sqrt{3}\cos(\theta)}$; (b) $\dfrac{(2 - \sqrt{3}\cos(\theta)(\sqrt{3}\cos(\theta)) - \sqrt{3}\sin(\theta)(\sqrt{3}\sin(\theta))}{(2 - \sqrt{3}\cos(\theta))^2}$.

(c) Set the numerator in part (b) equal to zero and use the identity $\cos^2 + \sin^2 = 1$. You get $2\cos(\theta) - \sqrt{3} = 0$, that is, $\cos(\theta) = \sqrt{3}/2$, so that $\theta = \pi/6$ radians (or $30°$, that is, $1/12$ revolution).

(d) angular velocity is $\pi/2$ rad/sec, so at $t = 1/3$ sec; (e) $t = 2$ sec; (f) $t = 1$ sec.