Name_____

Math 124 **Second Midterm** **8:30-9:50, Feb. 20, 2018**

(80 minutes — 100 points)

DIRECTIONS: No calculator or other electronic device is permitted. You may have 1 sheet of notes in your own writing. Please show all your work — **no credit** for an answer without a correct method except in #5(d)–(f). Leave your answers in exact form (for example, $10 \ln 2$, $\sqrt{79}$, and $\frac{5}{6}\pi$ are in exact form).

1. (15 points) A boat is being pulled to shore. The rope attaches to a point on the boat that is 1 meter about sea level, and goes through a point on the dock that is 6 meters above sea level. Let $a(t)$ be the distance of the boat from shore, and let $c(t)$ be the length of rope connecting the boat to the dock. At the moment when $a(t) = 12$ meters and $c(t) = 13$ meters, find the ratio of the rate at which the rope is being pulled in (in m/sec) to the speed of the boat (in m/sec). Please show your work clearly.

2. (20 points) Let C be the curve consisting of all points (x, y) whose distance from $(0, 10)$ is twice its distance from $(5, 0)$. Notice that $(8, 4)$ is on C, because its distance from $(0, 10)$ is 10 and its distance from $(5, 0)$ is 5. Find a $y = mx + b$ formula for the line that best approximates the curve C near the point $(8, 4)$.

3. (20 points) Let L be the limit

$$\lim_{h \longrightarrow 0} \frac{(4+h)^{(4+h)/2} - 16}{h}.$$

(a) L is the derivative of what function $f(x)$ at what point (x, y)?

(b) Find L by finding the derivative in part (a). You may use derivative formulas.

4. (15 points) Use the tangent line approximation to find $\text{Arcsin}(\frac{\sqrt{2}}{2} + h)$ in terms of h. A multiple of π should appear in your answer.

5. (30 points in all) The parametric equations for the path of a point P that's located between the center C and the rim of a wheel are $x(\theta) = r\theta - a\sin(\theta)$, $y(\theta) = r - a\cos(\theta)$. (Here we assume that the wheel is rolling down the x-axis and that P is directly below C when it passes over the origin.)

A big wheel of radius 2 meters is rolling to the right along the x-axis without slipping or skidding. At the moment when it passes over the origin, a pebble P gets stuck in a spoke directly under the center at a distance of $\sqrt{3}$ meters from the center.

(a) (8 points) Find a formula in terms of θ for the slope of the path of P when the wheel has rotated through θ radians.

(b) (8 points) Find the derivative with respect to θ of the slope of the path of P.

(c) (8 points) During the first revolution of the wheel (that is, $0 \le \theta \le 2\pi$), for what value of θ is the slope of the path of P the greatest?

Now suppose that the wheel's speed is π m/sec, starting at time $t = 0$ when it passes the origin. In parts (d)–(f) you are not required to show work, and there is no partial credit within a part.

(d) (2 points) Find the first time when the slope of the pebble's path is maximal.

(e) (2 points) Find the first time when the pebble's horizontal velocity is maximal,

(f) (2 points) Find the first time when the pebble's vertical velocity is maximal.