Similar Triangles Examples

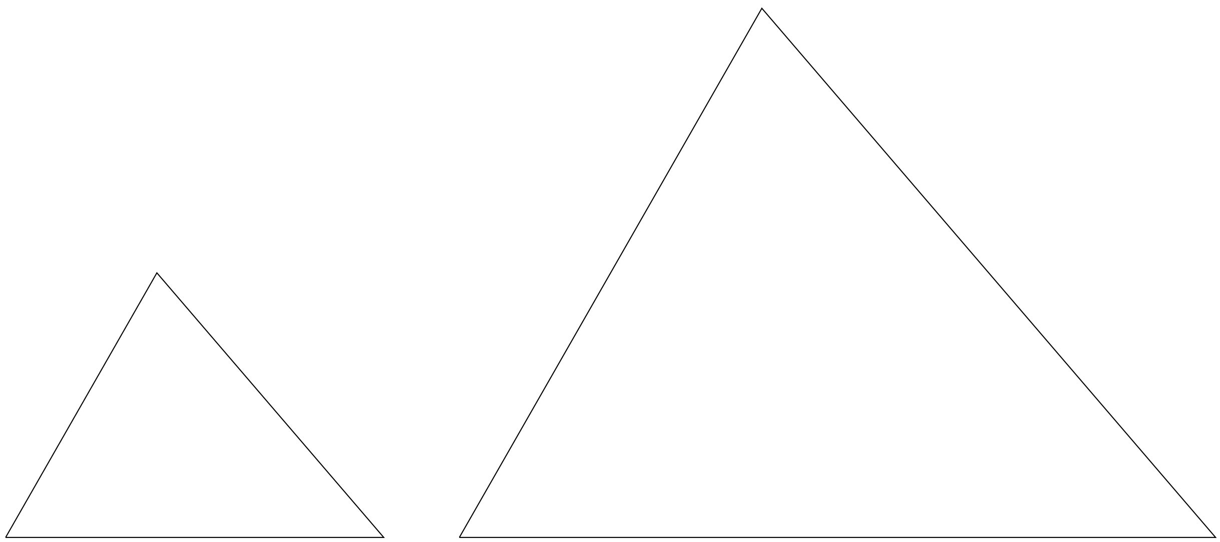
The method of similar triangles comes up occasionally in Math 120 and later courses.

In one phrase, the idea of the method is this: the ratio of corresponding sides of triangles with the same shape are equal.

Triangles have the same shape if they have the same angles. Since the angles in a triangle sum to $180° = \pi$ radians, if two angles are known, the third is determined. As a result, if two triangles have *two* angles in common, they have the third as well, and so have the same shape.

Two triangle that have the same shape are called *similar*.

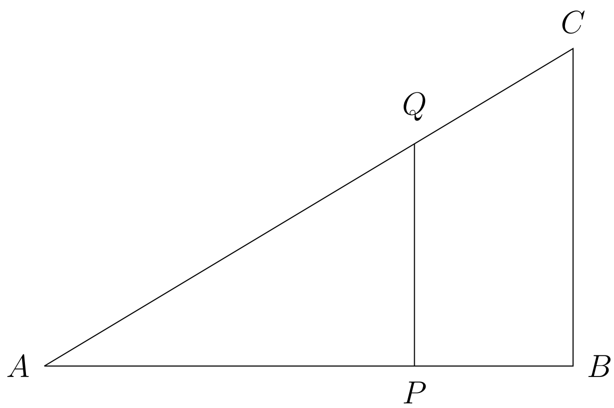
For example, in the picture below, the two triangles are similar.



You could check with a protractor that the angles on the left of each triangle are equal, the angles at the top of each triangle are equal, and the angles on the right of each triangle are equal.

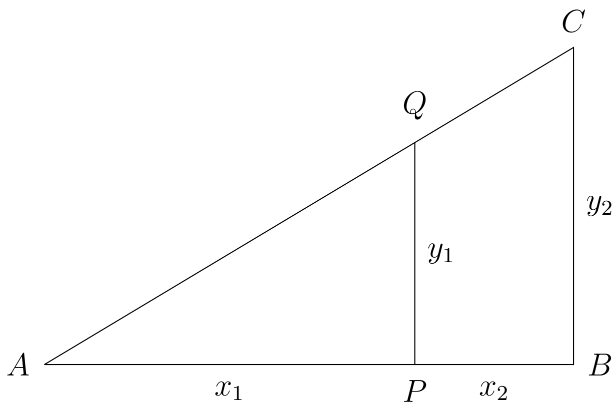
As a result, the triangles are *similar* and this means that they have the same shape. This means that one is a scaled up version of the other (in fact, the large triangle is exactly the smaller one scaled up by a factor of 2: the sides of the large triangle are twice as long as the corresponding sides of the smaller triangle). Note that when we scale up, or scale down, a triangle, the angles do not change.

The most common way to we see similar triangle in Math 120 or in calculus courses is when we have to right triangles that overlap, like this:



With B a right angle, this figure shows two right triangles: ABC and APQ. Since both triangles share angle B, and angle A, they also have their third angles in common (the angles at C and Q). As a result, the two triangles are similar; they have the same shape.

If we label the lengths of the legs of the triangles like this:



then we can use the idea of similar triangles to make some statements about ratios of sides.

If we take the legs of the smaller triangle APQ, and consider their ratio

$$\frac{y_1}{x_1},$$

then, because triangle ABC is similar, its corresponding ratio

$$\frac{y_2}{x_1 + x_2}$$

is the same. That is,

$$\frac{y_1}{x_1} = \frac{y_2}{x_1 + x_2}. \tag{1}$$

(Notice here that x_2 is not, by itself, the length of any sides of the triangles, but $x_1 + x_2$ is the length of the horizontal leg of ABC.)

Similarly, we can set up ratio between corresponding sides of the triangles. We have

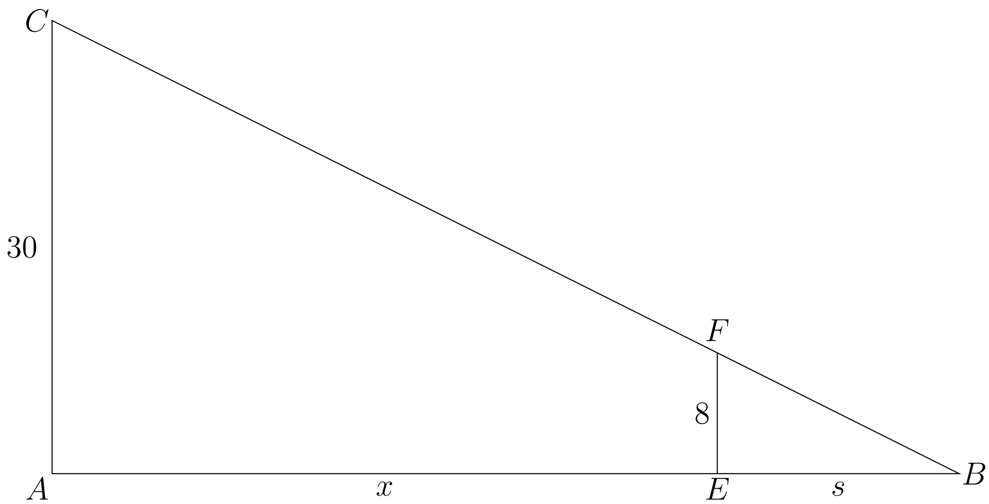
$$\frac{y_2}{y_1} = \frac{x_1 + x_2}{x_1}. \tag{2}$$

Notice that will just a little rearrangement, equation (2) is identical to equation (1). So in practice, both equations will get us to the same result.

We can use similar triangles to find unknown quantities, and to determine relationships between quantities.

Example 1: Suppose an emu is 8 feet tall and is walking away from a street light that is 30 feet tall. At night, the emu will cast a shadow on the ground. What is the relationship between the length of the shadow and the distance from the emu to the street light?

If we let s be the length of the emu's shadow, and x be the horizontal distance from the emu to the light post, we can make the following figure:



Triangles ABC and EBF are both right triangles since A and E are right angles (we may assume that the street light and the emu are vertical). Since the two triangles share angle B, they thus have two angles in common, and hence have all their angles in common. Thus, the triangles are similar. So, we know that ratios of corresponding sides are equal:

$$\frac{s}{8} = \frac{x+s}{30}$$

From this, we can conclude that

$$\frac{s}{8} = \frac{x}{30} + \frac{s}{30}$$
$$s\left(\frac{1}{8} - \frac{1}{30}\right) = \frac{x}{30}$$
$$s\frac{11}{120} = \frac{x}{30}$$
$$s = \frac{4}{11}x$$

This final equation shows the simple relationship between the length of the shadow and the distance from the emu to the street light. Note that this equally says that

$$x = \frac{11}{4}s$$

so one quantity is easily determined from the other. For example, if the emu is 22 feet from the street light, then we can see that its shadow is

$$s = \frac{4}{11}(22) = 8$$

feet long.

Here is a more elaborate example.

Suppose, instead of a street light, there is a hot air balloon vertically rising at the rate of 1.3 feet per second. There is a lantern on the bottom of the balloon's gondola that shines brightly downward in all directions. At a certain instant, the lantern is 10 feet off the ground, and the emu is 15 feet away horizontally from the point on the ground directly below the lantern. The emu walks away from this point at 4 feet per second.

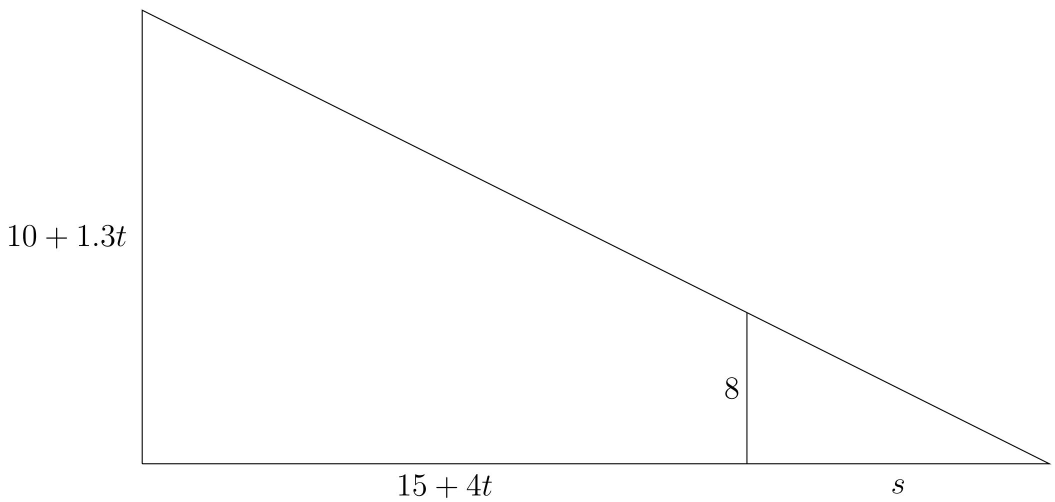
Let's find an expression for the length of the emu's shadow t seconds after this instant.

We may make a couple of statements.

First, t seconds later, the lantern's height will be $10 + 1.3t$ feet.

Second, t seconds later, the emu will be $15 + 4t$ feet away from the point on the ground directly below the lantern.

With this in mind, we may make the following figure:



Using the similar triangles idea again, we may say

$$\frac{10 + 1.3t}{15 + 4t + s} = \frac{8}{s}.$$

Solving for s, we find

$$s = \frac{120 + 32t}{2 + 1.3t}.$$

So that, say, 10 seconds after the initially described instant, the shadow is

$$\frac{88}{3} = 29.3333....$$

feet long.

Note this is a fairly complex relationship between s and t. You might be interested in considering: is the length of the shadow increasing or decreasing, or does it change from one to the other? In the long run, how long is the shadow going to be?