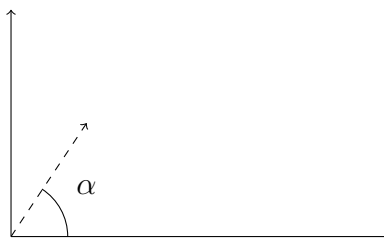


## Worksheet for Week 11: Maximizing functions

The best way to study for the Math 124 final is to solve challenging problems. In this worksheet, you'll solve a difficult problem that incorporates several topics from the class: position and velocity, parametric equations, derivatives, and maximizing functions by checking for critical points and also checking the endpoints.

You're on the planet Zed, which has no air and has gravitational constant  $-g$ . (Earth's gravitational constant is about  $-9.8m/s^2$ .) You throw a ball at speed  $v$  from the origin on a coordinate plane, where the horizontal axis lies along the ground of Zed. (See the picture.) The angle  $\alpha$  is the angle above the horizontal that the ball is thrown. Suppose  $0 \leq \alpha \leq \pi/2$ .



Let  $x(t)$ ,  $y(t)$  denote the position of the ball at time  $t$ , where  $t = 0$  is the instant the ball was thrown in the air. Then  $x'(t)$ ,  $y'(t)$  denote the horizontal and vertical velocities as functions of  $t$ .

In this situation (airless planet with gravitational constant  $-g$ ), we have the formulas

$$y(t) = -\frac{1}{2}gt^2 + y'(0)t + y(0),$$
$$x(t) = x'(0)t + x(0).$$

1. What are  $x'(0)$ ,  $y'(0)$ ,  $x(0)$  and  $y(0)$ ? Use these numbers and the formulas above to find equations for  $x'(t)$  and  $y'(t)$ .

2. When does the ball hit the ground?

3. When does the ball reach the peak of its trajectory?

4. How far from the origin is the ball when it hits the ground?

5. Which angle  $\alpha$  will maximize the distance the ball travels?

6. In this problem, you'll figure out when the ball is farthest from the origin. The answer will depend on the angle  $\alpha$ .

- (a) Find a formula  $F(t)$  for the **square of** the distance from the ball to the origin at time  $t$ . (Maximizing the square of the distance is the same as maximizing the distance, and this way is less messy.)

- (b) Find an expression for the non-zero critical numbers of  $F(t)$ .
- (c) For some values of  $\alpha$ , there are no non-zero critical numbers to check. For which  $\alpha$  are there **no** non-zero roots of  $F'(t)$ ?
- (d) For the values of  $\alpha$  you found in part (c), when is the ball the farthest from the origin?

(e) For which angle(s)  $\alpha$  is there only one non-zero critical number of  $F(t)$  to check?

(f) For the values of  $\alpha$  you found in part (e), when is the ball the farthest from the origin? Be sure to check your answer.

- (g) Are there any angles  $\alpha$  so that the ball is farthest from the origin while it's in the air?