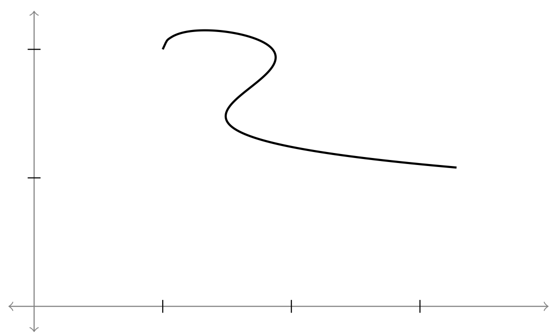
Worksheet for Week 4: Velocity and parametric curves

In this worksheet, you'll use differentiation rules to find the vertical and horizontal velocities of an object as it follows a parametric curve. You'll also get a preview of how to find tangent lines to parametric curves.

1. A bug is running around on a window. If we put x and y axes on the window, then the bug follows the path given by the parametric equations

$$(x(t), y(t)) = \left(\frac{1}{20}te^t + e^t - t^e,\ t^{2/3} - t + 2\right).$$

   The bug's path is shown below, for $0 \le t \le 3$.



   Use the equations for the bug's path to answer the following questions.

   (a) The horizontal velocity of the bug at some time t is given by the derivative $x'(t)$. What is a formula for $x'(t)$?

(b) Now find a formula for $y'(t)$, the vertical velocity of the bug.

(c) Find a point on the path where the vertical velocity y' is zero.

(d) Now look back at the picture of the bug's path on the first page. What can you say about the tangent line to the path at the point you found in Part (c)? Explain why your answer makes sense, given what you know about y' there.

(e) Suppose (a, b) is a point where the horizontal velocity x' is zero, and the vertical velocity y' is *not* zero. What do you predict the tangent line looks like at this point? Why?

2. Now suppose a spider is following the path given by the parametric equations

$$(x(t), y(t)) = \left(t - 2, t^{4/3} - 2t\right).$$

(a) At which point (a, b) is the vertical velocity of the spider equal to 0?

(b) If a curve is given parametrically and if t_0 is a time where $x'(t_0) \neq 0$, then the slope of the tangent line to the curve at the point $(x(t_0), y(t_0))$ is given by

$$\frac{y'(t_0)}{x'(t_0)}.$$

(You will see this again in class.)

Using this information, find the point (a, b) on the spider's path where the tangent line is perpendicular to the line $y = -5x - 12$.