Efficient Computation of Maximal Independent Sets in Unstructured Multi-Hop Radio Networks

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Algorithms for Ad Hoc and Sensor Networks...

Excerpts from a typical paper

Algorithm 2 $\mathrm{LP_{MDS}}$ approximation (Δ known)

$x_i := 0;$
for $\ell := k-1$ to 0 by -1 **do**
    $(*\ \tilde{\delta}(v_i) \le (\Delta+1)^{(\ell+1)/k},\ z_i := 0\ *)$
    for $m := k-1$ to 0 by -1 **do**
        $(*\ a(v_i) \le (\Delta+1)^{(m+1)/k}\ *)$
        send color_i to all neighbors;
        $\tilde{\delta}(v_i) := \left|\{j \in N_i \mid \mathrm{color}_j = \text{'white'}\}\right|;$
        if $\tilde{\delta}(v_i) \ge (\Delta+1)^{\ell/k}$ **then**
            $x_i := \max\left\{x_i, \frac{1}{(\Delta+1)^{m/k}}\right\}$
        fi;
        send x_i to all neighbors;
        if $\sum_{j \in N_i} x_j \ge 1$ **then** $\mathrm{color}_i :=$ 'gray' **fi**;
    od
    $(*\ z_i \le 1/(\Delta+1)^{(\ell-1)/k}\ *)$
od

Assumptions

6: **send** color$_i$ to all neighbors;

How do you know your neighbors ???

How can you exchange data with them ???

→ Collisions (Hidden-Terminal Problem)

Most papers assume that ...

1) ... there is a MAC-Layer in place!

2) ... nodes know their neighbors!

This assumption may make sense in well-established, structured networks,...

...but it is certainly invalid during and shortly after the deployment of ad hoc and sensor networks.

Assumptions...

Excerpts from a typical paper

Algorithm 2 LP_{MDS} approximation (Δ known)

$x_i := 0;$

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      $x_i := \max\left\{x_i, \frac{1}{(\Delta+1)^{m/k}}\right\}$

    fi;

    send x_i to all neighbors;

    if $\sum_{j \in N_i} x_j \geq 1$ **then** $\text{color}_i := \text{`gray'}$ **fi**;

  od

  $(*\ z_i \leq 1/(\Delta+1)^{(\ell-1)/k}\ *)$

od

More technicalities...

$$2:\ \textbf{for}\ \ell := k-1\ \textbf{to}\ 0\ \textbf{by}\ -1\ \textbf{do}$$

How do nodes know when to start the loop ???

What if nodes join in afterwards ???

→ Asynchronous wake-up!

Paper assumes that there is a global clock and synchronous wake-up!

This assumption may make sense in well-established, structured networks,...

...but it is certainly invalid during and shortly after the deployment of ad hoc and sensor networks.

Deployment

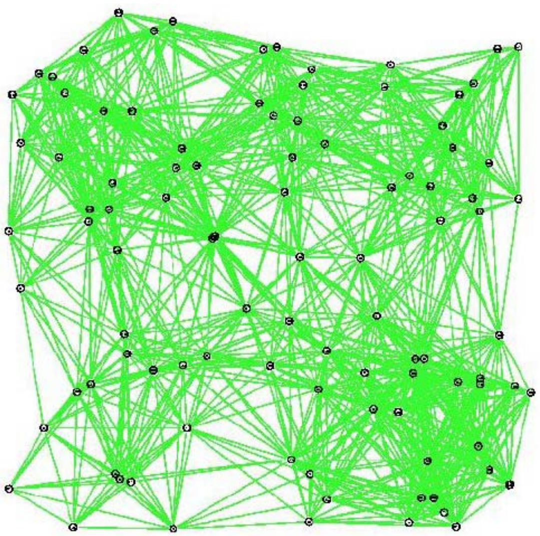
- Absence of a-priori knowledge
  - → Neighbors are unknown
  - → Even number of neighbors is unknown
- No built-in infrastructure
  - → No existing MAC-Layer
  - → No reliable point-to-point connections
  - → Collisions, Interference...
- Asynchronous wake-up

→ Nodes have to structure their network !

Initialization-Gap

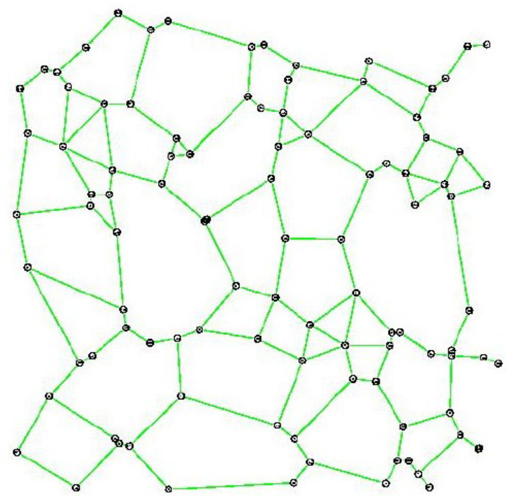
Reality starts like this:

Algorithms assume this:



Self-Organization

„Initialization“



Deployment and Initialization

- Initialization in current systems often very slow (Bluetooth)

- Ultimate Goal: Come up with an efficient MAC-Layer quickly.
- Theory Goal: Design a *provably* fast and reliable initialization algorithm.

We must make realistic assumptions!

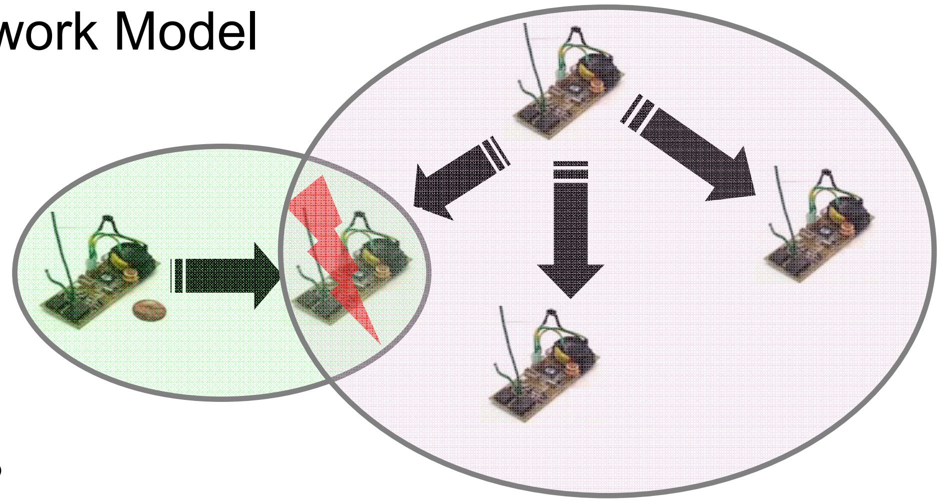
We need to define a model capturing the characteristics of the initialization phase.

Overview

- Introduction → Initialization-Gap
- Model
- Clustering
- Algorithm & Analysis
- Conclusions & Open Problems

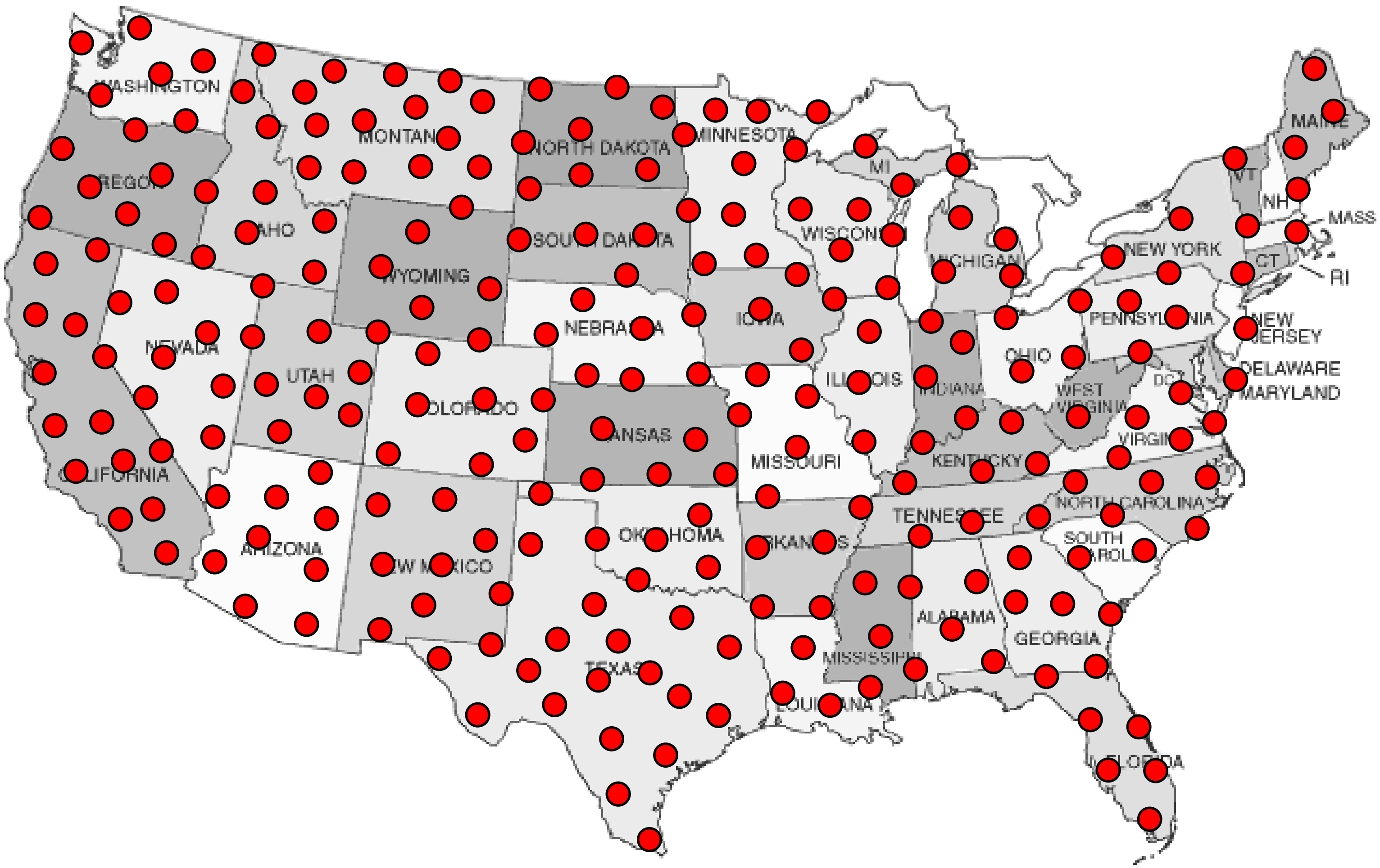
Unstructured Multi-Hop Radio Networks – Model (1)

- Introduced by [Kuhn, Moscibroda, Wattenhofer, MOBICOM 2004]
- Adaptation of classic Radio Network Model



- Multi-Hop
  - Hidden-Terminal Problem
- Node distribution...?
  - How do real networks look like?

Like this?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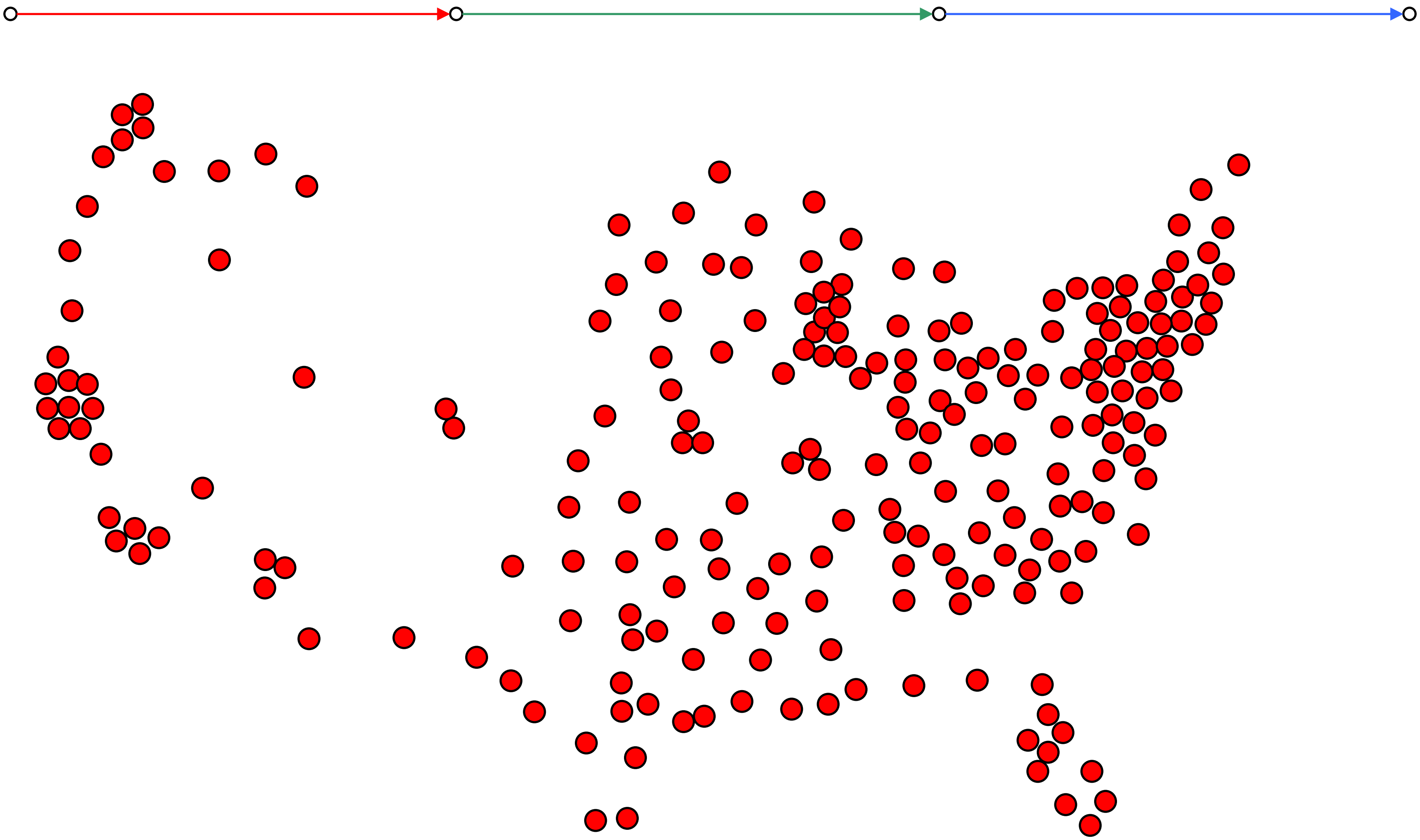
NEW JERSEY
DELAWARE
MARYLAND
MI
DC

Or rather like this?



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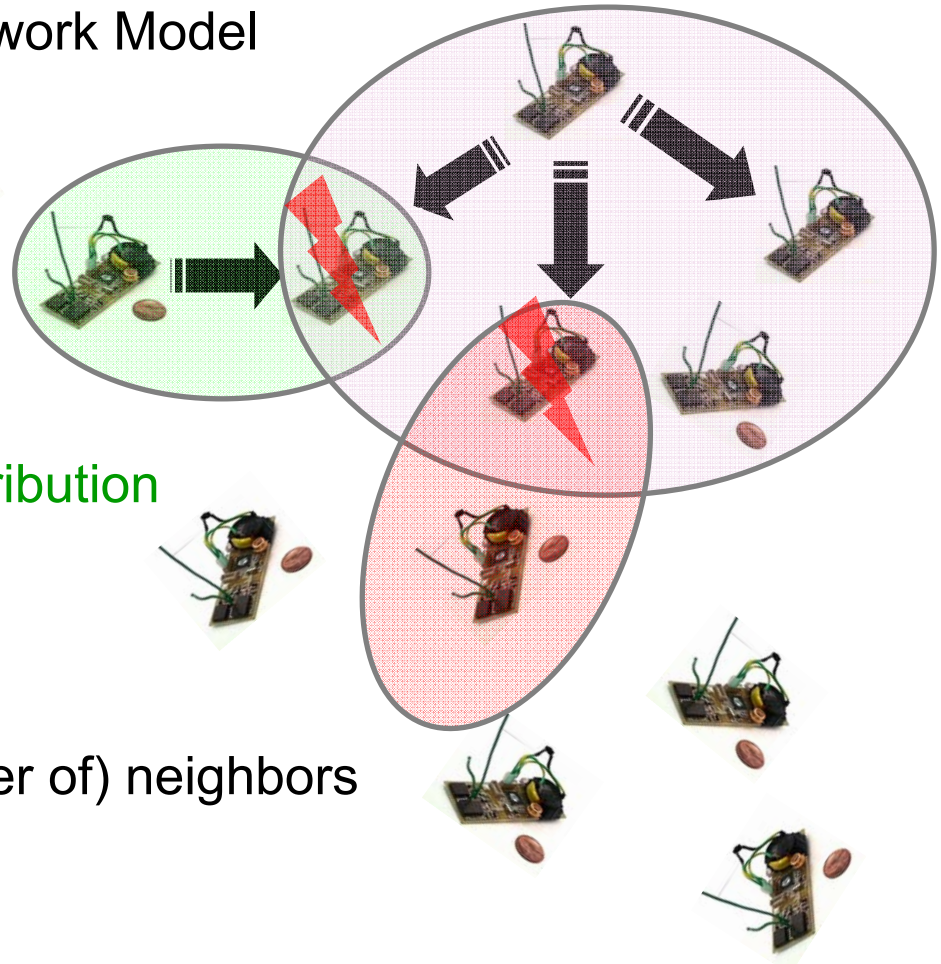
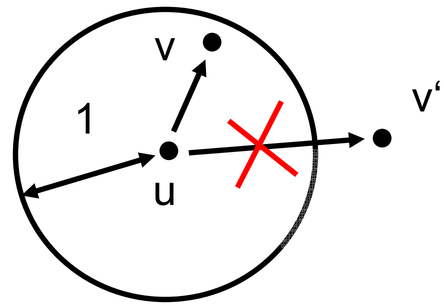


- Multi-Hop
  - Hidden-Terminal Problem
- Arbitrary (worst-case) node distribution
  - No assumption of uniformity!
- No collision detection
  - Not even at the sender!
- No knowledge about (the number of) neighbors
- Asynchronous Wake-Up
  - No global clock!

Unstructured Multi-Hop Radio Networks – Model (2)

- Unit Disk Graph (UDG) to model wireless multi-hop network
  - Two nodes can communicate iff Euclidean distance is at most 1



- Upper bound N for number of nodes in network is known
  - This is necessary due to Ω(n / log n) lower bound [Jurdzinski, Stachowiak, 2002]

Q: Can we efficiently (and provably!) compute an *initial structure* in this harsh model?
Q: Can we efficiently (and provably!) compute an *MAC-Layer* in this model?

A: Hmm, we…
A: Yes, we can!

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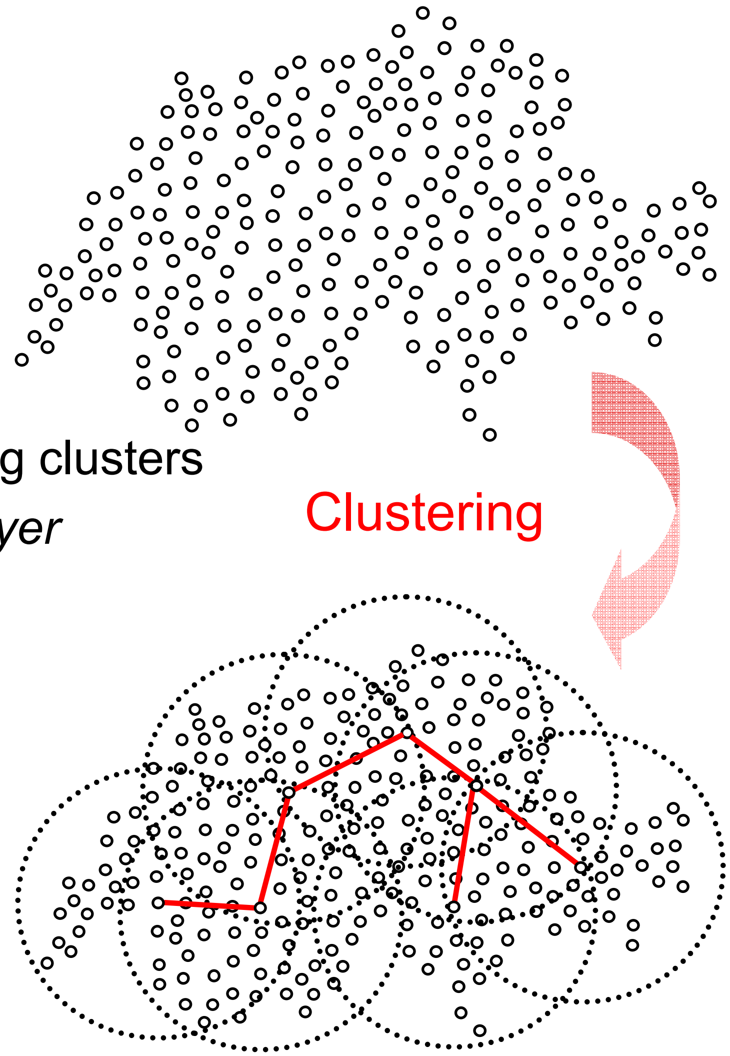
The Importance of Being Clustered...

- Clustering
  - Virtual Backbone for efficient routing
    → *Connected Dominating Set*
  - Improves usage of sparse resources
    → *Bandwidth, Energy, ...*
  - Spatial multiplexing in non-overlapping clusters
    → *Important step towards a MAC Layer*



Clustering brings structure into Chaos!

Clustering: Dominating Set

- Choose clusterhead such that:

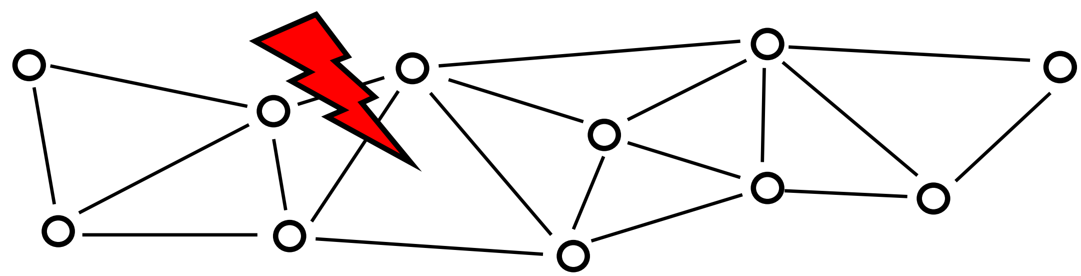
  Each node is either a clusterhead or has a clusterhead in its communication range.

- When modeling the network as a graph G=(V,E), this leads to the well-known Dominating Set problem.

Dominating Set:

- A Dominating Set DS is a subset of nodes such that each node is either in DS or has a neighbor in DS.
- Minimum Dominating Set MDS is a DS of minimal cardinality.

Interference between clusterheads



Clustering – Dominating Set

Dominating Set: provides a good initial structure !

Can be solved efficiently !

→ See our MOBICOM paper...

There may be interference between clusterheads!

→ We don't want neighboring clusterheads!

Clustering – Maximal Independent Set

- Choose clusterhead such that:

  1) *Each node is either a clusterhead or has a clusterhead in its communication range.*

  2) *No two clusterheads are within each other's communication range.*

  → This avoids interference at clusterheads!

- When modeling the network as a graph G=(V,E), this leads to the Maximal Independent Set problem.

Maximal Independent Set:

– A Maximal Independent Set MIS is a dominating set where no two dominators are neighbors.

Yet Another Clustering Algorithm...???

Recently, we gave a O(1) approximation algorithm for dominating set in time $O(\log^2 n)$ in the same model.

[Kuhn, Moscibroda, Wattenhofer, MOBICOM 2004]

- There are many other existing clustering algorithms for ad hoc and sensor networks
  - [Kutten, Peleg, Journal of Algorithms 1998], [Jia, Rajaraman, Suel, PODC 2001]
  - [Gao, et al., SCG 2001], [Wan, Alzoubi, Frieder, INFOCOM 2002 & MOBIHOC 2002]
  - [Chen, Liestman, MOBIHOC 2002], [Kuhn, Wattenhofer, PODC 2003]
  - .....

- Q: Why yet another clustering algorithm ?
- A: Other algorithms - with theoretical worst-case bounds - make too strong assumptions! (see previous slides...)

→ Not valid during initialization phase!

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MIS Algorithm - Results

- With three communication channels

Algorithm computes a correct MIS in time $O(\log^3 n / \log\log n)$, w.h.p.

- Idea: Simulate three channels with a single channel.

Algorithm compute a correct MIS in polylogarithmic time even with a single communication channel, w.h.p.

MIS Algorithm – Basic Idea

- Use 3 independent communication channels Γ_1, Γ_2, and Γ_3.
  → Then, simulate these channels with a single channel.

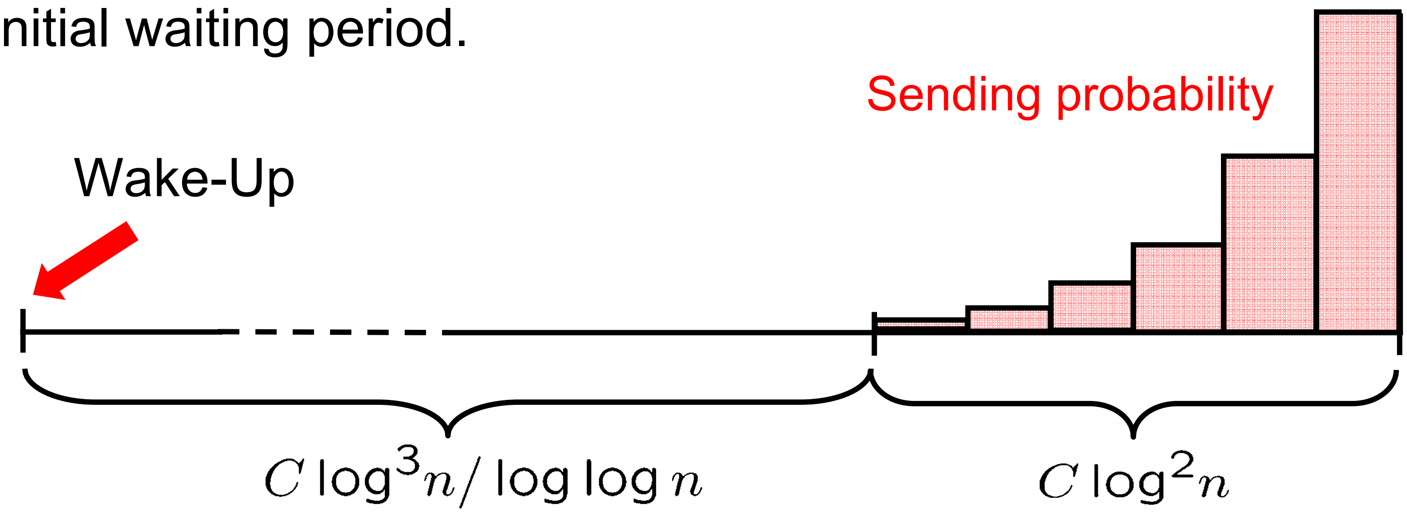
- Nodes compete for becoming candidates on Γ_1.
  → Dominating Set Algorithm → Candidate Election
  → Idea: Reduce number of potential MIS nodes
  from n to $O(\log^2 n / \log\log n)$ nodes w.h.p.

- Candidates compete for joining the MIS on Γ_2.
  → MIS Algorithm

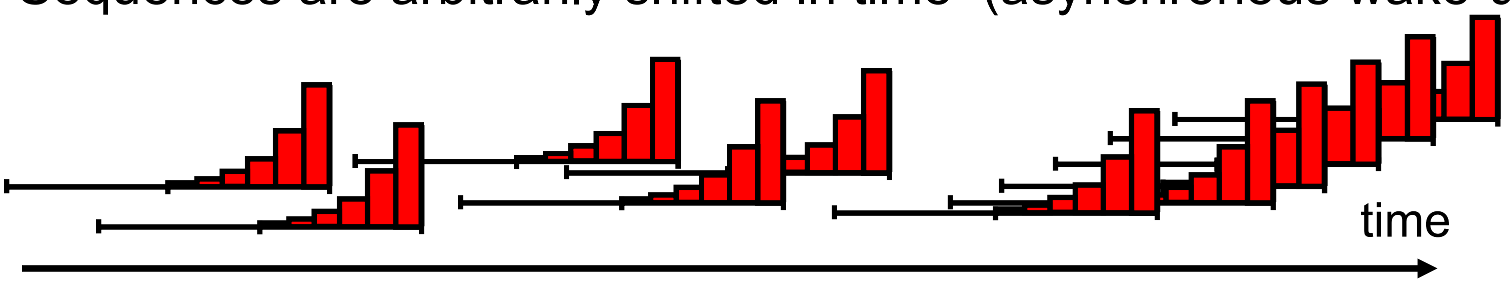
- MIS nodes send on Γ_3.
  → Inform nodes which have woken up late

Candidate Election – Basic Structure

- Each node's sending probability increases exponentially after an initial waiting period.



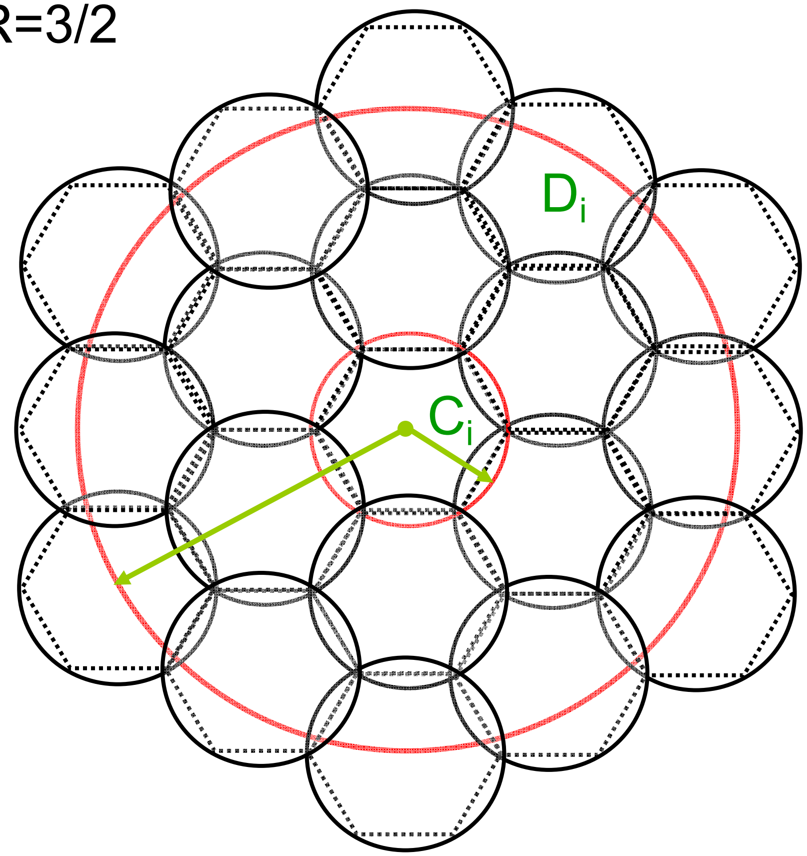
- When sending a message → become candidate
- When receiving a message (without collision!) → restart algorithm!
- Sequences are arbitrarily shifted in time (asynchronous wake-up)



Candidate Election - Analysis

- Cover the plane with (imaginary) circles C_i of radius r=1/2
- Let D_i be the circle with radius R=3/2

- A node in C_i can hear all nodes in C_i
- Nodes outside of D_i cannot interfere with nodes in C_i



Result:

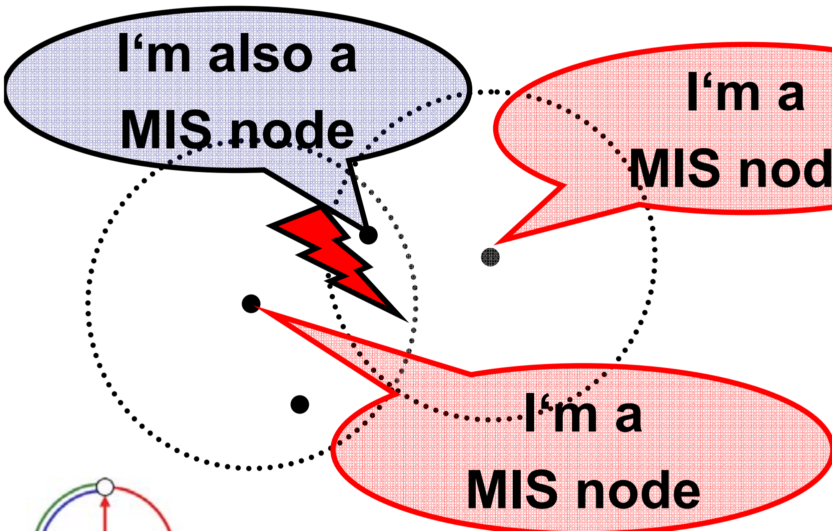
Algorithm produces at most $O(\log^2 n / \log\log n)$ candidates in each C_i w.h.p.

MIS Node Election

- Given the $O(\log^2 n / \log\log n)$ candidates per C_i.
- We want to select MIS nodes from candidates.

1.Simple Idea:

- Each candidate sends with certain probability
- Sender becomes MIS node
- Node that receives message stop competing



[Wan, Alzoubi, Frieder]

Doesn‘t work!

→ Collisions!

MIS Node Election

2. Better Idea:

- Each active candidate has counter c(v).
- Active candidates send with prob. $\Theta(\log\log n / \log^2 n)$.
- When sending first time

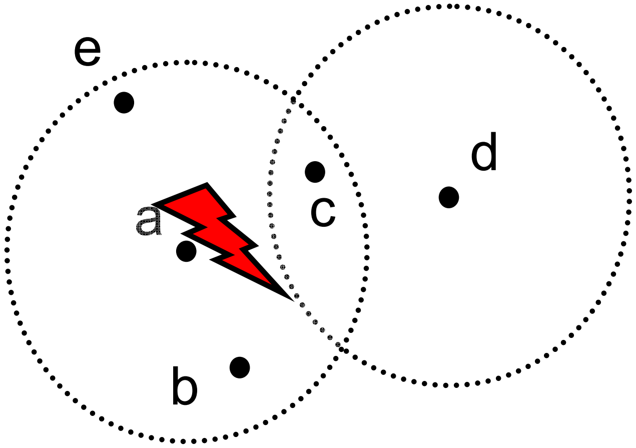
  → start incrementing counter in each time-slot
- Node that receives message, restart algorithm
- When counter reaches $\Theta(\log^3 n / \log\log n)$

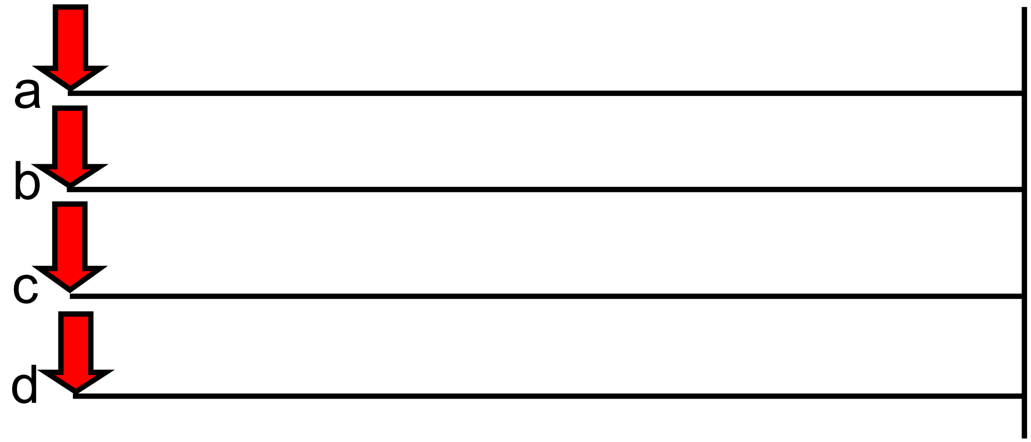
  → join MIS

  → start sending on channel Γ_3 with prob. 1/6

Intuition: Each node hears every other node (without collision) at least once!

MIS Node Election





$$\Theta\left(\frac{\log^3 n}{\log\log n}\right)$$

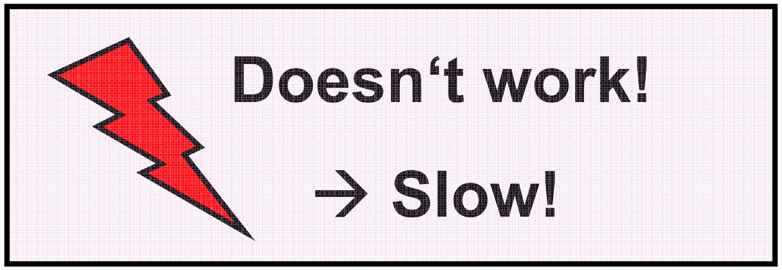
Positive:

→ No neighboring candidates join MIS

Negative:

→ New candidates can „kill“ so far successful candidates

→ Progres is too slow !!!



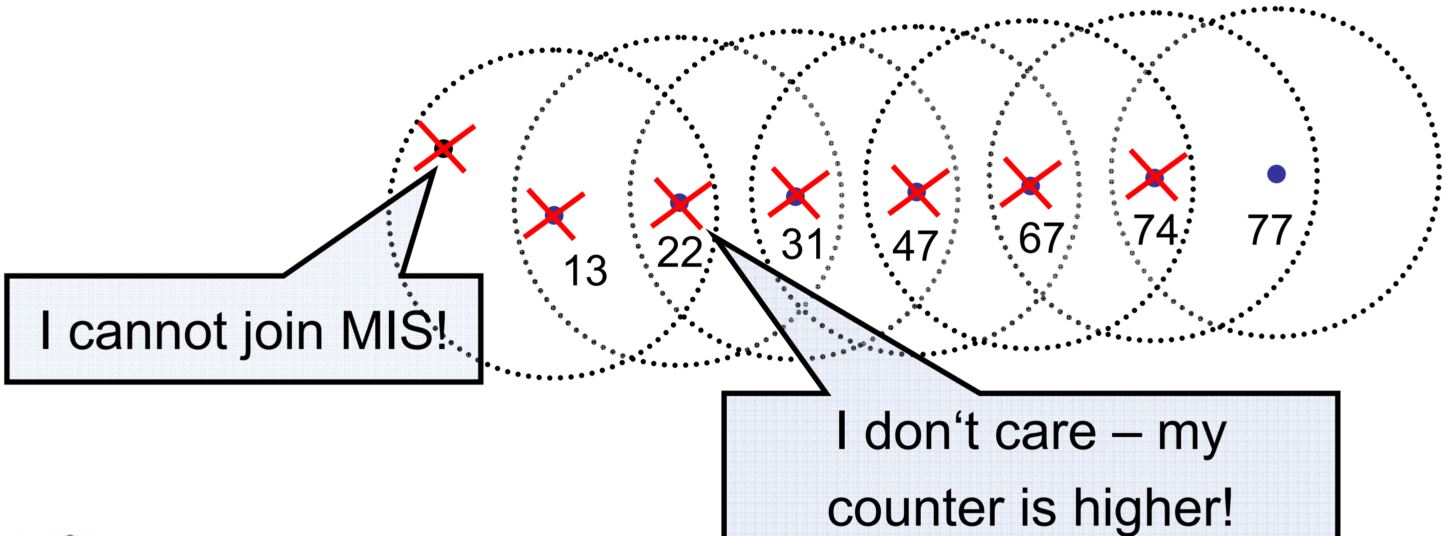
MIS Node Election

3. Improvement:

- Only nodes with **higher counter value** can „kill“ other active candidates!

Problem : Long cascading chains of restarts...

→ progress still slow in some regions!



MIS Node Election

- Goal: Every node restarts at most a constant number of times.
  → Must avoid long cascading chains of restarts!
- Intuition: Avoid that two candidates join MIS at the same time!
  → Then MIS nodes send on Γ_3 to inform candidates!

4. New Idea:

- Candidates that receive a message from a candidate with equal counter resets counter to 0.
  → Only candidates with **equal counter** can stop each other.

MIS Node Election

Let t* be the first time-slot when a candidate appears in C_i.
It can be shown that...

→ Within time t*+O(log n), a candidate sends without collision!

→ After that, this candidate cannot be stopped anymore!

→ **Only constant number of restarts!**

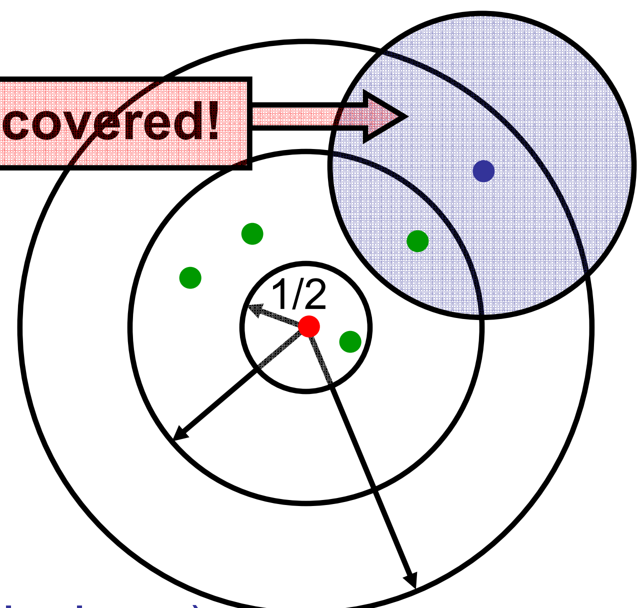
This region is covered!

t_0: u wakes up

$t_1 := t_0 + O(\log^2 n)$: at least one candidate emerges

$t_2 := t_1 + O(\log n)$: one candidate can send without collision...

... and joins MIS at $t_2 + O(\log^3 n / \log\log n)$



MIS Node Election

Candidates can have counter very close to each other!

When a node joins MIS

→ not enough time to inform neighboring candidates!

→ Feasibility not guaranteed anymore!

4. Final Idea:

- Candidates that receive a message from a candidate with similar counter resets counter to - $\Theta(\log n)$.

→ Only candidates with **similar counter** (within $\Theta(\log n)$) can stop each other

Enough time to inform neighbors!

This works!

Results

- Some nasty details have been left out...
- Running time of algorithm is $O\left(\frac{\log^3 n}{\log\log n}\right)$
- Solution with a single channel is possible within O(polylog(n)).
- Idea:
  - Each node simulates each of its multi-channel time-slots with O(polylog(n)) single-channel time-slots.
  - It can be shown that result remains the same.

Algorithm compute a correct MIS in polylogarithmic time even with a single communication channel, w.h.p.

Conclusion and Outlook

- Initialization of ad hoc and sensor network of great importance!
- Relevant model must be considered!

MOBICOM 2004:

- A model capturing the characteristics of the initialization phase
- A fast algorithm for computing a good dominating set from scratch

In this paper:

- A fast algorithm for computing more sophisticated structures (MIS)




GOAL

A fast algorithm for establishing a MAC Layer from scratch!