Recap: Waves

- **Traveling Transverse Wave:**

→ displacement: $y(x,t) = y_m \sin[kx \mp \omega t]$

→ velocity of particles in medium: $v_y(x,t) = \frac{dy}{dt} = \mp \underbrace{\omega y_m}_{v_m} \cos[kx - \omega t]$

not the same!

→ wave speed = "speed of crest": $v_{wave} = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f$

- **Sound Waves:**

→ Longitudinal Displacement: $s(x,t) = s_m \cos[kx - \omega t]$

↕ 90° out of phase

→ causes pressure wave: $\Delta p(x,t) = \Delta p_m \sin[kx - \omega t]$

→ wave speed: $v_{sound} = \sqrt{B/\rho}$ } depends on medium

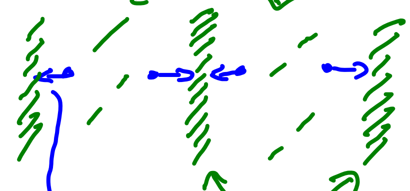
→ Sound Intensity: $I = \frac{\text{acoustic power}}{\text{area}} = \frac{1}{2}\rho \omega^2 s_m^2 v_{sound} = \frac{\Delta p^2}{2\rho v}$

→ Sound Level: $\beta = (10\,dB)\log_{10}(I/I_0)$

→ Beats: $f_{beat} = |f_1 - f_2|$ } 2 sources of $f_1 \neq f_2$

Recap: Waves

- **Sound Waves: (traveling)**



→ Longitudinal Displacement: $s(x,t) = s_m \cos[kx - \omega t]$

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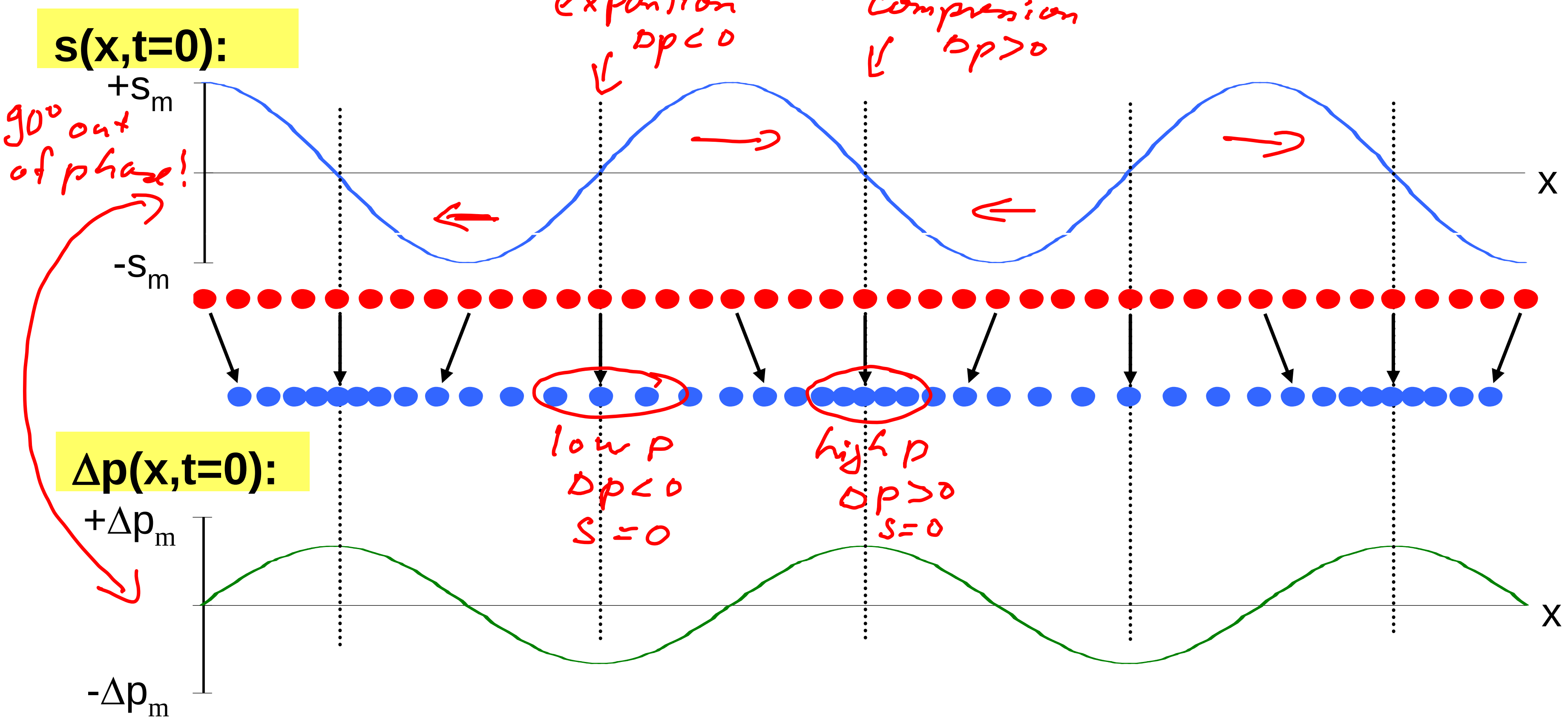


→ Standing Waves

Consider two traveling waves of equal amplitude and frequency, but traveling in opposite direction:

$y_1(x,t) = y_m \sin[kx - \omega t]$ $\longrightarrow$ $+x$

$y_2(x,t) = y_m \sin[kx + \omega t]$ $\longleftarrow$ $-x$

⇒ total displacement: (use principle of superposition)

⇒ add wave amplitudes:

$$y(x,t) = y_1(x,t) + y_2(x,t)$$

$$= y_m[\sin(kx - \omega t) + \sin(kx + \omega t)]$$

$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)$

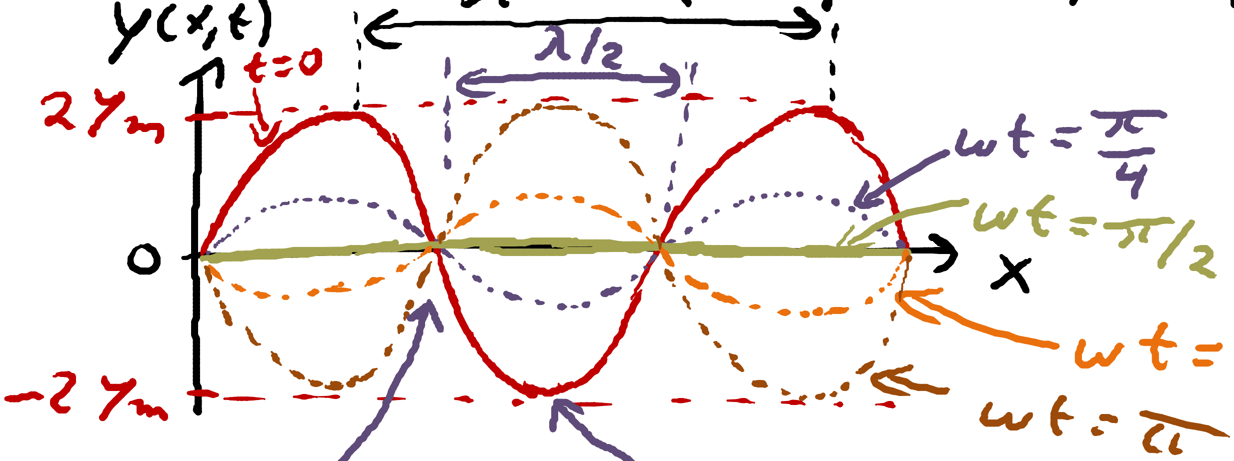
$$= [2y_m \sin(kx)]\cos(\omega t)$$

note: not of the form $\sin(kx \pm \omega t)$

⇒ not a traveling wave

⇒ standing wave

$$y(x,t) = \underbrace{[2y_m \sin(kx)]}_{\text{amplitude }(x)} \underbrace{\cos(\omega t)}_{\text{oscillation}}$$



standing wave! (crests don't move)

nodes: (no motion) at

$$x = n\frac{\lambda}{2}$$

$n = 0, 1, 2, 3, \ldots$

antinodes: (max. oscillation amplitude) at

$$x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}$$

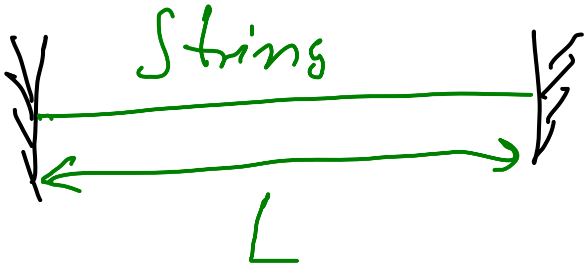
$n = 0, 1, 2, 3 \ldots$

- spacing between nodes $= \lambda/2$
- spacing between antinodes $= \lambda/2$
- spacing from node to antinode $= \lambda/4$

- Standing wave:
  - crests are at fixed positions
  - each point in medium undergoes SHM
  - Amplitude of SHM varies sinusoidally with x
  - Phase of SHM is same for all x ($\cdot \cos(\omega t)$)
    $\phi = \omega t$

- Traveling waves:
  - crests move / propagate with V_{wave}
  - each point in medium undergoes SHM
  - Amplitude of SHM is same for all x
  - Phase of SHM increases linearly with x
    $\sin(kx - \omega t)$, ϕ_0 (under kx)

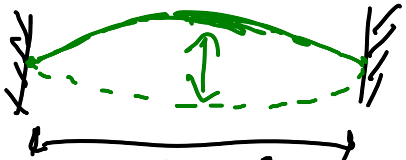
→ Standing Waves on a String –



- string clamped at each end
  - ⇒ ends don't move
  - ⇒ ends must be displacement node!

⇒ at certain frequencies ("resonant frequencies") we can excite standing wave patterns ("resonant oscillation modes") on the string!

• $n=1$: lowest frequ. mode : "fundamental mode"



$L = \lambda_1/2$

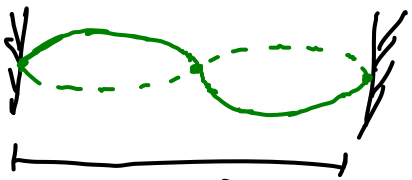
$\lambda_1 = 2L$

$f_1 = \dfrac{v}{\lambda_1} = \dfrac{v}{2L}$

2 nodes

1 antinode

• $n=2$: "second harmonic"



$L = 2\,\lambda/2$

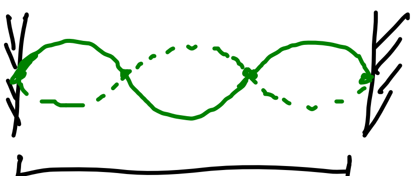
$\lambda_2 = L$

$f_2 = \dfrac{v}{\lambda_2} = \dfrac{v}{L} = 2 f_1$

3 nodes

2 antinodes

• $n=3$: "third harmonic"



$L = 3\,\lambda/2$

$\lambda_3 = \dfrac{2}{3} L$

$f_3 = \dfrac{v}{\lambda_3} = \dfrac{3}{2}\dfrac{v}{L} = 3 f_1$

4 nodes

3 antinodes

distance between nodes $= \lambda/2$ (always!)

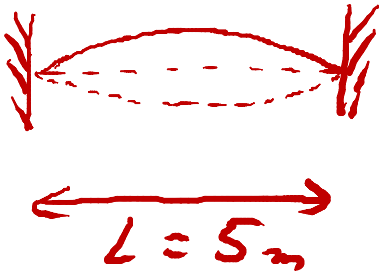
general for string:

$L = n\,\lambda_n/2, \quad n = 1, 2, 3, \ldots$

$\lambda_n = 2L/n$

$f_n = \dfrac{v}{\lambda_n} = \dfrac{n v}{2L} = n f_1$

What is the wavelength of the **lowest** frequency standing wave on a **5 m** long **string**?



$n = 1$

$$5m = \frac{\lambda_1}{2} \Rightarrow \lambda_1 = 10m$$

λ ~ ?

A. 2.5 m

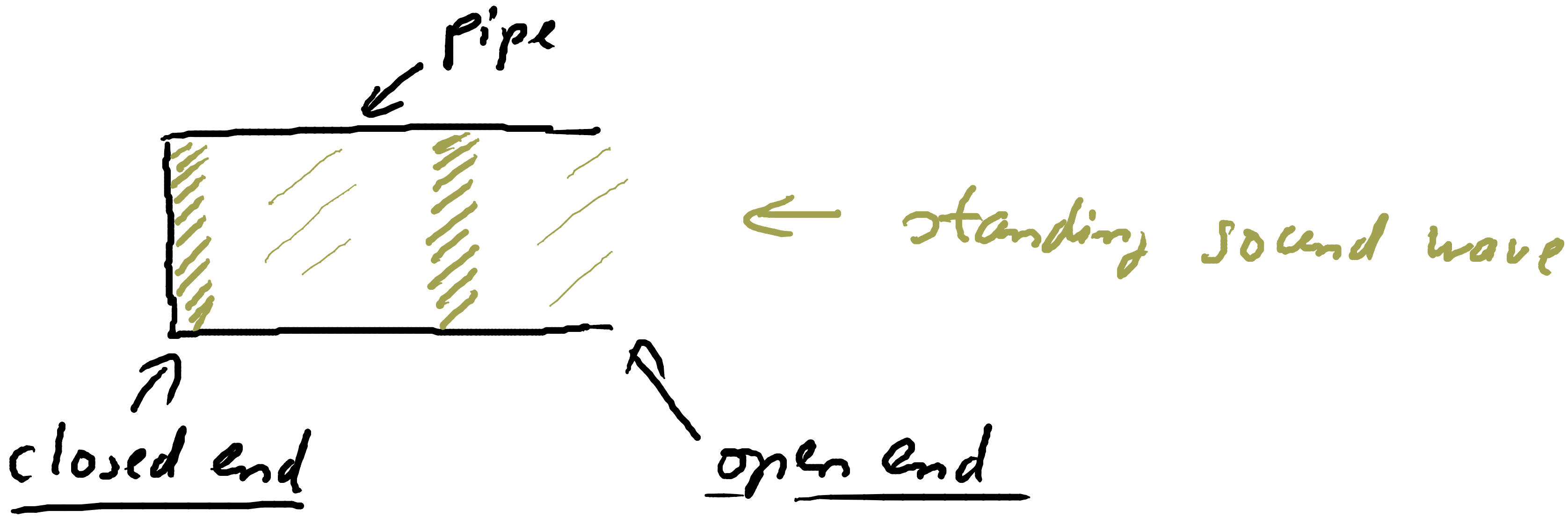
B. 5 m

C. 7.5 m

D. 10 m

E. 15 m

→ Standing Sound Waves in Pipes



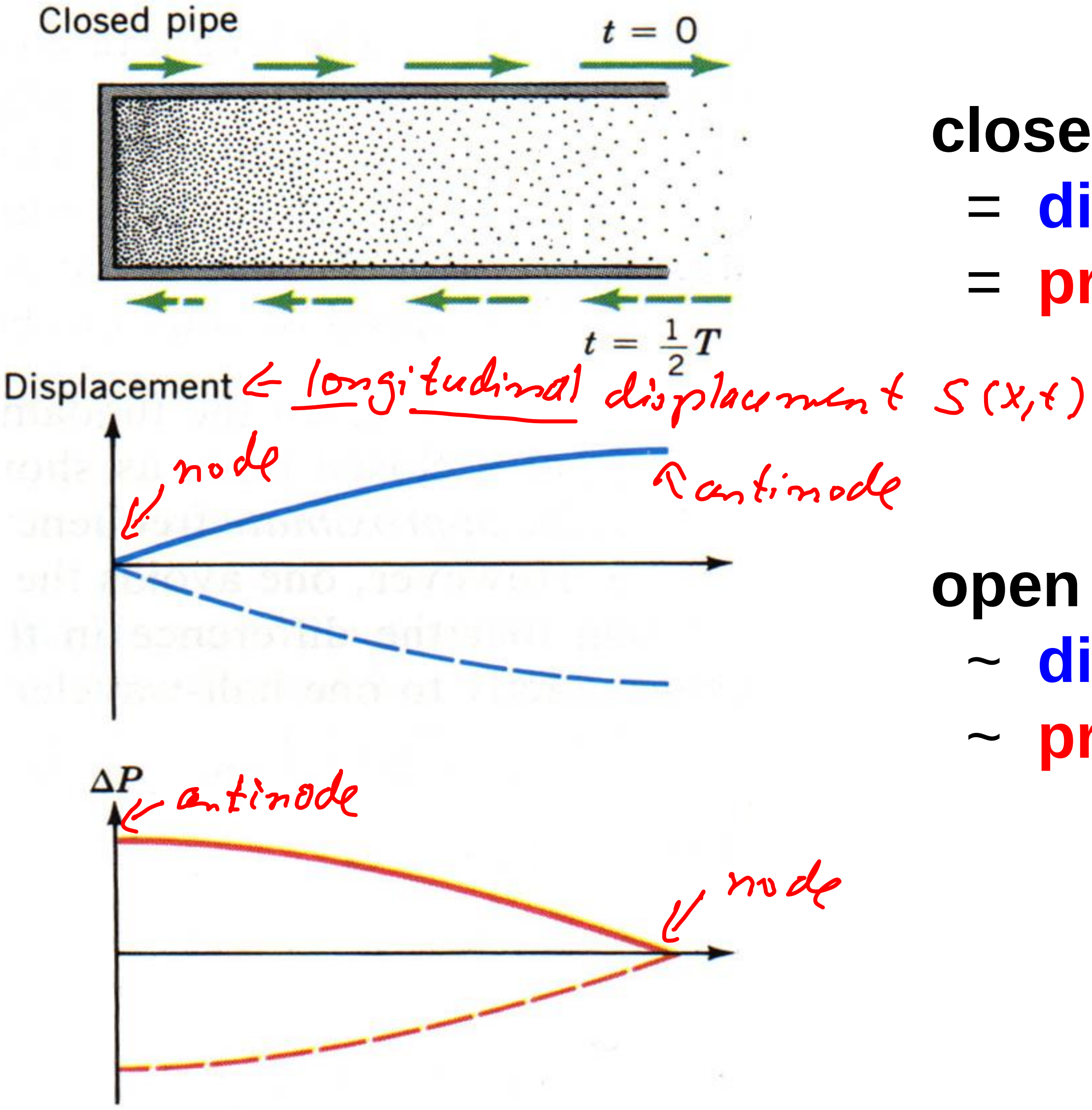
closed end
- displacement node
- pressure antinode

↑ 90° out of phase

open end
- pressure node ($\Delta p = 0$)
- displacement antinode

⇒ resonance at certain frequencies → standing waves at these frequ.

Standing Sound Waves in Pipes:



closed end

= **displacement node**

= **pressure antinode**

open end

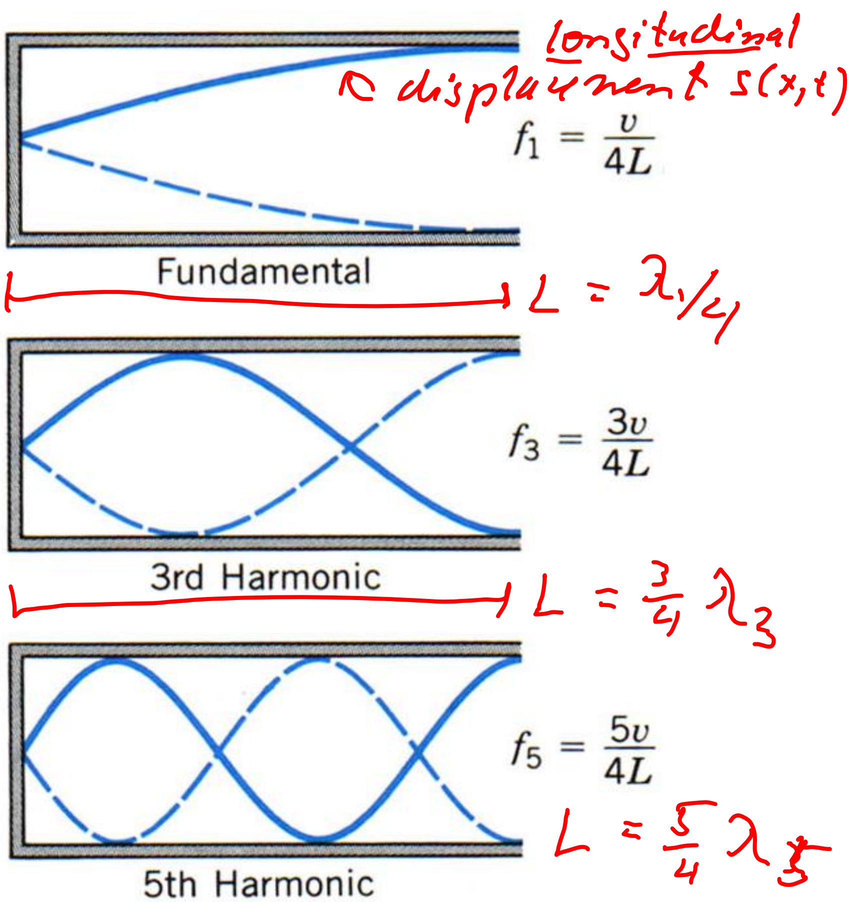
~ **displacement antinode**

~ **pressure node**

pipe closed at one end:

$\lambda_n = 4L / n, \quad f_n = nv / 4L$

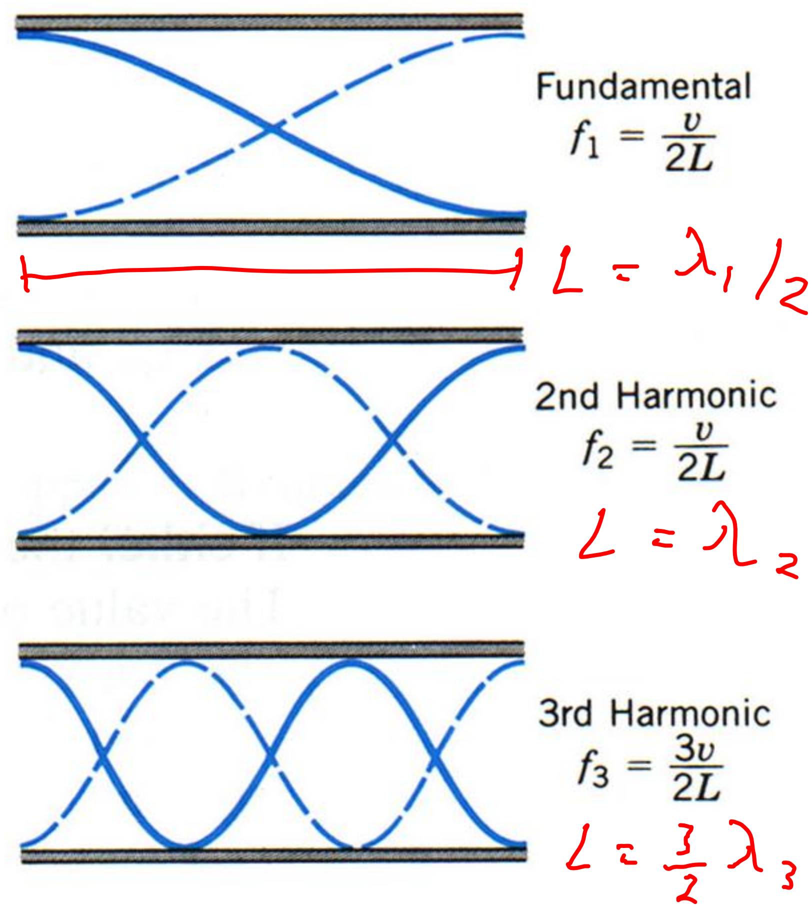
n=1, 3, 5, . . .



pipe open at both ends:

$\lambda_n = 2L / n, \quad f_n = nv / 2L$

n=1, 2, 3, . . .



Musical Scales:

- **1 octave** = ***multiplicative* factor of 2** in frequency

A_3=220 Hz, A_4 = 440 Hz, A_5=880 Hz

- Each **octave** has **12 notes**.

- The frequency f of each note is $2^{1/12}$ times the frequency of the preceding note:

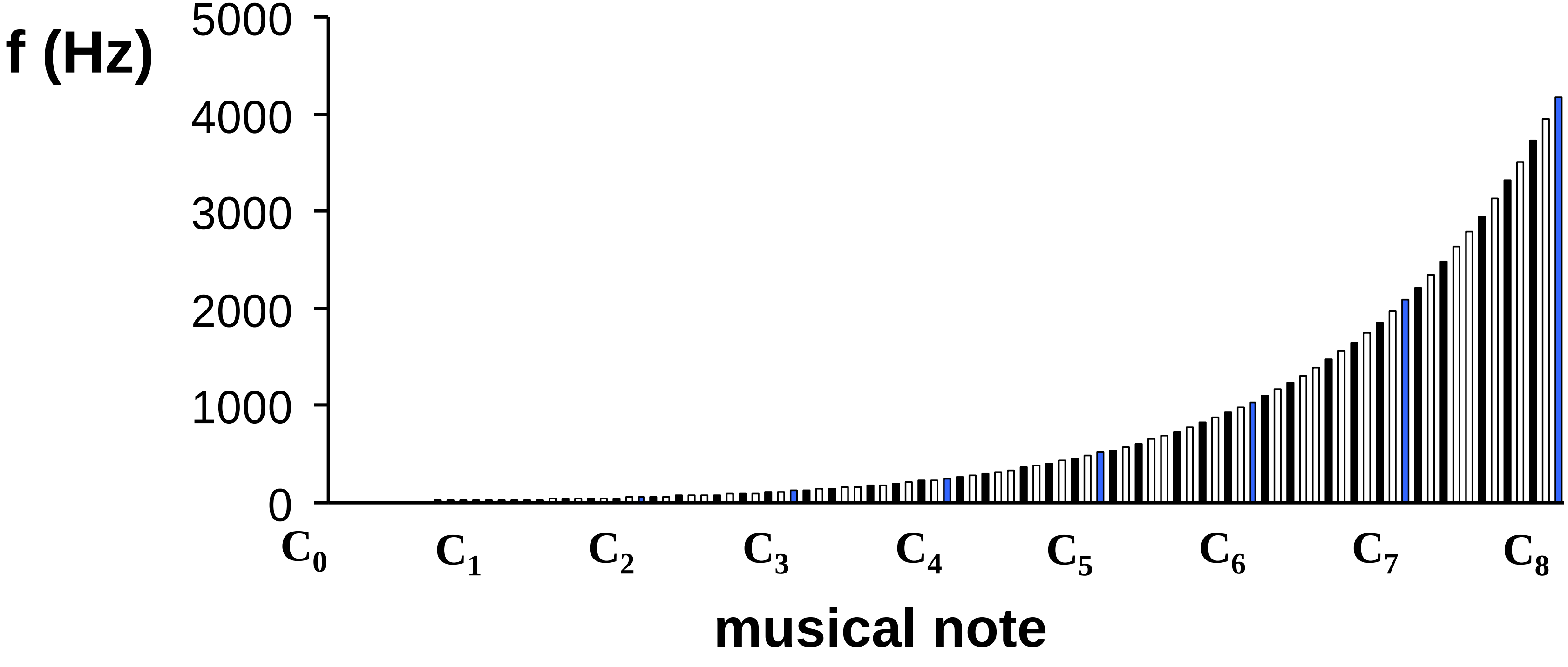
$$\mathbf{f_{n+1} / f_n = 2^{1/12} = 1.059463}$$

$$\Rightarrow \quad f_{n+12} / f_n = (2^{1/12})^{12} = 2$$

∴ Equal separation in number of notes corresponds to **equal multiplicative factors** in frequency.

⇒ **Note frequencies increase exponentially with note number.**

Equal Tempered Chromatic Scale:

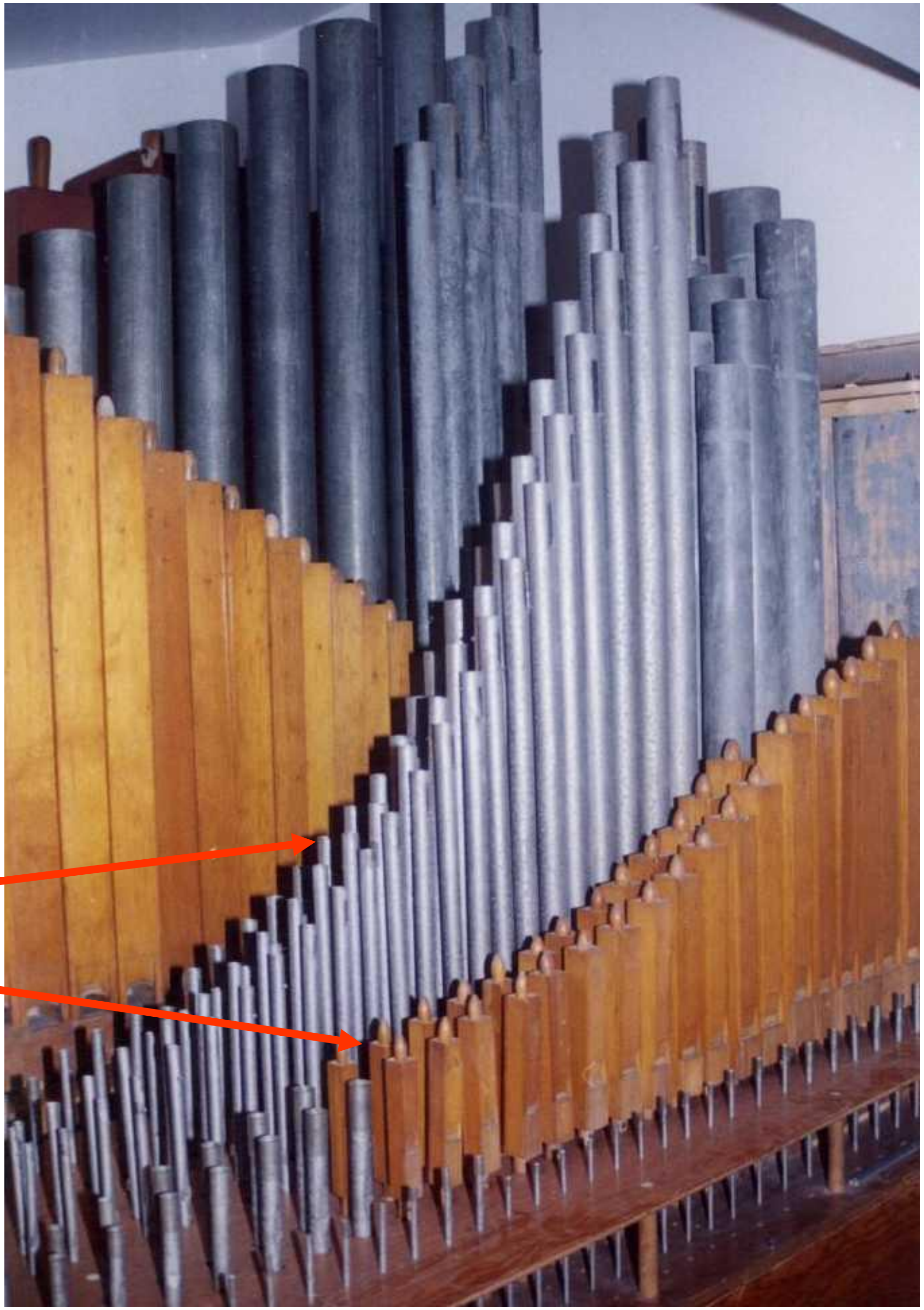


Sage Chapel Organ:

For pipe organs,
pipe length L $\propto$1/f

Pipe length increases exponentially!

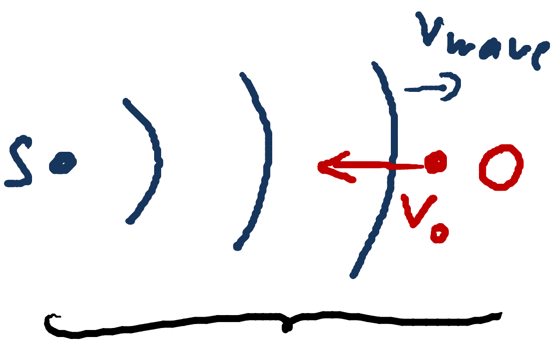
Ranks of pipes



→ Doppler Effect:

Frequency of waves measured by an observer depends on the motion of source and the observer (relative to medium)!

① Source is stationary, observer is moving:



intercepts more crests / time

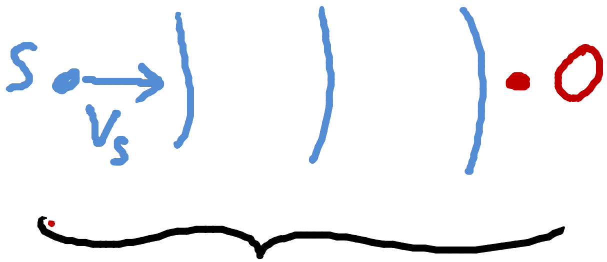
$\Rightarrow f'_{measured} > f_{source}$

$$f'_{\text{heard by observer}} = f_{source}\,\frac{v_{wave} \pm v_o}{v_{wave}}$$

v_o: speed of observer > 0

- use "+" sign if observer moves towards source
- use "−" sign if observer moves away from source

② Observer stationary, source moving:



λ gets smaller

$$\lambda' = \lambda - V_s T$$

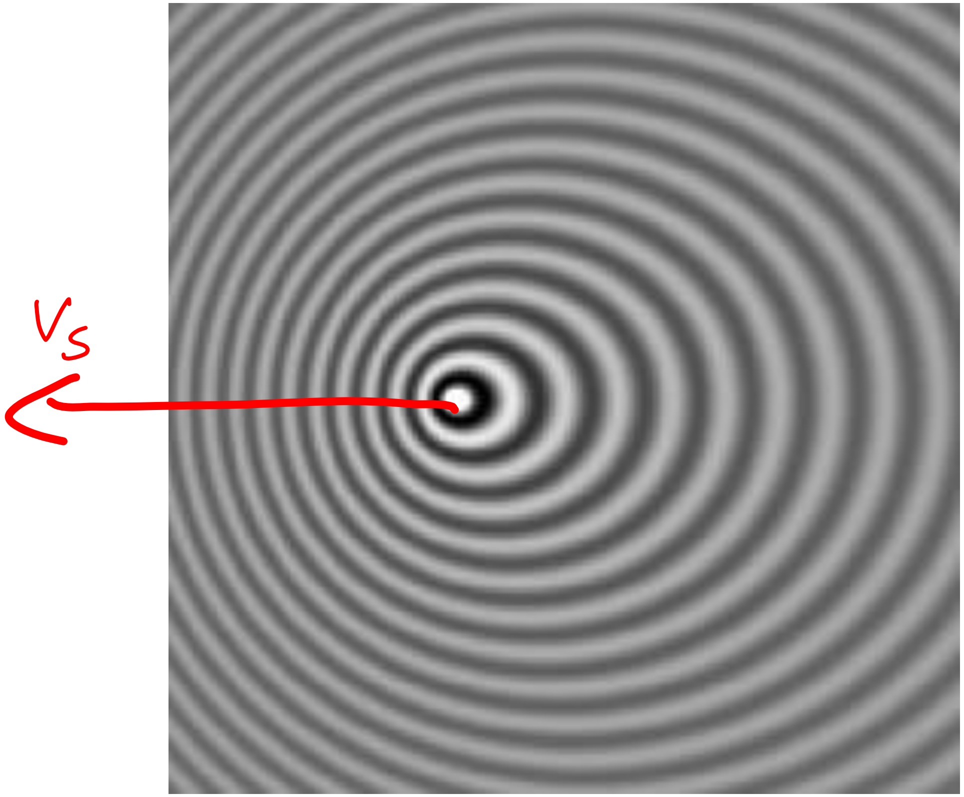
$$f'_{\text{measured by observer}} = \frac{V_{wave}}{\lambda'} = f_{source} \frac{V_{wave}}{V_{wave} \mp V_s}$$

V_s = speed of source > 0

- use "+" sign if source is moving away from observer
- use "−" sign if source is moving towards observer

In either case, if the distance between the observer and source is decreasing with time, $f_{measured}$ goes up!

Doppler Effect for a Moving Source



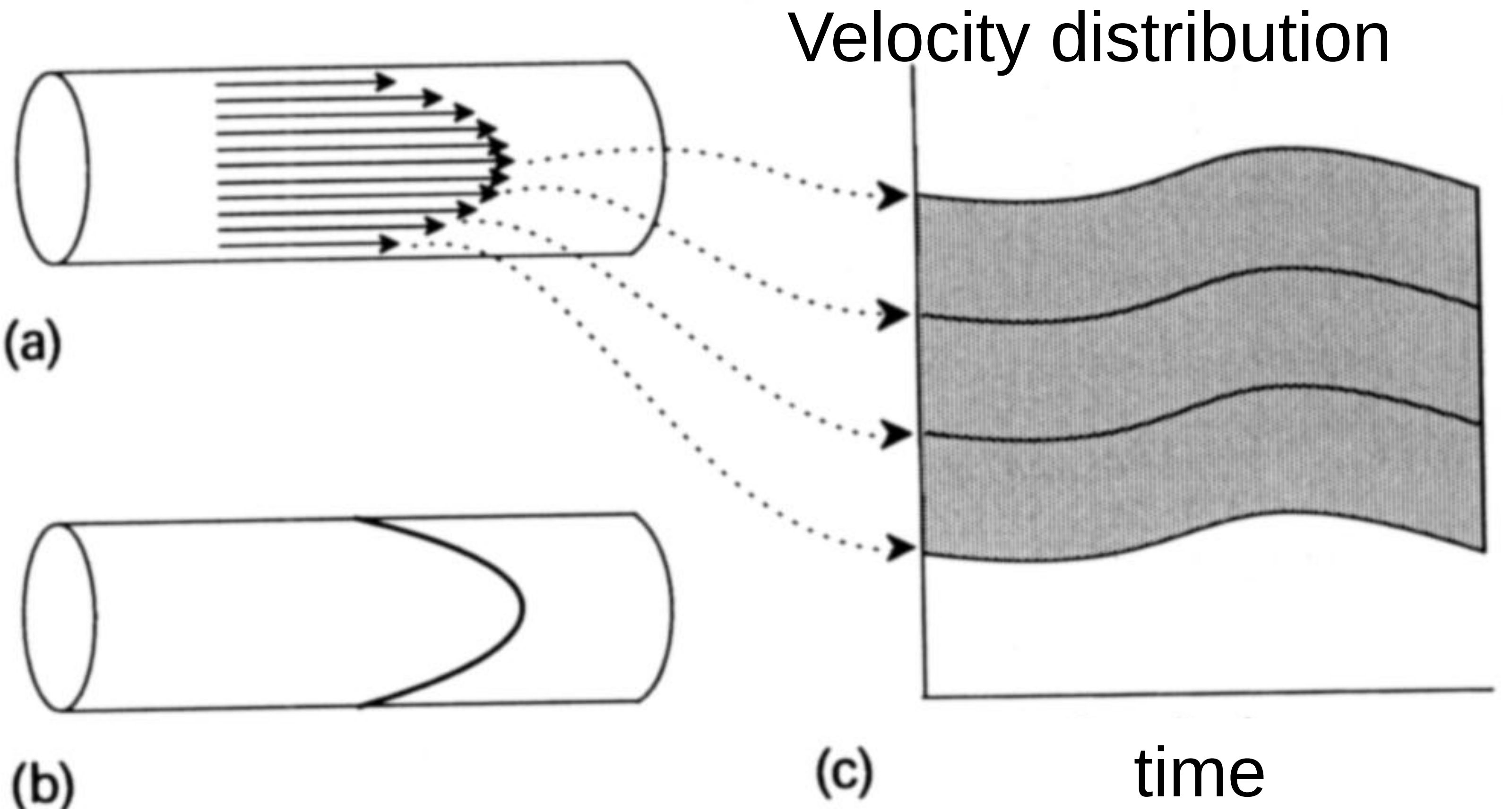
③ both moving:

$$f'_{\text{measured by observer}} = f_{\text{source}} \frac{V_{\text{wave}} \pm V_o}{V_{\text{wave}} \mp V_s}$$

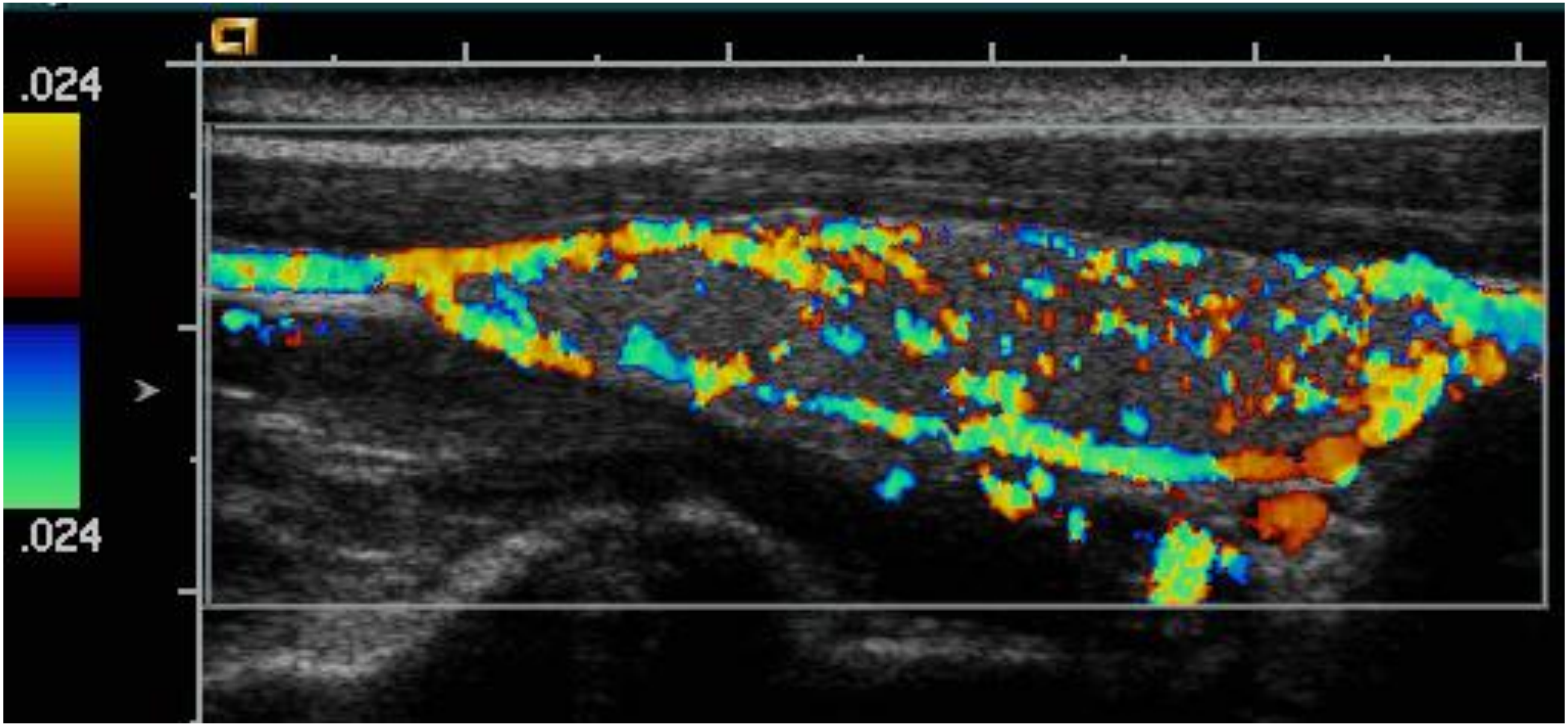
Doppler Effect: Applications

Doppler Ultrasound Imaging

- Used to image blood flow and flow profiles.
- **Flow velocity ∝ Doppler frequency shift**



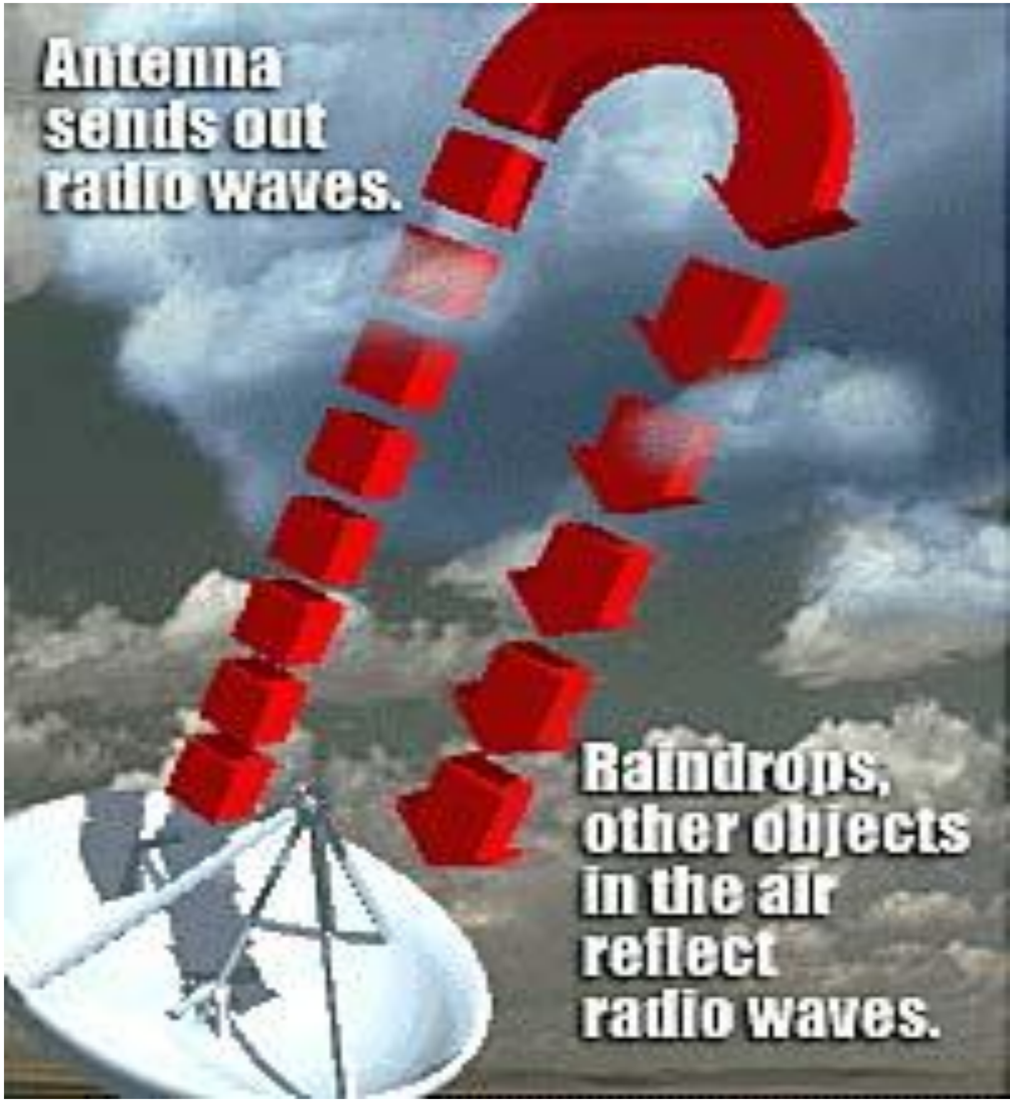
Blood flow in a healthy thyroid gland:



Doppler Weather Radar

- Radio waves are trans-mitted by an antenna that scans the sky and measures the **amplitude and frequency** of reflected waves.

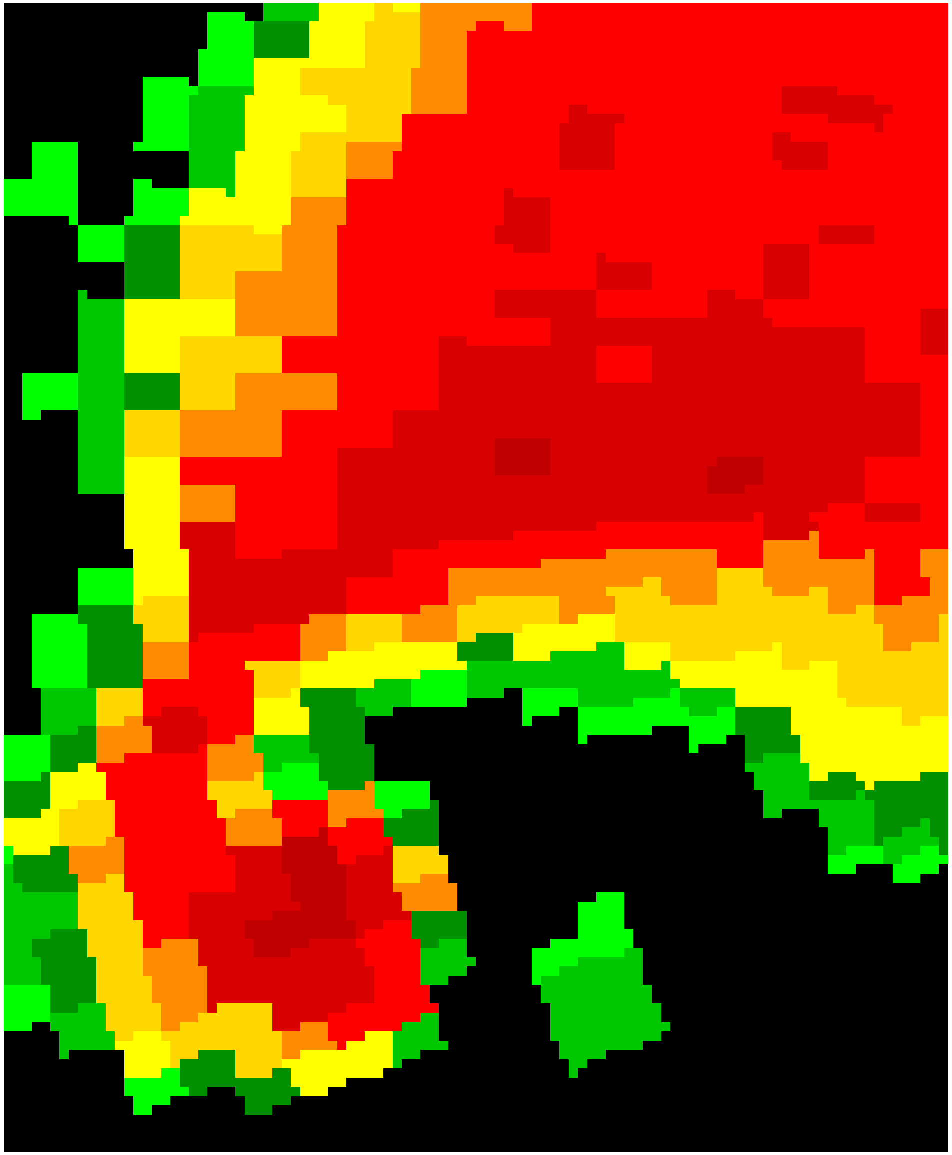
- Detects precipitation ***intensity*** **as well as** ***motion*** caused by winds.



Doppler Weather Radar

- Detects precipitation precipitation ***intensity*** as well as ***motion*** caused by winds.
- **Heavy precipitation** produces **larger reflected signals**.
- **Larger raindrops** and hail fall faster (have a larger terminal speed) and produce **larger Doppler shifts.**
- **Winds** change direction of motion of precipitation and **produce Doppler shifts.**

Signatures of a Tornado:



"Hook" in the radar intensity map:

Indicates that precipitation is following a curved path.

Astronomical Redshifts

Because the Universe is **expanding**, other galaxies are moving away from us. The **farther they are away, the faster they are moving**.

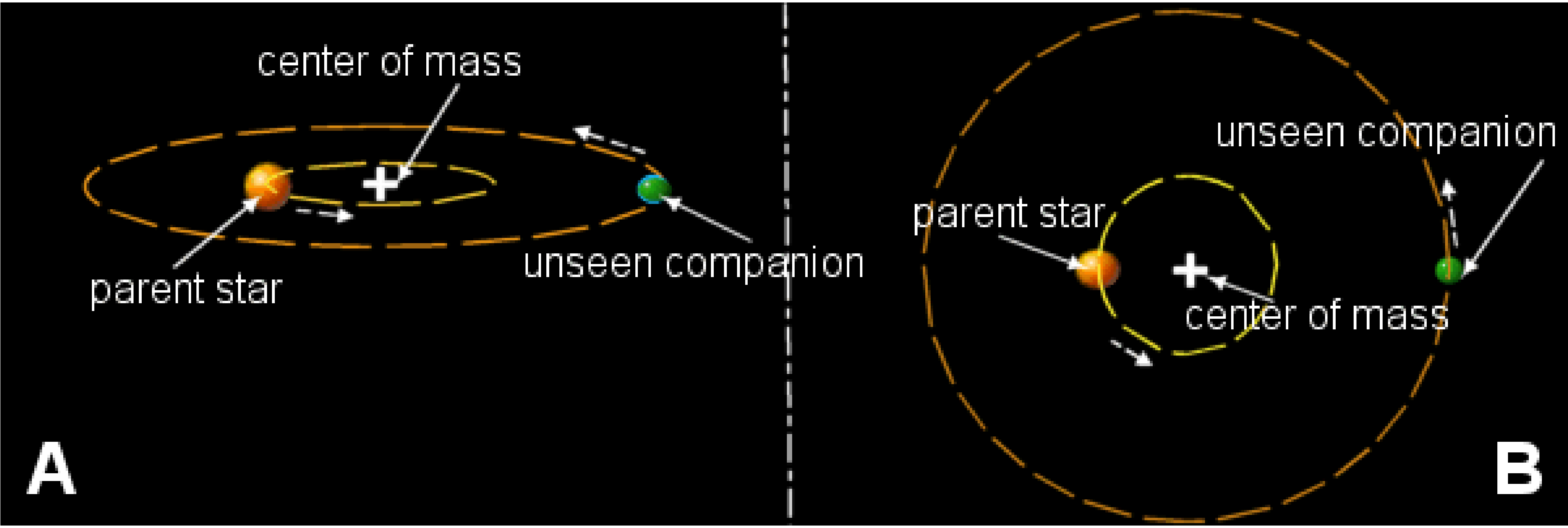
z = Red shift = (observed λ - rest λ)/(rest λ)

Visible spectrum: 400 nm (violet) to 750 nm (red)

Quasars: **$z=5.82 \Rightarrow \lambda'=6.52\lambda$**

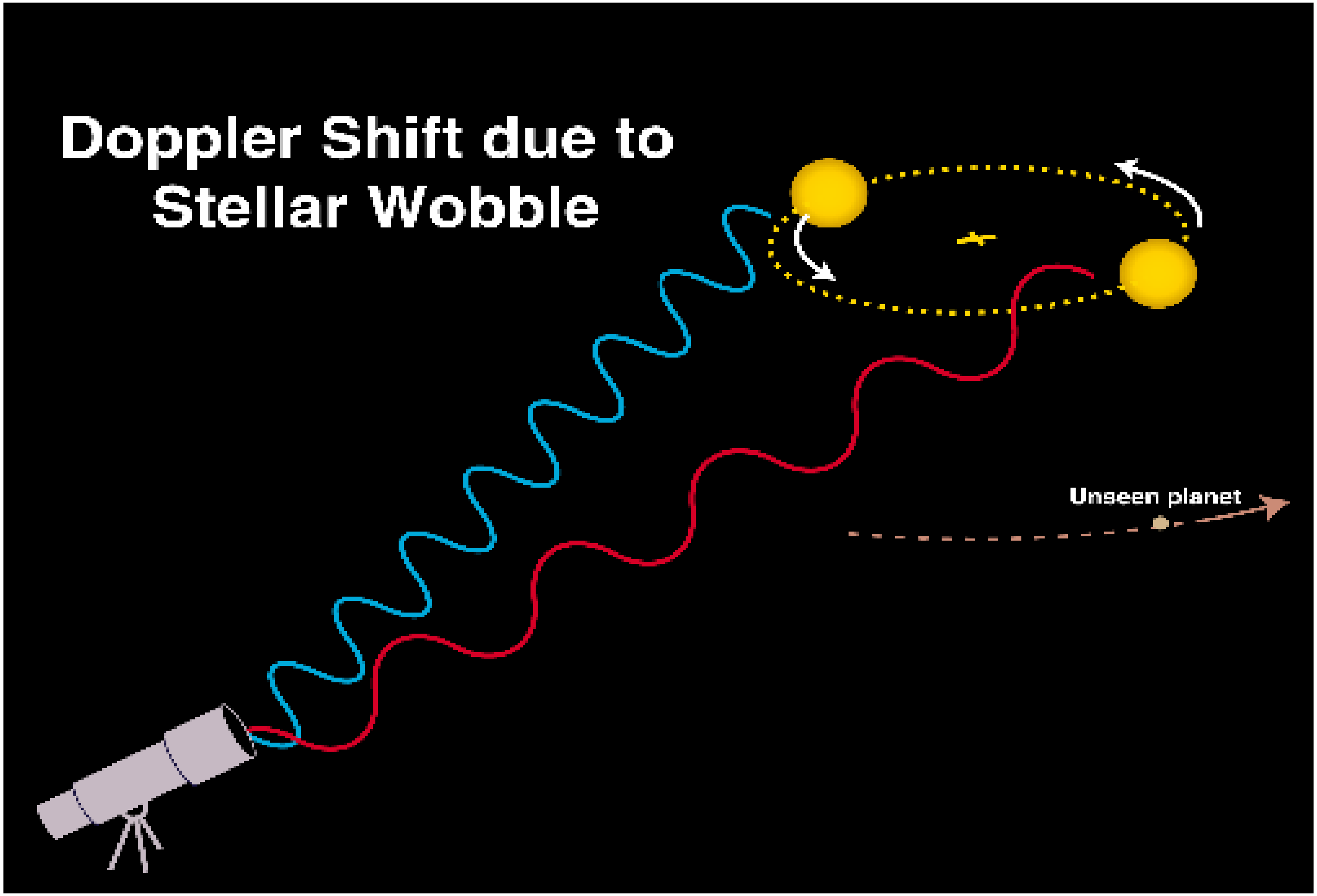
$\therefore$ Hydrogen spectral line at 121.6 nm **(ultraviolet)** shifts to 829.1 nm **(infrared)**.

Detecting Planets Around Other Stars

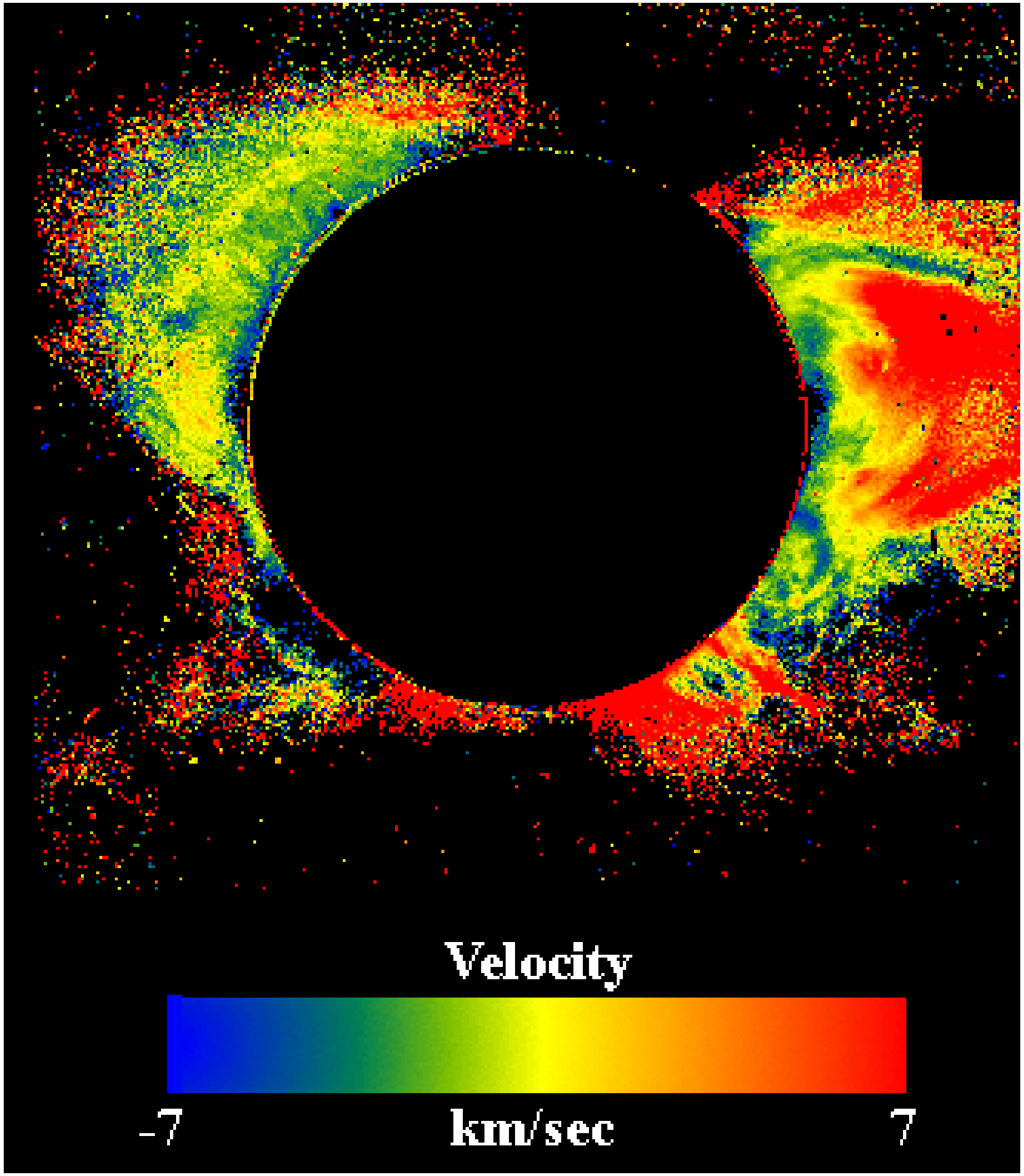


A star and its planet orbit about their common center of mass.

Detecting Planets Around Other Stars



Can detect changes in stellar velocity of 3 m/s!



Solar coronal plasma velocity towards or away from the satellite measured by Doppler shift