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Duverger's Law in the Laboratory

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Abstract

In this paper, we conduct a laboratory experiment to test the robustness of Duverger's law and its extension "M+1 rule." The M+1 rule states that, in an M-member electoral district with the single nontransferable vote, votes concentrate on M+1 candidates, and Duverger's law is the case of M=1. Our experimental results support the comparative statics of the M+1 rule so that votes concentrate on a smaller number of candidates under M=1 than under M=2. Whether the M+1 rule itself is supported or not depends on what type of index we use to measure the effective number of candidates. Some degree of variety is also observed between experimental sessions. Nonetheless, on average, our analysis with Molinar's index supports the M+1 rule well.

Key Words: Duverger's law, M+1 rule, laboratory experiment

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1. Introduction

Duverger's law (Duverger, 1954) is the most famous law in political science. The law states that the single-member electoral district system leads to a two-party system. Its application to the electoral-district level is that two candidates gather a large share of votes in each single-member district. The M+1 rule (Reed, 1990) is an extension of Duverger's law to M-member districts with the single nontransferable vote ($M \geq 1$). The empirical literature examined whether Duverger's law holds using electoral data from various countries (e.g., Chhibber and Kollman, 2004), and Reed (1990) and others tested the M+1 rule using Japanese data. Not all but many empirical studies supported these laws.

This paper also tests the M+1 rule for the cases of M=1 (Duverger's law) and M=2 at the district level, but our method is laboratory experiment. As explained in Section 2, several studies examined Duverger's law in laboratories. However, since their main purposes are not the examination of Duverger's law itself, their experimental design is relatively easy for the law to hold, and hence their tests are not sufficient to judge whether Duverger's law holds in the laboratory. Our experiment is designed to eliminate something helpful for the law to hold as severely as possible. In this sense, our experiment can be regarded as the severest test of Duverger's law and the M+1 rule.

Our experimental result supports the comparative statics of the M+1 rule so that the smaller number of candidates tend to gather votes under M=1 than under M=2. Whether the M+1 rule itself is supported or not depends on what type of index we use to measure the effective number of candidates. Some degree of variety is also observed between experimental sessions. Nonetheless, on average, our analysis with Molinar's (2001) index supports the M+1 rule well. In the literature of experimental social sciences, it is frequently observed that experimental data do not support theoretical predictions exactly (experiment vs. theory), but that they support the comparative statics of the theories well (experiment vs. experiment). The results of our experiment also show this tendency.

Why laboratory? The empirical literature tries to find whether Duverger's law holds in the actual politics, and hence they use data from actual elections. Our interest is somewhat different: we would like to know the robustness of Duverger's law and the M+1 rule. The robustness is examined in terms of the following three aspects. First, we would like to know the external validity of Duverger's law and the M+1 rule by examining them in the laboratory. Secondly, we would like to know whether the laws hold beyond the political context so that we may be able to apply the laws to any other academic fields. For this purpose, we conduct our experiment in an abstract context without using terms such as election, vote, candidate, and others related to politics. Finally, we would like to know whether the laws hold even without the strategic behaviors of politicians. For this purpose, in our experiment, candidates are exogeneously given only as alternatives for voters to choose, and subjects play the roles of voters only. If the laws hold under such a setup, then we may be able to say that the laws are so robust that they are highly expected to hold in the actual elections even when there exist various additional and complex factors that affect the electoral outcomes.

This paper is organized as follows. In Section 2, we review the related literature to clarify the purpose of our experiment. Section 3 explains our experimental design, and Section 4 shows the experimental results. Section 5 concludes the paper.

2. Related Literature

The theoretical literature is divided in two groups according to who are assumed to be players in the game, candidates or voters.[1] Reed (2003) states that the major effects of Duverger's law are produced by candidate retirements, and conducts a computer simulation in which a simple candidacy rule realizes the M+1 rule. Wada (1996) also constructs a game-theoretic model in which three parties, left, middle and right, make decisions on whether to run a candidate in a single-member district under the mixed electoral system. He shows that Duverger's law does not necessarily hold if candidacy itself gives parties some amount of benefit possibly due to a contamination effect (i.e., running a candidate in a single-member district itself works as an advertisement that attracts some votes for the party in the proportional representation bloc).

Mathematical proofs of Duverger's law and the M+1 rule are made by Palfrey (1989) and Cox (1994) respectively. They assume that only voters are players. Each voter takes account of how his or her vote affects the voting outcome and maximizes his or her expected utility obtained from the voting outcome. Hence, if his or her most-preferred candidate is less likely to compete for the seats, he or she casts his or her ballot for the second-preferred candidate. This type of strategic voting leads to only M+1 candidates getting votes in an M-member district. However, Clough's (2007) agent-based simulation, in which all voters are strategic, results in more than two parties in single-member plurality systems if each voter does not know all the other voters' strategies but knows his or her neighbors' strategies only.

The experimental literature also designs laboratory experiments in which subjects play the roles of voters. To our best knowledge, there is no voting experiment at this point of time in which subjects also play the roles of candidates or politicians. The following three experiments are closely related to ours.

2.1 Forsythe, Myerson, Rietz and Weber (1993)

Forsythe, Myerson, Rietz and Weber (1993) use a payoff matrix such as Table 1.[2] Three candidates, A, B and C, compete for one seat, and there are three groups of voters, Orange (4

[1] Morelli (2004) is an exception. In his model, all of parties, candidates and voters are strategic decision makers.

[2] Using the same structure of payoffs, this research group conducts a series of experiments (e.g., Forsythe, Rietz, Myerson and Weber (1996), Rietz, Myerson and Weber (1998), and Gerber, Morton and Rietz (1998)).

voters), Green (4 voters) and Blue (6 voters). In case that candidate A wins, for example, orange, green and blue voters obtain \$1.20, \$0.90 and \$0.40 respectively. If all voters vote for their most-preferred candidates sincerely, then candidates A, B and C obtain 4, 4 and 6 votes respectively. Hence, under the plurality rule, candidate C wins although he or she is the Condorcet loser. Forsythe et. al's interest is in what kind of information, such as pre-election polls or repetition of elections, is necessary for the split majority (i.e., orange and green voters) to coordinate their voting behaviors, so that they concentrate their votes on one of candidates A and B in order to win against blue voters who support candidate C. In this setup, voting for candidate C is the unique weakly dominant strategy for blue voters. In their experiment, in fact, most subjects who were assigned the roles of blue voters voted for candidate C. Hence, if orange and green voters can successfully coordinate their voting behaviors, then candidate C and one of candidates A and B obtain votes, which can be regarded that Duverger's law holds.

	A	B	C
Orange 4 voters	**1.20**	**0.90**	**0.20**
Green 4 voters	**0.90**	**1.20**	**0.20**
Blue 6 voters	**0.40**	**0.40**	**1.40**

Table 1. Forsythe, Myerson, Rietz and Weber's (1993) Payoff Matrix

It seems relatively easy for Duverger's law to hold under the payoff matrix in Table 1 because only orange and green voters have to coordinate their behaviors: given that what blue voters can do is to vote for candidate C sincerely, this is like a coordination game played between two blocs of voters, orange and green. That is, the payoff matrix in Table 1 can be transformed into the one in Table 2. If the two blocs can reach one of two pure-strategy Nash equilibria, (A, A) and (B, B), then it is regarded that Duverger's law holds.

		Green	
		A	**B**
Orange	**A**	**1.20, 0.90**	**0.20, 0.20**
	B	**0.20, 0.20**	**0.90, 1.20**

Table 2. Coordination Game Between Orange and Blue Blocs

2.2 Bassi (2008)

Bassi (2008) uses the payoff matrix in Tables 3 and 4 to compare three voting rules, the plurality rule, approval voting, and the Borda rule. In the payoff matrix in Table 3, the Condorcet winner is candidate D, and the loser is candidate B. Under the plurality rule, the Nash equilibrium which survives the iterated elimination of weakly dominated strategies is that two voters who prefer A and the voter who prefers D vote for candidate A, while two voters who prefer C vote for candidate C. Since two candidates, A and C, gather votes, Duverger's law can be regarded to hold in this Nash equilibrium. In the sense that only one subject's strategic behavior (i.e., voting for the second-preferred candidate) is necessary for Duverger's law to hold, this voting environment is in favor of Duverger's law. Of 10 rounds in Bassi's experiment, in fact, Duverger's law was supported in 4 rounds when subjects were inexperienced, and in 9 rounds when subjects were experienced.

	A	B	C	D
2 voters	1.40	1.00	0.20	0.60
2 voters	0.60	0.20	1.40	1.00
1 voter	1.00	0.20	0.60	1.40

Table 3. Bassi's (2008) Payoff Matrix with Condorcet Winner Treatment

Note: The grey cells represent the Nash-equilibrium strategies for each type of voter.

Another payoff matrix used in Bassi's experiment is the one in Table 4. In this payoff matrix, the Condorcet winner does not exist while the loser is candidate D. In the same type of Nash equilibrium as Table 3, two voters who prefer B vote for candidate B, while two voters who prefer A and the voter who prefers C vote for candidate C. This Nash equilibrium is also regarded to correspond to Duverger's law. In this setup, two voters, instead of one voter in Table 3, need to vote strategically for Duverger's law to hold. Hence, Duverger's law seems less likely to hold in the setup of Table 4 than Table 3. Of 10 rounds of Bassi's experiment using the payoff matrix in Table 4, in fact, Duverger's law was supported in 4 rounds when subjects were inexperienced, and 6 rounds when subjects were experienced.

	A	B	C	D
2 voters	0.60	1.40	0.20	1.00
2 voters	1.40	0.20	1.00	0.60
1 voter	0.60	1.00	1.40	0.20

Table 4. Bassi's (2008) Payoff Matrix with Cycle Treatment

Note: The grey cells represent the Nash-equilibrium strategies for each type of voter.

2.3 Endersby and Shaw (2009)

Endersby and Shaw (2009) conduct a series of classroom voting experiments, in which the overall winning student receives a few extra credit points in the course as an incentive, instead of monetary payment. They define payoffs for students on a one-dimensional (linear) policy space such as Figure 1. That is, there is a line segment, and the left end is 1 while the right end is 11. Eighteen to Seventy five students participate in each election, and each student is assigned one of the integers 1 to 11 (i.e., ideal point) so that the overall distribution of ideal points is approximately uniform over the line segment. According to elections, 2, 3, 5 or 7 candidates are located at different integers, and one of the candidates is the Condorcet winner except for the case of 2 candidates. After two pre-election polls are held, each student is asked to vote for one of the candidates under the plurality rule, in order to minimize the distance between his or her ideal point and the winning candidate's location.

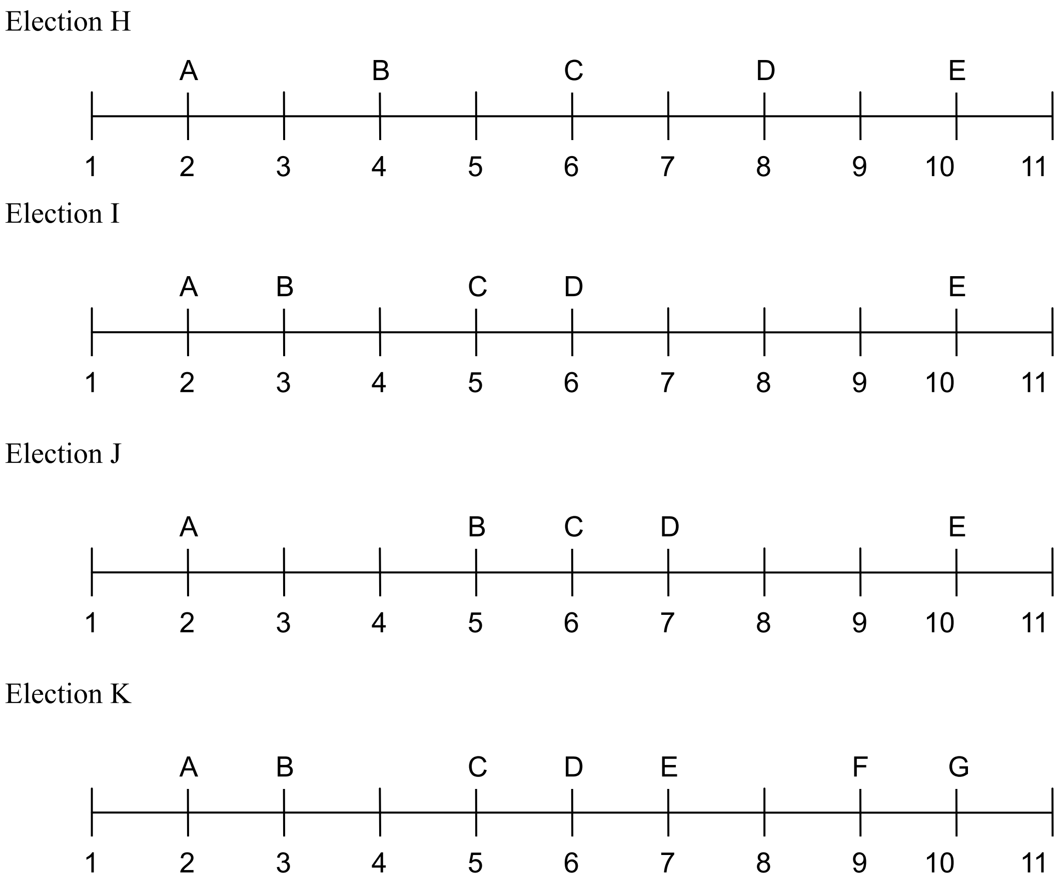

Figure 1. Candidate's Locations in Endersby and Shaw's (2009) Experiment

Note: These graphs are a part of Table 2 in Endersby and Shaw (2009).

Figure 1 describes the candidates' locations in Endersby and Shaw's (2009) four elections with 5 or 7 candidates. Table 5 transforms Elections H (symmetric case with 5 candidates) in Figure 1 into a matrix form so that we can compare the voter preferences between the above two experiments (i.e., Forsythe et. al. (1993) and Bassi (2008)) and Endersby and Shaw's (2009). The number in each cell represents the distance between each voter's ideal point (the first column) and each candidate's location. The smaller distance is better for each voter. In comparison with Tables 1, 3 and 4 used by Forsythe et al. (1993) and Bassi (2008), it seems much harder for voters to coordinate their behaviors under Table 5. However, in Endersby and Shaw's (2009) classroom experiment, votes concentrated on two candidates in not all but most elections. Duverger's law seems to hold if voter preferences are expressed on one dimension and if the Condorcet winner exists.

Voters' Ideal Points	Candidates				
	A	B	C	D	E
1	1	3	5	7	9
2	0	2	4	6	8
3	1	1	3	5	7
4	2	0	2	4	6
5	3	1	1	3	5
6	4	2	0	2	4
7	5	3	1	1	3
8	6	4	2	0	2
9	7	5	3	1	1
10	8	6	4	2	0
11	9	7	5	3	1

Table 5. Distance between Voters' Ideal Points and Candidates' Locations in Election H of Endersby and Shaw (2009)

From the above related experiments, we understand that Duverger's law tends to hold in the laboratory when the coordination of voter behaviors is relatively easy in the sense that (i) the Condorcet winner and/or the loser exists (*payoff matrix*), (ii) a pre-election poll is held or elections are repeated (*information*), and (iii) the number of voters is small (*number of voters*). In our experiment, (i) neither the Condorcet winner nor the loser exists, (ii) elections are repeated for 40 rounds, and (iii) the number of voters is 20 (relatively large). That is, we eliminate the two factors that make it easier for Duverger's law to hold, *payoff matrix* and *number of voters*, whereas *information* (i.e., repetition of elections) is kept.

3. Experimental Design

3.1 Preferences

We use the revenue matrix such as Table 6. We use the term "revenue" instead of "payoff" here because, as explained in the next subsection, we introduce a cost of voting. In our experiment, we derive a (net) payoff for each voter by subtracting the cost of voting from the revenue. Twenty voters are equally divided into 4 groups, Red, Blue, Yellow and Green. Group members are fixed over all the rounds in each session. The revenue matrix is symmetric in the sense that the voting environment is exactly the same for every voter. That is, the preference order for each group of voters is symmetric, and so there exists neither majority nor minority: each of 4 candidates, A, B, C and D, is most preferred by one group of voters. Under M=2, each voter's total revenue is calculated as the sum of revenues obtained from the two winners. For example, if candidates A and B win, then red voters receive 50 yen (=30+20). We assume that any tie is broken randomly.

	A	B	C	D
Red 5 voters	30	20	10	0
Blue 5 voters	0	30	20	10
Yellow 5 voters	10	0	30	20
Green 5 voters	20	10	0	30

Table 6. Revenue Matrix

3.2 Voting Cost

In our previous experiment (i.e., Hizen, Kurosaka and Ushizawa, 2010), we observed, using the same structure of revenues as Table 6, that votes are less likely to converge to a small number of candidates if abstention is not allowed and if voting is costless. This is possibly because any voting behavior is optimal for each subject as long as his or her vote is not pivotal, so that he or she can keep voting for his or her most-preferred candidate even though his or her most-preferred candidate is highly expected to lose the election. Hence, we introduce a cost of voting and allow subjects to abstain. That is, if a subject chooses to vote for one of the four candidates, then he or she must pay 5 yen as a voting cost. If he or she abstains, there arises no cost.[3]

3.3 Nash equilibria that are consistent with the M+1 rule

In general, if a voting cost is introduced, then multiple mixed-strategy Nash equilibria tend to

[3] In Forsythe et. al's (1993) experiment, abstention is also allowed but voting incurs no cost. Hence, abstention is a weakly dominated strategy for every voter. Bassi (2008) and Endersby and Shaw (2009) do not allow voters to abstain. Although their experiments do not include voting costs, Duverger's law tends to hold possibly because other factors mentioned in Section 2 are more or less in favor of Duverger's law.

exist.[4] Pure-strategy Nash equilibria also exist in our setup, and the following two pure-strategy Nash equilibria under M=1 are consistent with Duverger's law. First, red and blue voters vote for candidate B, while yellow and green voters vote for candidate D. In this Nash equilibrium, candidates B and D are in a tie for the single seat getting 10 votes respectively. Hence, each of candidates B and D wins with probability 0.5, and each of blue and green voters receives 15 yen as the expected payoff (i.e., $0.5 \times 30 + 0.5 \times 10 - 5 = 15$), while each of red and yellow voters receives 5 yen (i.e., $0.5 \times 20 + 0.5 \times 0 - 5 = 5$). If one of the red voters abstained, instead of voting for candidate B, for example, then he or she could save his or her voting cost, 5 yen. However, since his or her abstention leads to candidate D's win with certainty, his or her revenue decreases to 0 yen. Hence, such a deviation is not beneficial for him or her. Similarly, the second Nash equilibrium is that green and red voters vote for candidate A, while blue and yellow voters vote for candidate C. In this way, subjects in our experiment need to find out with which group they coordinate their behaviors. We would like to examine whether these types of Nash equilibria are realized in the laboratory.

Note that sincere voting, under which all voters vote for the most-preferred candidates sincerely so that each candidate gets 5 votes, is not a Nash equilibrium because any voter can increase his or her expected payoff from 10 to 15 by switching unilaterally to the second-preferred candidate (i.e., from $0.25 \times 30 + 0.25 \times 20 + 0.25 \times 10 + 0.25 \times 0 - 5 = 10$ to $20 - 5 = 15$).

Under M=2, on the other hand, there does not exist any pure-strategy Nash equilibrium that is consistent with the M+1 rule, in which three candidates obtain almost the same number of votes. This is because our experiment has 20 voters who are divided into 4 groups: neither 20 nor 4 can be divided by 3 exactly.[5] Instead, sincere voting is a pure-strategy Nash equilibrium, in which 4 candidates share votes equally so that each candidate wins a seat with probability 0.5. Therefore, the M+1 rule seems more difficult to hold under M=2 than under M=1, but the comparative statics tells that the larger number of candidates tend to gather votes under M=2 than under M=1. We examine this comparative-statics prediction by experiment.

3.4 Sessions

We had 8 sessions as described in Table 7. Twenty subjects participated in each session, and hence the total number of subjects was 160. Two sessions (Sessions 7 and 8) consisted of subjects who have ever experienced the paper-based version of this experiment (i.e., *experienced subjects*). Six sessions (Sessions 1 to 6) consisted of subjects who did not have such experiences (i.e., *inexperienced subjects*).

Each session consisted of 40 rounds under M=1 and another 40 rounds under M=2.

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[4] See Palfrey and Rothenthal (1983) for the case of two groups of voters with two candidates.

[5] If we have 6 candidates and 6 groups of voters in the voting game, then the pure-strategy Nash

Four of eight sessions had the M=1 treatment first and the M=2 treatment second (i.e., Session 1, 2, 3 and 7), while the other 4 sessions had the M=2 treatment first and the M=1 treatment second (i.e., Sessions 4, 5, 6 and 8).

	Inexperienced subjects	Experienced subjects
M=1 first, M=2 second	Sessions 1, 2 and 3	Session 7
M=2 first, M=1 second	Sessions 4, 5 and 6	Session 8

Table 7. Number of Sessions

3.5 Experimental Procedures

The experiment was held on July 22 and 23, 2010 in a laboratory of the Center for Experimental Research in Social Sciences, Hokkaido University, Japan. Subjects were recruited on campus. More than half of them were first-year undergraduate students from various academic disciplines. The experiment was programmed and conducted with the software z-Tree (Fischbacher, 2007).

Randomly Seated → Instruction → 3 minutes → First 40 rounds → Additional instruction → 3 minutes → Second 40 rounds → Tasks of Lottery Selection → Payment

Table 8. Experimental Procedures

Each session proceeded in the order described in Table 8. In each session, 20 subjects were seated in front of laptop PCs separated by boards. Their seats were determined randomly by lot when they entered the laboratory. Each PC was assigned one of the four groups (i.e., red, blue, yellow and green) in advance, but the group assignment was shown on the PC screen at the beginning of the first round. Instruction was conducted by PowerPoint presentation with artificial voice software. In our experiment, we did not use terms such as election, vote, candidate and others related to politics. After the instruction, 3 minutes were given to subjects to make sure of the rules and consider how to behave in the experiment.

In each round, subjects were asked to choose one of alphabets A, B, C, D and X, where X represented abstention. If a subject chose one of A, B, C and D, then he or she paid 5 yen. Choosing X incurred no costs. After all subjects made decisions, the voting result appeared on each subject's PC screen. The result included the number of subjects who chose each alphabet, the winning alphabet(s), and his or her own revenue, cost and net payoff. When each

equilibria exist under M=2 in which 3 candidates get votes from 2 groups of voters respectively.

subject made decisions, he or she could see the election history, including the number of subjects who chose each alphabet, winning alphabet(s), his or her own choice, and his or her net payoff in each of the previous rounds (see Appendix for the PC screens).

After the first 40 rounds, an additional instruction was given for the second 40 rounds, and 3 minutes were given again. After the second 40 rounds, subjects performed the tasks of lottery selection. This was held to examine the relationship between subjects' attitude toward risk and their behaviors.[6] At the end of the experiment, each subject received the sum of net payoffs obtained in the total of 80 rounds, as well as an additional payoff obtained from the lottery selection. The largest earning realized in the experiment was 2,102 yen while the smallest earning was 1,000 yen. The average was 1,611 yen. Each session took about 85 minutes.

4. Experimental Results

In this section, we analyze the experimental data to examine whether Duverger's law and the M+1 rule hold in the laboratory. We first focus on how many candidates can be regarded to have competed for the seats. We then analyze how subjects behaved.

4.1 The Laakso and Taagepera Index

There are various measures for how many candidates are regarded to have competed for seats (i.e., *effective number of candidates*). The index used most frequently in political science is the one proposed by Laakso and Taagepera (1979) as

$$LT \equiv \frac{1}{\sum_{i \in N} s_i^2},$$

where N is the set of candidates, and $s_i \in [0,1]$ is the share of effective votes (excluding abstention) for candidate $i \in N$, so that $\sum_{i \in N} s_i = 1$. For example, if only one candidate gets all the effective votes, then we have LT = 1. If 4 candidates share effective votes equally, then we have LT = 4.

Figure 2 shows the fluctuations of the LT index (vertical axis) in each of the 8 sessions over the 40 rounds (horizontal axis) for M=1 (red curve) and M=2 (blue curve). There seem to be no clear differences between inexperienced sessions (Sessions 1 to 6) and experienced sessions (Sessions 7 and 8), but there is some degree of variety between sessions regardless of the subjects' experiences. For example, the LT index for M=1 in Session 6 converges to 2 exactly, whereas the LT indices for M=1 and M=2 in Session 4 move more or

[6] The details of the tasks of lottery selection are explained in Subsection 4.3.

less similarly. However, a common observation to all the sessions is that, for both M=1 and M=2, the LT index starts with a relatively high value in the early rounds, and it decreases, accompanied by fluctuations, to some extent. In not all but many rounds of the 8 sessions, the LT index for M=1 takes a lower value than the LT index for M=2.

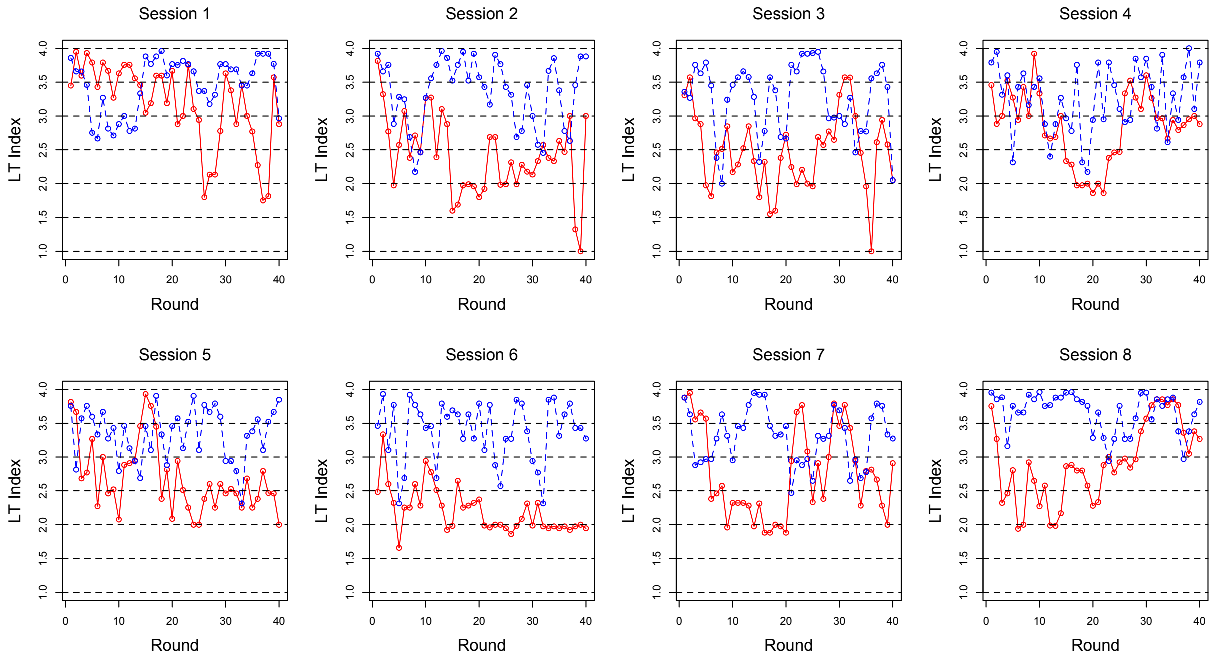

Figure 2. Fluctuations of the LT Index in the 8 Sessions

Why did the LT index fluctuate so wildly? One possible answer is as follows. Our experiment includes a cost of voting. This gives subjects an incentive to avoid wasting their votes. There are two types of behaviors for subjects to avoid wasting votes. First, subjects abstain or vote for the second-preferred candidate if their most-preferred candidate is less likely to attract enough number of votes. Such strategic abstention and voting decrease the effective number of candidates to M+1. Second, subjects also abstain to free-ride the votes of the other subjects if their most-preferred candidate is highly expected to win a seat. This second type of strategic behavior prevents a candidate from obtaining a large share of votes over a long period, which increases the effective number of candidates to M+1. When these two types of strategic behaviors are well balanced, the M+1 rule holds in that round. In the actual experiment, however, it is not easy for all the 20 subjects in each session to have the same expectation about the other subjects' behaviors, and hence the above two types of strategic behaviors are not necessarily balanced. If the two strategic incentives overcome each other round by round, then the effective number of candidates can fluctuate wildly.

Figure 3 shows the fluctuations of the average of the LT index (vertical axis) among the 8 sessions over the 40 rounds (horizontal axis) for M=1 (red curve) and M=2 (blue curve). Red and blue bars express the standard deviations for M=1 and M=2 respectively. On average, the LT index for M=1 converges to about 2.5, whereas the LT index for M=2 converges to about 3.4.

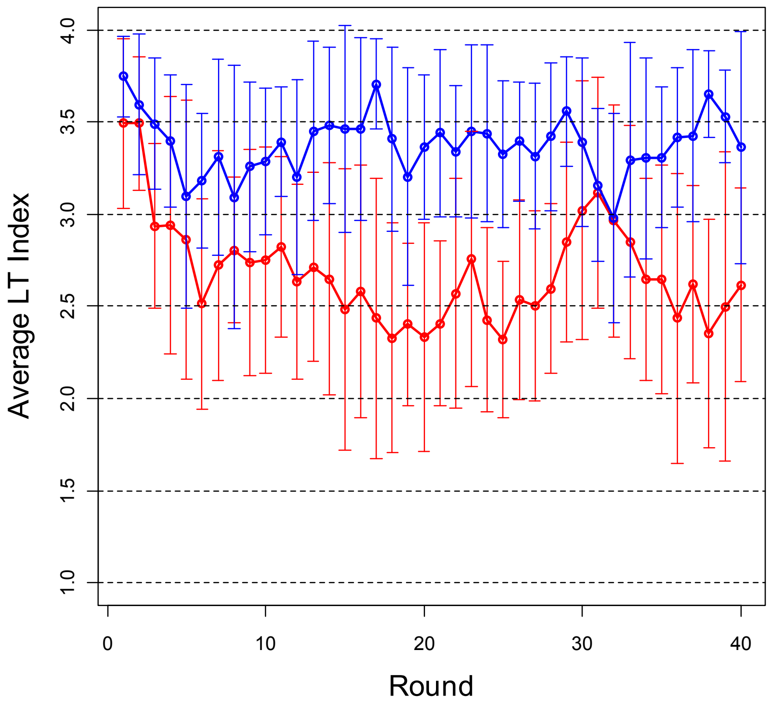

Figure 3. Fluctuations of the Average of the LT Index among the 8 Sessions

Next, following Reed's (2000) analysis on the Japanese election data, we describe the dynamics of the LT index in the form of return maps (Figure 4) and transition matrices (Table 9). Figure 4 plots points based on the LT indices in rounds t (horizontal axis; t = 1, …, 39) and t+1 (vertical axis) for M=1 (left) and M=2 (right) using all the data from 8 sessions. Points below (above, respectively) the 45-degree line imply that the LT index decreased (increased) when the experiment proceeded from round t to round t+1. The solid line in Figure 4 is the regression line. The determination coefficient is $R^2 = 0.530$ for M=1 and $R^2 = 0.213$ for M=2.

The regression line for M=1 intersects the 45-degree line around the point (2.5, 2.5), and its slope is 0.723 (t=18.704, P<0.001). This implies that the LT index tends to increase (decrease, respectively) when it is smaller (larger) than 2.5 in the previous round. In other words, the LT index for M=1 tends to be attracted to 2.5, which is higher than the prediction of Duverger's law.

The regression line for M=2 also shows similar properties to the case of M=1. That is, it intersects the 45-degree line around (3.3, 3.3), which is higher than the prediction of the M+1 rule. Its slope is 0.459 (t=9.167, P<0.001). Many points concentrate near (4.0, 4.0).

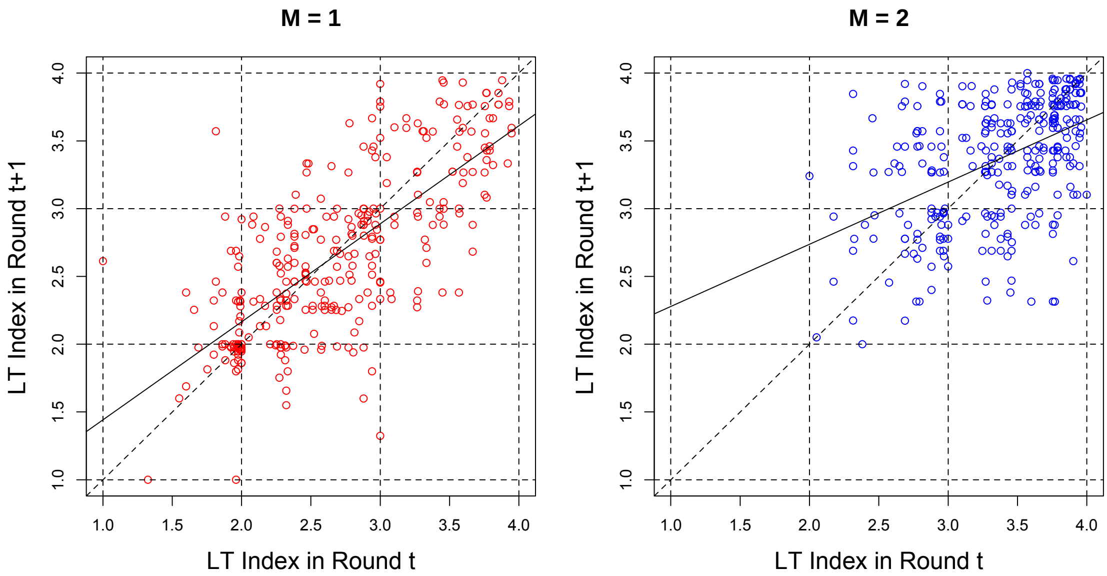

Figure 4. Return Maps of the LT Index from All the Sessions

If we round off the LT indices in Figure 4 to integers, we obtain Table 9. For M=1 (upper table), the transition from LT=2 in round t to LT=2 in round t+1 occupies the largest share (30.45%) of the data. This is a partial support for the difficulty in getting out of the vote distribution with LT=2 once subjects reach there. Even if subjects get out from LT=2 to LT=3, they are more likely to return to LT=2 (13.14%) than to go up to LT=4 (5.77%). For M=2 (lower table), the transitions from LT=3 to LT=3 and from LT=4 to LT=4 occupy almost the same shares (30.45% and 30.77%, respectively). That is, both the predictions of the M+1 rule (LT=3) and pure-strategy Nash equilibrium (LT=4) seem reasonable.

M=1	LT Index	Round t 1	2	3	4
Round t+1	1	0.32%	0.32%	0.32%	0.00%
	2	0.00%	30.45%	13.14%	0.32%
	3	0.32%	12.18%	22.76%	6.73%
	4	0.00%	0.32%	5.77%	7.05%

M=2	LT Index	Round t 1	2	3	4
Round t+1	1	0.00%	0.00%	0.00%	0.00%
	2	0.00%	1.28%	2.88%	0.96%
	3	0.00%	3.21%	30.45%	15.06%
	4	0.00%	0.64%	14.74%	30.77%

Table 9. Transition Matrices of the LT Index from All the Sessions

4.2 Molinar's Index

Molinar's index (Molinar, 2001) is another measure for the effective number of candidates used frequently in political science. Using data from Japan, France and some other countries, Molinar shows that Molinar's index describes the actual party system in each country more appropriately than the LT index. Here we also conduct the same analysis as Subsection 4.1 by using Molinar's index. Following Reed (2000), we extend Molinar's index to the case of $M \geq 1$ such as

$$Molinar = M + LT \frac{\sum_{j \notin W} s_j^2}{\sum_{i \in N} s_i^2},$$

where M is the number of seats, LT is the LT index, and W is the set of winners (i.e., the winner for M=1 and two winners for M=2). The first term, M, means that Molinar's index counts each winner as one effective candidate. Note that even if the vote distribution among candidates remains the same, Molinar's index takes different values if the number of seats is different. This might be a disadvantage of Molinar's index when it is applied to the case of $M \geq 1$.

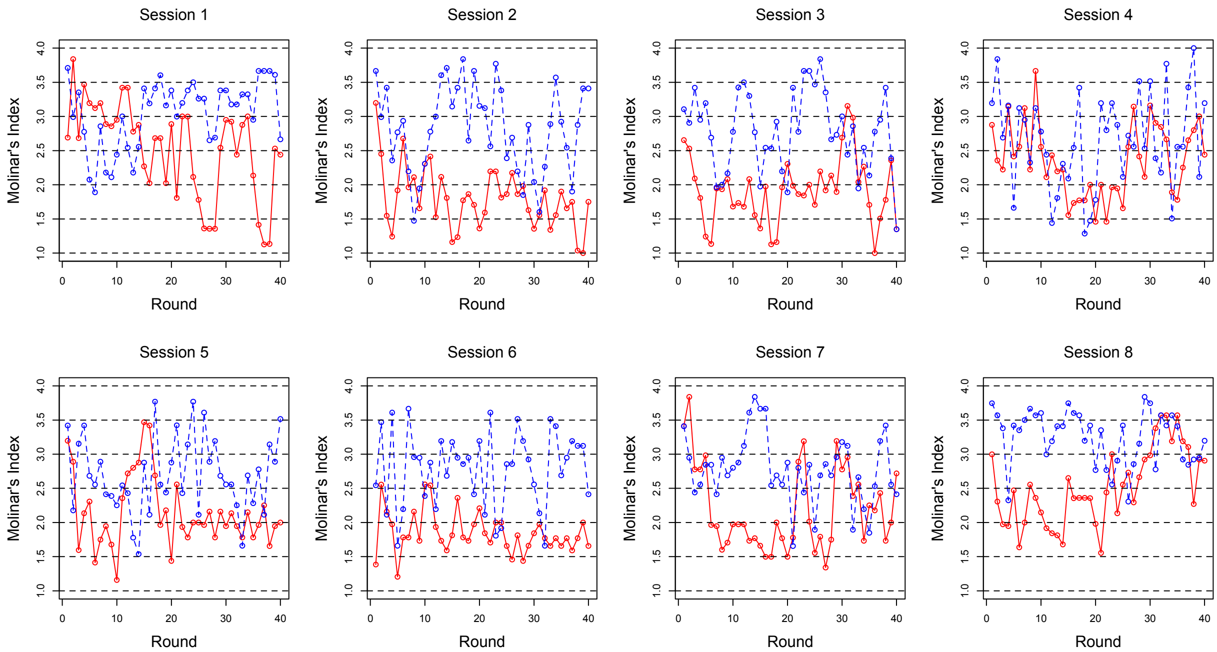

Figure 5. Fluctuations of Molinar's Index in the 8 Sessions

Figure 5 shows the fluctuations of Molinar's index in each of the 8 sessions. The vertical axis is the only difference between Figures 2 and 5. In Figure 5, Molinar's indices for both M=1 (red curve) and M=2 (blue curve) in each session fluctuate wildly as in Figure 2, but they are smaller than the LT indices in Figure 2.

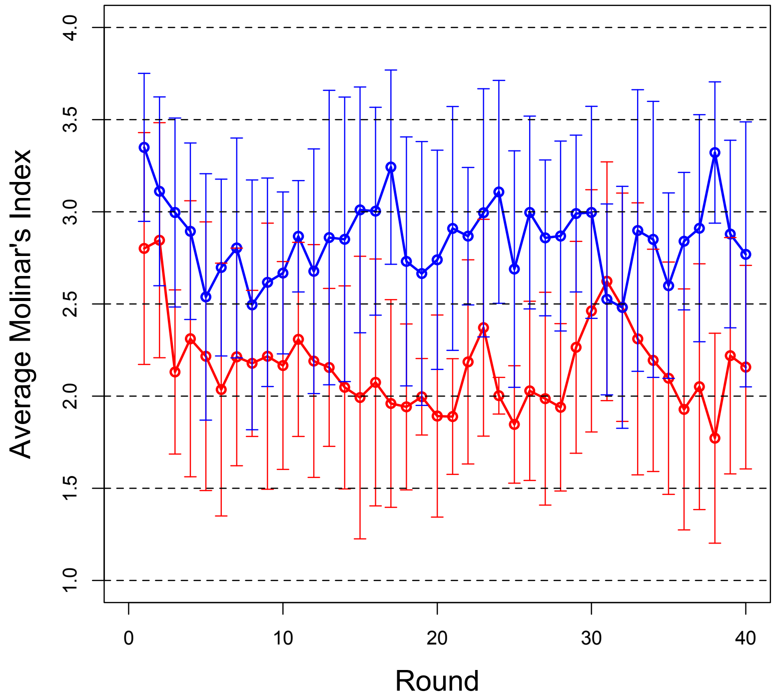

Figure 6. Fluctuations of the Average of Molinar's Index among the 8 Sessions

Figure 6 shows the fluctuations of the average of Molinar's index among the 8 sessions. The vertical axis is the only difference between Figures 3 and 6. On average, Molinar's index for M=1 (red curve) converges to 2, whereas the index for M=2 (blue curve) converges to 3 or lower.

Figure 7 provides the return maps with Molinar's index. The determination coefficient of the regression line is $R^2 = 0.408$ for M=1 and $R^2 = 0.111$ for M=2. Its slope is 0.629 (t=14.607, P<0.001) for M=1 and 0.331 (t=12.233, P<0.001) for M=2. The regression line intersects the 45-degree line slightly above the point (2.0, 2.0) for M=1 and around (2.8, 2.8) for M=2. We can observe some degree of convergence of points around (2.0, 2.0) for M=1 and (3.0, 3.0) for M=2.

Table 10 provides the transition matrices with Molinar's index. In comparison with Table 9 (i.e., transition matrices with the LT index), the transition from 2 to 2 for M=1 (upper table) and the transition from 3 to 3 for M=2 (lower table) occupy the largest shares in Table 10.

The LT index and Molinar's index show the similar fluctuations over rounds, but their absolute values are different. The M+1 rule is more supportable when we measure the effective number of candidates by Molinar's index than by the LT index.

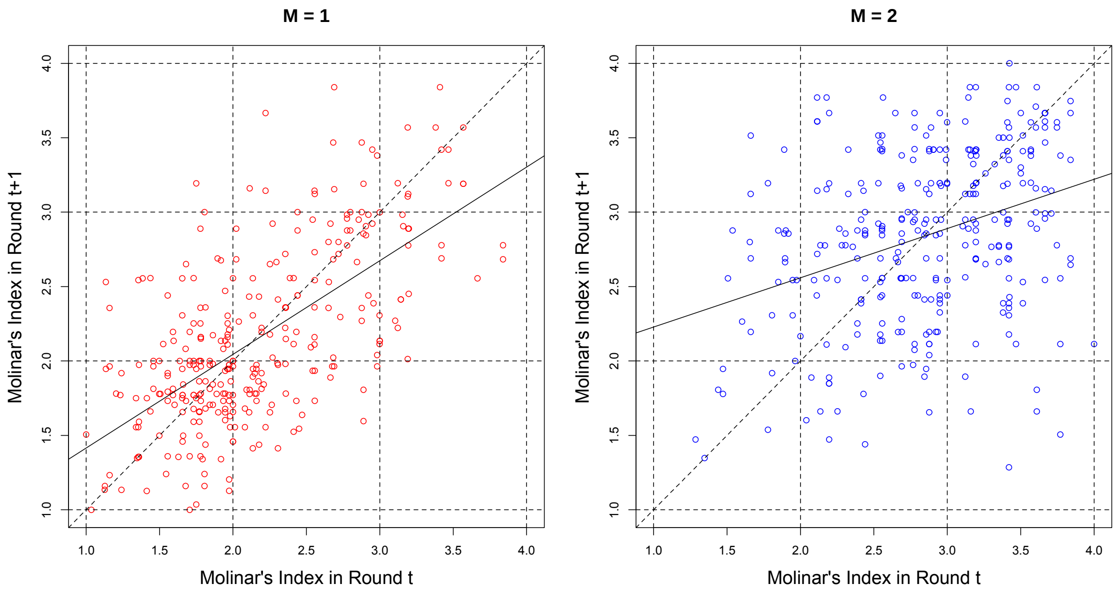

Figure 7. Return Maps of Molinar’s Index from All the Sessions

M=1	Molinar's Index	Round t 1	2	3	4
Round t+1	1	3.21%	7.37%	0.00%	0.00%
	2	6.09%	47.12%	9.94%	0.00%
	3	1.28%	7.05%	14.42%	1.60%
	4	0.00%	0.32%	1.28%	0.32%

M=2	Molinar's Index	Round t 1	2	3	4
Round t+1	1	0.64%	0.64%	0.32%	0.00%
	2	0.96%	7.69%	14.10%	1.92%
	3	0.00%	13.46%	37.18%	8.33%
	4	0.00%	2.24%	7.37%	5.13%

Table 10. Transition Matrices of Molinar’s Index from All the Sessions

4.3 Individual Behaviors

In Subsections 4.1 and 4.2, we analyzed the aggregate outcomes to examine whether Duverger’s law and the M+1 rule have held in the laboratory. In this subsection, we focus on the individual behaviors of subjects.

4.3.1 The Change of Individual Behaviors

We use the transition matrix again to see whether subjects changed their behaviors round by round. In Table 11, "Most," "Second," "Third," and "Least" represent voting for the most, second, third, and least preferred candidates respectively. The percentage in each cell expresses what percent of each behavior in round t+1 follows what type of behavior in round t, which is the only difference from the previous transition matrices. For example, 67.21% in the cell (Abstain, Abstain) means that, among the subjects who abstained in round t+1, 67.21% also abstained in round t. We can observe that abstention and voting for the most-preferred candidate tend to be chosen repeatedly. Strategic behaviors (i.e., voting for the second or third preferred candidate) in round t+1 are less likely to follow the same behaviors in round t. These observations imply that when subjects take strategic behaviors, they tend to change their behaviors more frequently than when they abstain or vote sincerely.

		Round t				
		Abstain	Most	Second	Third	Least
Round t+1	Abstain	67.21%	23.50%	7.93%	1.17%	0.19%
	Most	17.37%	76.42%	5.08%	0.79%	0.34%
	Second	26.53%	23.14%	46.72%	2.73%	0.88%
	Third	25.85%	24.88%	19.02%	27.80%	2.44%
	Least	31.48%	22.22%	22.22%	7.41%	16.67%

Table 11. Transition Matrix of Subjects' Behaviors

4.3.2 A Typical Session

We can see in Figures 2 and 5 that the effective number of candidates for M=1 converges to about 2 in Session 6 in terms of both the LT index and Molinar's index. As an example, therefore, let us see the subject behaviors for M=1 in Session 6. Figure 8 describes the four groups' behaviors respectively for M=1 in Session 6. The horizontal axis represents the rounds, while the vertical axis represents what percent of subjects in each group took each type of behavior, that is, voting for the most-preferred candidate (dark grey), voting for the second one (light grey), voting for the third one (grey), voting for the least one (black), and abstaining (white). Clearly, red and yellow voters voted for the most-preferred candidates A and C sincerely, whereas blue and green voters voted for the second-preferred candidates A and C strategically. As a result, only candidates A and C could win under M=1 in Session 6. We may be able to say that strategic voting results in worse outcomes for the voters who behave strategically.

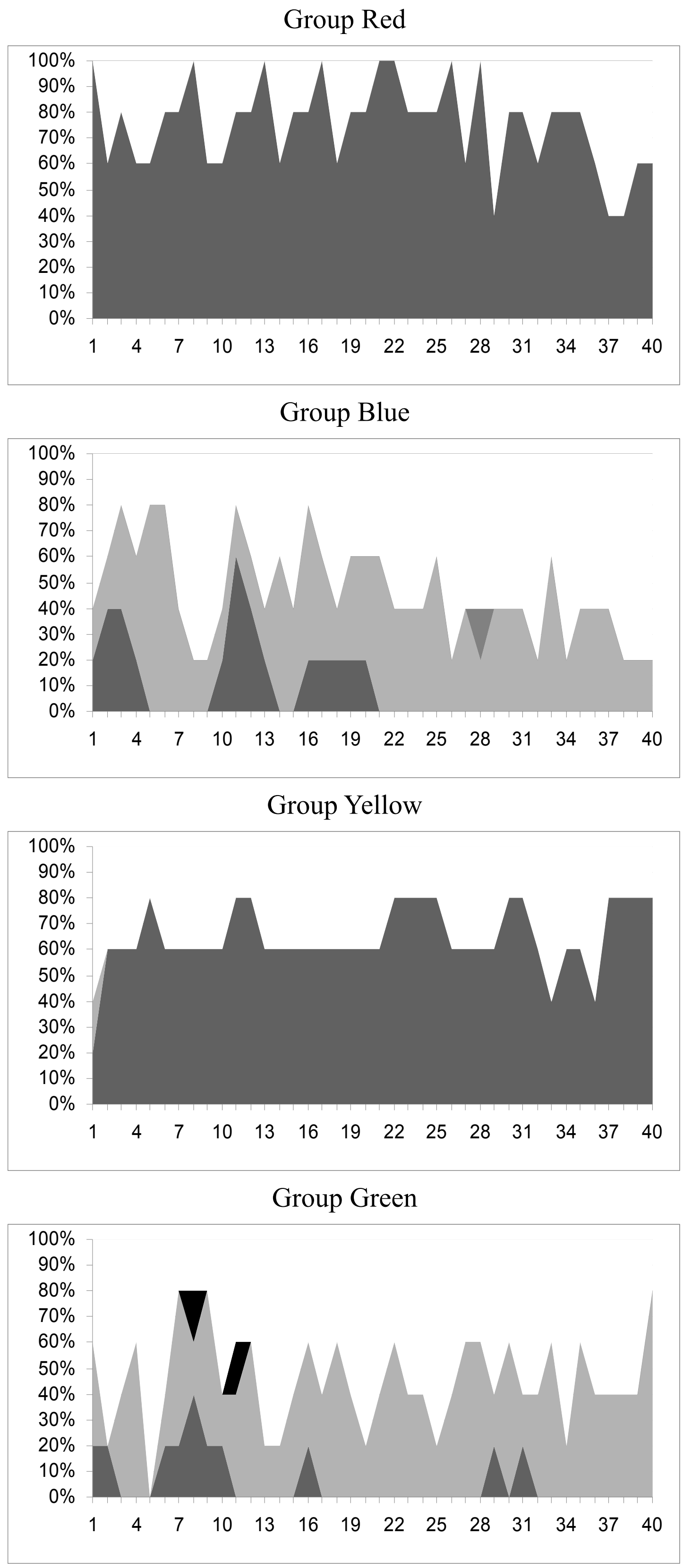

Figure 8. Voting Behaviors of Each Group in Session 6 for M=1

Note: Dark grey (light grey, grey, black, respectively) – the percentage of subjects who voted for the most (second, third, least) preferred candidate; White – the percentage of subjects who abstained.

4.3.3 Attitude toward Risk and Strategic Behaviors

What types of subjects were more likely to behave strategically? After the second 40 rounds in each session, subjects performed 11 selection tasks such as Table 12. There are two alternatives, A and B. Alternative A, "a lottery that gives you 30 yen with prob. 50%, while 10 yen with prob. 50%," is common to all the 11 tasks. Alternative B is different between tasks, and it is a certain amount of money. After all the subjects finish the 11 tasks, one of them is chosen randomly and independently, which determines each subject's payoff from the 11 tasks, in addition to the payoffs from the 80 elections. This task of lottery selection is based on Holt and Laury (2002). We measure each subject's attitude toward risk by the number of his or her answering "B."

Task 1	A: a lottery that gives you 30 yen with prob. 50%, while 10 yen with prob.50%	B: 10 yen
Task 2	A: a lottery that gives you 30 yen with prob. 50%, while 10 yen with prob.50%	B: 12 yen
Task 3	A: a lottery that gives you 30 yen with prob. 50%, while 10 yen with prob.50%	B: 14 yen
Task 4	A: a lottery that gives you 30 yen with prob. 50%, while 10 yen with prob.50%	B: 16 yen
Task 5	A: a lottery that gives you 30 yen with prob. 50%, while 10 yen with prob.50%	B: 18 yen
Task 6	A: a lottery that gives you 30 yen with prob. 50%, while 10 yen with prob.50%	B: 20 yen
Task 7	A: a lottery that gives you 30 yen with prob. 50%, while 10 yen with prob.50%	B: 22 yen
Task 8	A: a lottery that gives you 30 yen with prob. 50%, while 10 yen with prob.50%	B: 24 yen
Task 9	A: a lottery that gives you 30 yen with prob. 50%, while 10 yen with prob.50%	B: 26 yen
Task 10	A: a lottery that gives you 30 yen with prob. 50%, while 10 yen with prob.50%	B: 28 yen
Task 11	A: a lottery that gives you 30 yen with prob. 50%, while 10 yen with prob.50%	B: 30 yen

Table 12. Tasks of lottery selection

Spearman's rank correlation coefficient is significant at the 5% level or lower between the attitude toward risk (i.e., the number of answering "B") and the following three types of behaviors. First, the attitude toward risk and sincere voting are negatively correlated (Spearman's coefficient is -0.164, $P<0.05$). Second, the attitude toward risk and voting for the second-preferred candidate are positively correlated (Spearman's coefficient is 0.181, $P<0.05$). Lastly, the attitude toward risk and voting for the least-preferred candidate are positively correlated (Spearman's coefficient is 0.204, $P<0.001$). From these significant correlations, we find that risk-averse subjects tended to vote strategically. This is understandable because risk-averse voters prefer the win of their second-preferred candidate with certainty to the tie among several candidates.

4.3.4 Time for Decision Making

Finally, we examine how much time subjects spent for their decision making. Figure 9 describes the fluctuations of time (vertical axis) subjects spent for their decision making in each round (horizontal axis). The time is calculated by taking the average of time (seconds) among 20 subjects in each round of each session. In each session, subjects were spending relatively long time in the early rounds, then their decision making became quicker, and the time stayed short in the later rounds. Remember that, as described in Table 7, Sessions 7 and 8 consisted of experienced subjects. In fact, their time for decision making is relatively short in comparison with other sessions.

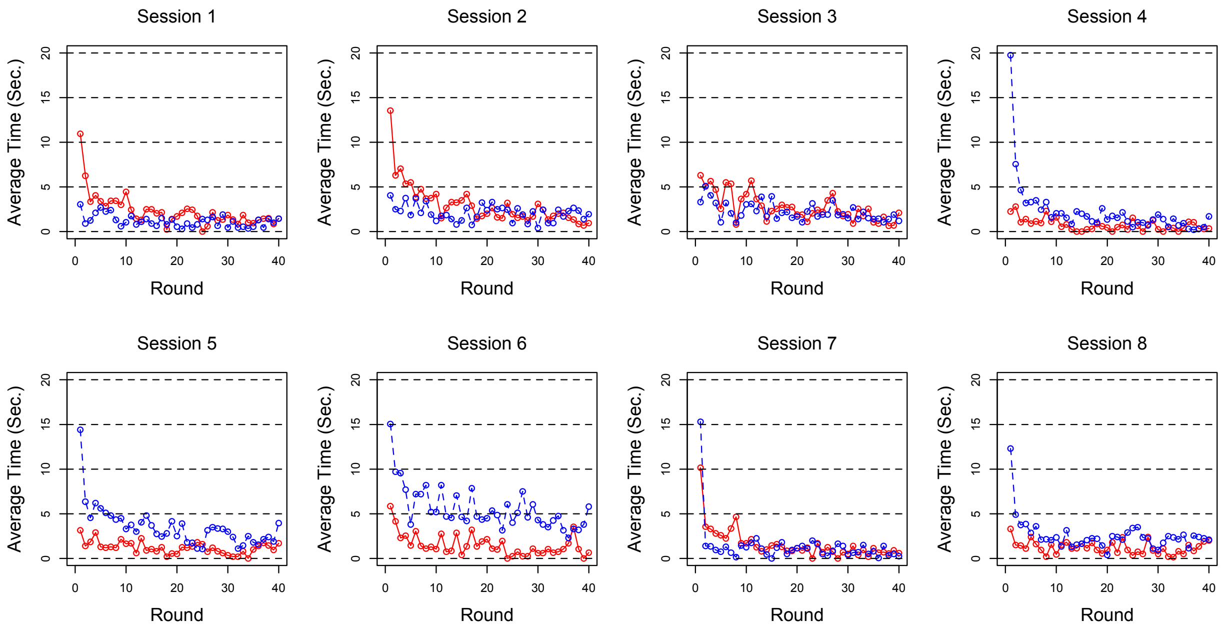

Figure 9. Fluctuations of Time for Decision Making in the 8 Sessions

Sessions 1, 2, 3 and 7 had the M=1 treatment first while Sessions 4, 5, 6 and 8 had the M=2 treatment first. Figure 10 describes the average time for decision making among the 4 sessions with the same order of treatments (i.e., Sessions 1, 2, 3 and 7 in the left figure, and Sessions 4, 5, 6 and 8 in the right figure). When the M=1 treatment is first (left figure), time for decision making is longer for M=2 (blue curve) than for M=1 (red curve) in the early rounds, but after that, time becomes almost the same between M=1 and M=2. When the M=2 treatment is first, on the other hand, time for M=2 keeps longer than time for M=1 even in the later rounds. One possible explanation is the difference of difficulty in learning how to vote. The first 40 rounds are expected to take longer time than the second 40 rounds because subjects are less experienced in making voting decisions in the first 40 rounds. In addition, the M=2 treatment is expected to take longer time than the M=1 treatment because how his or her own vote affects the voting outcome is more difficult for each subject to find out under M=2 than under M=1. Therefore, we can expect that subjects spend the longest time under the M=2 treatment in “M=2 first” sessions, while they require the shortest time under the M=1 treatment in “M=2 first” sessions. In “M=1 first” sessions, on the other hand, subjects can apply their experiences under M=1 to the play under M=2, which decreases the difference of time between the M=1 treatment and the M=2 treatment.

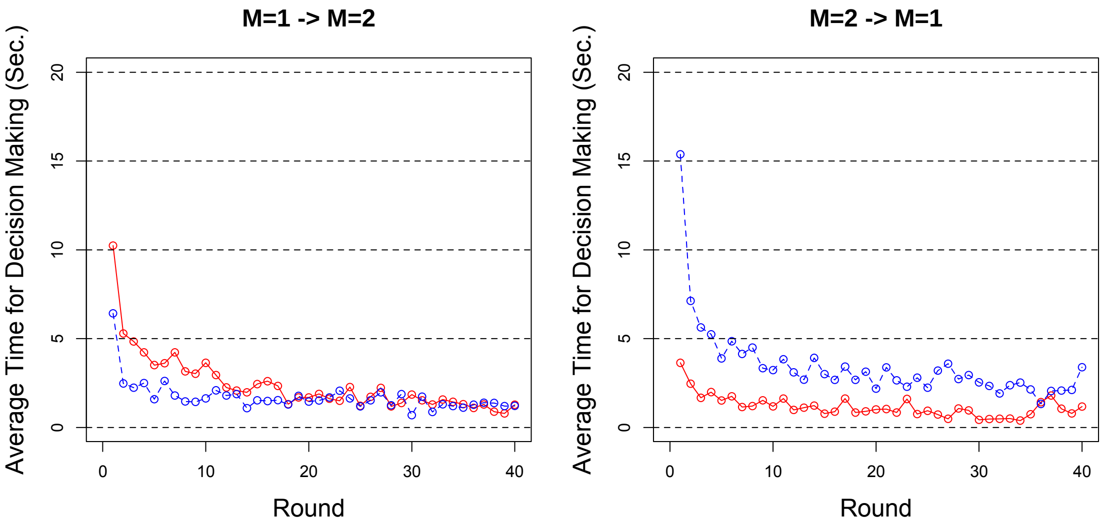

Figure 10. Fluctuations of the Average Time for Decision Making

5. Conclusion

We have conducted a laboratory experiment to test the robustness of Duverger's law and its extension "M+1 rule." Our experimental result supports the comparative statics of the M+1 rule so that the smaller number of candidates tend to gather votes under M=1 than under M=2. Whether the M+1 rule itself is supported or not depends on which index to use as the measure of the effective number of candidates. If we use the Laakso-Taagepera index, then the effective number of candidates does not converge to M+1 but stays larger. If we use Molinar's index, on the other hand, it converges around M+1 on average so that the M+1 rule is supported well. Although our experimental design is severe for the M+1 rule due to its symmetric revenue matrix and the relatively large number of voters, our experimental result supports the M+1 rule to a certain degree.

As mentioned in Subsection 4.1, the wild fluctuations of the effective number of candidates might be due to the voting cost. In our previous experiment (i.e., Hizen, Kurosaka and Ushizawa, 2010), sessions without voting costs resulted in a larger number of effective candidates, but we had only 10 rounds for M=1 and M=2 respectively in each session, which might be too short for the effective number of candidates to converge. Although it has both advantage and disadvantage in enhancing the realization of the M+1 rule, conducting the same type of experiment without voting costs can be a future task.

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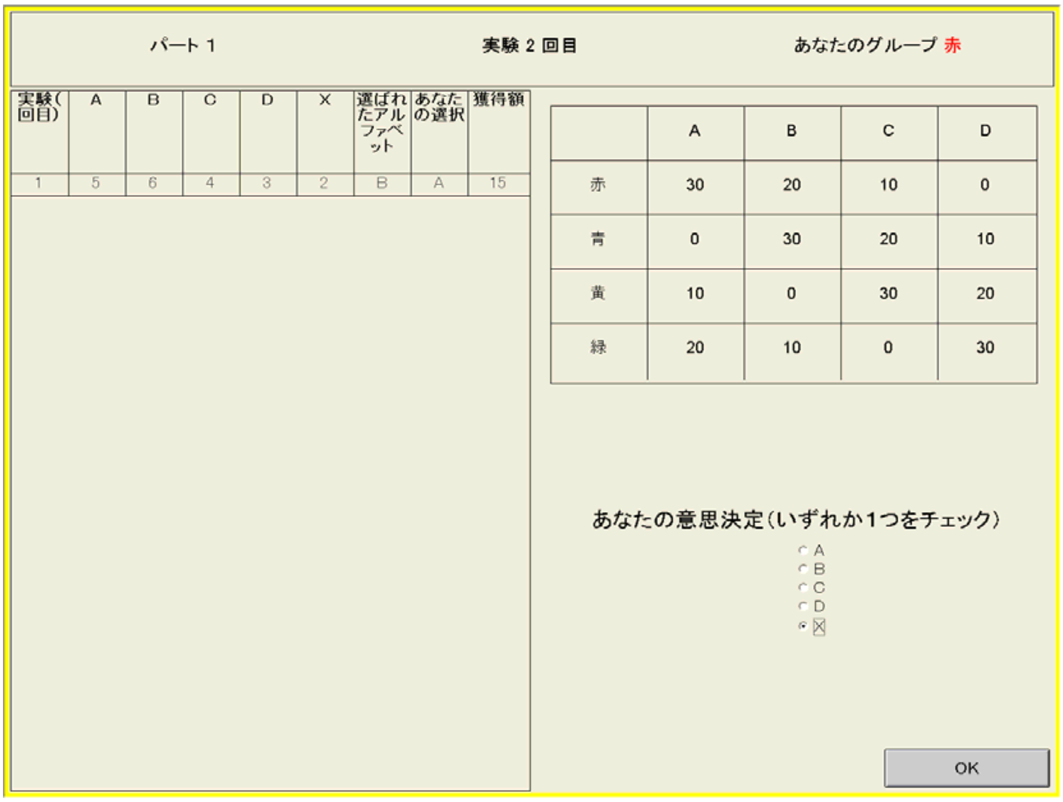
Appendix

The followings are the PC screens for M=1 in our experiment.

Decision making:



Voting Outcome: