CHAPTER 2

Solid Propellant Grain Design

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To design a solid propellant grain is to conceive and to define a grain which satisfies various requirements. This chapter describes the methods and procedures used today to design propellant grains. It describes and analyses:

- the various types of grain and the various families of propellant which are available and used today,
- the detailed requirements that a solid propellant grain must satisfy,
- the methods which are used to precisely define the propellant, the architecture and the configuration of the grain, and more specifically the methods used in order to ensure required ballistic performances though maintaining structural integrity of the grain (which is submitted to mechanical loads all through its life),
- an overview on a method of solid propellant grain reliability assessment.

The last section comprises a more specific treatment of some special cases.

1. Introduction

During the past 20 years, requirements on performance, reliability and cost of solid propellant rocket motors (and also on schedules and cost of development) have become more and more stringent. This, in turn, has a direct effect on solid propellant grain design methods and procedures, and on development program content.

The need for improved performance is the consequence of the need for longer ranges, higher velocities and more powerful payloads. The improvement of reliability originates from the need for higher availability of weapon systems, for lower malfunction probability and for longer service life. A decrease of duration and cost of a development program directly reduces the total cost of the program.

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During the same period of time, energetic, kinetic, mechanical and aging propellant properties have also been largely improved. Furthermore, the power of scientific computers has greatly increased, and the use of microcomputers has spread widely within the project manager's community.

Due to the pressure of competition (tactical missiles, space launchers) or for technical/political reasons (strategic missiles), the time assigned to designers for performing grain preliminary design* has decreased considerably.

It seems appropriate to present a synthesis of the various methods used today for designing solid propellant grains, within the larger frame of solid propellant rocket motor design.

Design of propellant grains involves vast knowledge and numerous techniques. This is due to the nature of propellants, the geometry and architecture of propellant grains and to their operation modes in rocket motors.

Grains are made of solid propellant put into a given configuration during manufacture; their surface is generally locally restricted or inhibited (to prevent ignition and combustion) by a flame-resistant adhesive material. Other parts of the grain may be bonded by a liner to the motor case (case-bonded grains).

Weights of propellant grains range from just a few grams to several metric tonnes, chamber pressures from a few tenths to more than 30 MegaPascal (MPa); operating times from a few milliseconds to a few minutes.

Manufacture, fielding, storage and operation of a propellant grain (within a rocket motor) involve numerous phenomena related to chemistry, thermodynamics, geometry, combustion, fluid dynamics, mechanics of continuous media, etc. In the present chapter, it is not possible to comprehensively analyse all the aspects of grain design which precisely define a propellant grain which can be industrially manufactured and which must satisfy requirements on storage and operation in various conditions. So it is assumed that the reader is familiar with the basic knowledge of solid propulsion (Chapter 1), internal ballistics and structural analysis (Chapters 4 and 6).

Main points discussed include:

- a description of various types of grain and associated propellants (including French terminology);
- an analysis of requirements for solid propellant grains;
- a review of mechanical and ballistic design methods used today, particularly in France;

*The result of a preliminary design is a first propellant grain definition which generally demonstrates how initial requirements may almost totally be satisfied. Additional modifications of the grain, often involving the use of large computer codes, are needed in order to establish the final design.

- a method of assessing propellant grain reliability;
- a description of some special designs used for very specific applications.

2. Description of Grain Geometries and Associated Propellants

In this section, various types of grain configurations and of propellants are presented, and also general principles on configuration and propellant selection.

2.1. GRAIN CONFIGURATIONS

There are two main types of grain architecture: free-standing grains and case-bonded grains. Grains of the first type are introduced into rocket motor cases (cartridge-loaded) after manufacture. Grains of the second type are bonded to the motor case during the casting (or injection) and curing steps of the propellant grain manufacturing process (Fig. 1).

There is not a single, well-defined, procedure for selecting a free-standing grain architecture or a case-bonded grain architecture for a given rocket motor, except when one of these two architectures is obviously most appropriate for a specific reason. Nevertheless, case-bonded grains generally give higher performances than free-standing grains for equal available volumes. However free-standing grains are largely widespread because this type of architecture may present significant advantages, for instance from the point of view of cost and of overall industrial management. Today, the trend is toward case-bonded architectures, due to the demand for higher performances.

2.1.1. Case-bonded grain configurations

When propellant grains have an outer diameter larger than 500 mm or a weight of more than 300 kg, they are almost always case-bonded. High-performance, middle-sized grains (outer diameter between 100 mm and

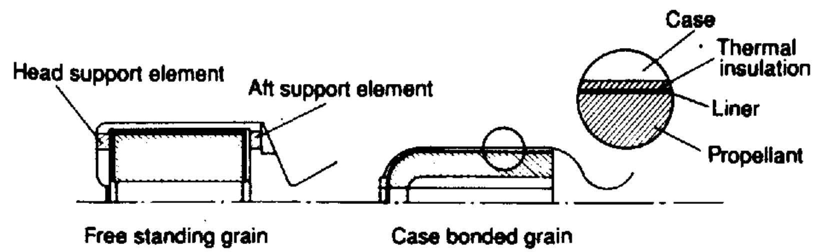

FIG. 2.1. Case bonded grain.

500 mm, weight between 10 and 300 kg) are case-bonded, but free-standing middle-sized grains are very common. For small rocket motors free-standing grains are generally used.

Case-bonded grains generally have a central port, the outer surface of the grain is bonded by a liner (and a thermal insulation) to the motor case. During firing, combustion of the propellant is initiated on the internal surface of the central port and proceeds radially toward the case (and to a certain extent longitudinally depending on the exact geometry). Exact grain geometry is obtained during manufacture of the grain either by direct casting in the case around the mandrel or by machining the port after casting and curing have been completed.

2.1.1.1. *Axisymmetric configurations*

AXIL: Axisymmetric grain with annular slots. The slots are circular; their axis is the same as the grain axis. They are located all along the central port (Fig. 2).

AXAR: Axisymmetric grain with annular slots. This configuration is similar to AXIL, except that the slots are located near the aft-end of the central port (Fig. 3).

CONOCYL (contraction of cone and cylinder): Axisymmetric grain with annular slots. The tips of the annular slots are inclined toward grain head-end so that a part of the grain is cone-shaped (Fig. 4).

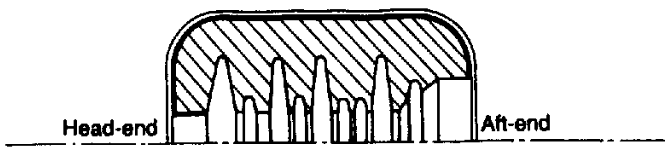

FIG. 2.2. Axil configuration.

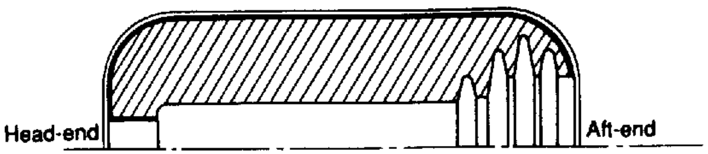

FIG. 2.3. Axar configuration.

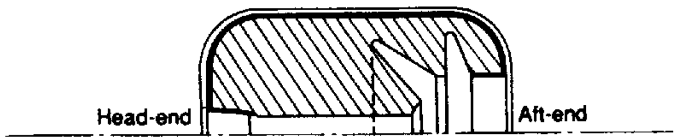

FIG. 2.4. Conocyl configuration.

2.1.1.2. *Cylindrical configurations*

STAR: The cross-section of the central port has the shape of an *n*-points star. The contour of the star is constant along the axis [in some cases it may be slightly tapered for manufacture practicality (Fig. 5)].

WAGON WHEEL: The cross-section of the central port looks like a wagon wheel (Fig. 6). Numerous parent configurations exist, such as dendrite, anchor and dogbone configurations.

Other configurations may be obtained, derived from some of the above-described configurations. For example, bipropellant star configuration (to eliminate sliver), or AXAR configuration having a stress-relieving annular slot in the head-end area. Full head-end web grains are also used. Simpler configurations such as internal-burning tube are commonly used; the ends are usually unrestricted to function as a burning surface control; they may also be partially restricted.

2.1.1.3. *Three-dimensional geometries*

The above-described configurations are considered as one- or two-dimensional, though of course being actually three-dimensional. They are either axisymmetric or cylindrical, with, often, an order "*n*" symmetry. It is therefore not too difficult to calculate burning area versus web burned or stress–strain field. Today, three-dimensional configurations are becoming more and more popular among the designer community; they are also much more difficult to design. Most of these configurations are referred to as

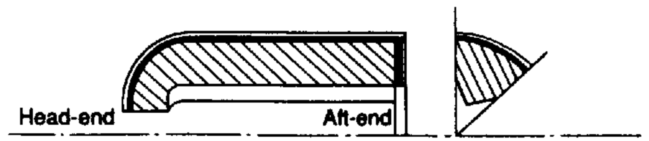

FIG. 2.5. Star configuration.

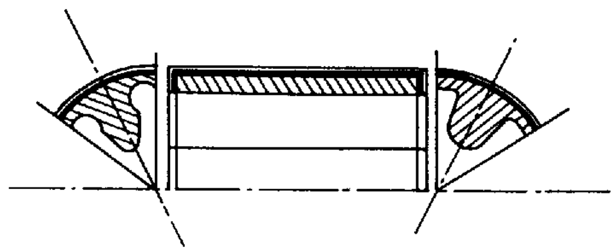

FIG. 2.6. Wagon wheel configuration.

"finocyl", which is a contraction of fin and cylinder. The fins may be located either at the head-end or at the aft-end of the grain (and sometimes at both ends); they merge into a central cylindrical port. They may have the shape of slots, which simplifies the geometry (Fig. 7).

Often, for stress-relieving, there are annular slots. These configurations require three-dimensional analysis for calculating burning area versus web burned, as well as stress-strain field or gas flow inside the central port.

2.1.1.4. *End-burning grains*

An end-burning configuration is not well adapted to case-bonded architecture because of problems of structural integrity. However, it is possible to manufacture such case-bonded grains using stress-relieving grain support and retention systems which allow thermal shrinkage due to propellant cooling after curing, though permitting pressure to equilibrate during firing.

2.1.2. *Free-standing grain configurations*

Free-standing grains are generally smaller than case-bonded grains. Because they are not bonded to the case wall, except sometimes locally, they allow configurations which cannot be obtained with case-bonded grains (for instance an internal-external burning tube).

Final checking of the grains is easier than in the case of case-bonded grains. They are loaded into the motor case during final assembly of the rocket motor. Various support systems may be used to ensure proper operation during firing. During missile service life it is often possible, if necessary, to replace the grain independently of other motor components.

2.1.2.1. *Cylindrical configurations*

Star, wagon wheel and tube configurations similar to those described above may be found for free-standing grains. Grain ends are generally simpler: they are plane and may be restricted or not. Rod and shell and cruciform type grains may also be found.

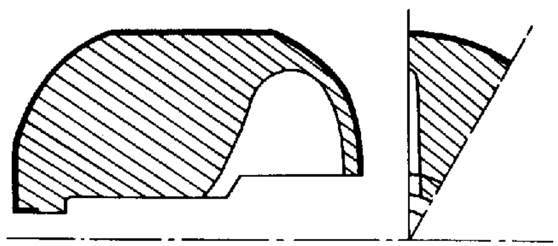

FIG. 2.7. Finocyl configuration.

2.1.2.2. *Configurations with evolving port cross-section*

To reach high-volume loading fractions for free-standing grains a configuration was developed: the cross-section of the central port is right circular in the forward section and becomes progressively star-shaped in the aft section of the grain (Fig. 8). In France this configuration is referred to as "trompette" (trumpet), though it does not much resemble the shape of a trumpet.

2.1.2.3. *End-burning grains*

The orientation of burning is totally in the longitudinal direction. This configuration is wide-spread because gas generation rate is almost constant, volumetric loading fraction is high and grain manufacture is easy. Side and head faces are restricted. Burning times are long and thrust levels are low or moderate. Thermal insulation and inhibitor play important roles, respectively, to protect the chamber walls from the continuous exposure to hot gas and to restrict the combustion to the desired area. They also generate pyrolysis gaseous products during firing, which must be taken into account in the total amount of gas generated by the grain. They are used mainly for the sustaining phase of the flight of some missiles. Anomaly of combustion may be observed on this type of grain, which is known as "coning".

2.1.3. *General principles for selection of grain configuration*

A practical procedure for selecting the grain configuration/propellant combination is discussed at Section 4. Hereafter only basic principles are discussed.

For selection of the grain configuration, the main factors which are taken into account are:

- volume available for the propellant grain;
- grain length to diameter ratio (L/D);
- grain diameter to web thickness ratio (D/e);
- thrust versus time curve: this gives a good idea of what should be the burning area versus web burned curve (neutral, regressive, progressive, dual-level);

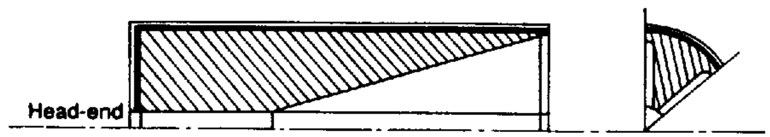

FIG. 2.8. Trumpet configuration.

- volumetric loading fraction: this can be estimated from required total impulse and actual specific impulse of available propellants;
- critical loads (thermal cycles, pressure rise at ignition, acceleration, internal flow);
- manufacture practicality, which depends on case geometry (some grain configurations are more or less easy to obtain);
- fabrication cost: this can be the critical factor for selecting a given configuration.

There is no definite procedure to select a grain configuration in order to satisfy a set of requirements, because there are often several technical solutions to the propulsion problem.

Practically, there are some general trends in selecting configurations, based on the shape of the burning area versus web burned curve (which is qualitatively close to the thrust versus time curve). Table 1 summarizes these trends. Table 2 presents the main characteristics of commonly encountered grain configurations.

2.2. PROPELLANT SELECTION

There are several solid propellant families which differ with respect to composition, manufacturing processes and ability to be processed into certain configurations. These families are comprehensively presented in Chapters 8, 9, 10 and 11.

2.2.1. *Propellant families*

Five families of propellants are commonly manufactured and used, and are described more specifically in other chapters of this book.

- Solventless extruded double-base propellants (EDB); the main ingredients of which are nitrocellulose and nitroglycerine. The configuration is obtained by extrusion through a die having the desired shape. The outer diameter is limited to about 300 mm because of equipment limitations. Additional grain machining may be performed.
- Cast double-base propellants (CDB); the ingredients are similar or parents to those of EDB propellants; they are obtained by casting a mixture of nitroglycerine and triacetin into a mold containing nitrocellulose-based casting powder.
- Composite modified cast double-base propellants (CMCDB), which are derived from CDB propellants by addition of RDX, HMX, or ammonium perchlorate and possibly nitroglycerin, in the casting powder.
- Composite propellants based on a non-energetic polymeric binder and on ammonium perchlorate, which may also contain aluminum powder.

TABLE 1 *Burning area neutrality versus grain configuration*

Burning area neutrality	Grain configuration	Remarks and comments
Good neutrality (less than 15% relative change in burning area)	Wagon wheel dendrite	Short burning times (≤ 3 s), low volumetric loading fraction
	Trumpet Tube and slots	A constant burning area versus web burned is obtained by progressive cylinder zone and degressive slots or trumpet zone
	Axil	Case-bonded grain, $L/D \leq 1$ for third stage of strategic missiles
	Finocyl Axar Conocyl	Case-bonded grains, $2 \leq L/D \leq 4$ for first and second stages of strategic missiles
	Tube (end faces may be restricted)	Volumetric loading fraction less than 0.8
	Bipropellant star	Two different propellants. Internal propellant has a higher burning rate than external propellant by a factor of about 2. It is a costly configuration requiring long manufacture cycles (two curing stages). But neutrality is kept, even for high volumetric fraction, and slivers are eliminated.
	End-burning	Long burning times, low or moderate thrust
	Remark: the lower the volumetric loading fraction, the better the neutrality. A constant burning area can be accurately obtained by adding axisymmetric slots in the central port or by restricting specific propellant surfaces. It raises grain cost because it requires additional phases during manufacture	
Dual level	Trumpet	The ratio of the two levels can be adjusted by varying the geometry of the aft-end section
	Slotted tube	Volumetric fraction may reach 0.88. L/D may reach 10
	Axar	Adjusted by number and geometry of the annular slots
	Bipropellant star	Adjusted by the geometry of the stars and propellant burning rates
	End-burning (with annular slots in the aft-end face)	Adjusted by the geometry of the annular slot(s) (boost with radial burning, sustain with end-burning)
Progressive	Tube (right circular port section)	The most common is the grain with restricted faces
	Star	With restricted end faces; slivers induce long tail-off
Regressive	Tube with internal External burning Star with internal-external burning	Unrestricted end faces and low to moderate L/D ratio
	End-burning with tapered aft-end	Long burning time, low to moderate thrust

TABLE 2 *Main characteristics of common grain configurations*

Configuration	Volumetric loading fraction	Web thickness	Burning area	Burning area neutrality	Sliver fraction	Web fraction	Comments
Star	0.75–0.90	intermediate	intermediate	good	5–10%	3.5–5.5	Case-bonded and free-standing grains
Trumpet	0.88–0.95	large	intermediate	excellent	0%	> 2	Free-standing grains
Slotted tube	0.75–0.85	large	large	good	0%	≃ 3	Case-bonded grains
Wagon wheel	0.5–0.7	small	very large	excellent	5–10%	6–12	Free-standing and case-bonded grains
Finocyl	0.85–0.95	large	large	good to excellent	0%	2–3	Mainly case-bonded grains, high L/D
Axil	0.88–0.93	intermediate	large	excellent	5%	3	Case-bonded grains, $L/D \simeq 1$
Axar	0.88–0.95	large	large	good to excellent	0%	2–3	Case-bonded grains, high L/D
Star (with full head-end web)	0.75–0.85	intermediate	intermediate	good	5–10%	3	Case-bonded grains
Bipropellant star	0.9	large	intermediate	excellent	< 5%	2.5–3	Case-bonded grains
End-burning grains	0.98–1	very large	small	excellent	0%	1	Low thrust

- High-energy propellants based on an energetic binder highly plasticized by a liquid nitric ester, and on RDX or HMX, which may also contain ammonium perchlorate and aluminum, which is called XLDB, for "crosslinked double-base", even if there is very little or no nitrocellulose in the binder and a very high level of energetic solids in the formulation.

There is a terminology commonly used in France for the last three of these propellant families, that will sometimes be used in the present book. It is based on the following principles: the name of a propellant is made up of a prefix, one consonant, and a suffix.

The prefix gives some information on the binder:

nitra energetic binder (usually containing nitric esters);
buta binder based on carboxy- or hydroxy-terminated polybutadiene;
iso binder based on polyurethane.

The middle letter indicates the nature of the main energetic filler.

l ammonium perchlorate;
m octogen (HMX) or hexogen (RDX);
p potassium perchlorate.

The suffix indicates the nature of the metallic fuel.

ane aluminum;
abe beryllium;
aze zirconium;
ite no metal added.

The most common of these propellants are:

- Nitramite* E: Nitrocellulose/nitroglycerine binder filled with RDX or HMX. "E" indicates that this family of propellants is obtained through a process very similar to the one used for manufacturing CDB propellants (known in France as "Epictete").
- Isolite*: Polyurethane binder and ammonium perchlorate.
- Isolane*: Polyurethane binder, ammonium perchlorate and aluminum.
- Butalite*: Polybutadiene binder and ammonium perchlorate.
- Butalane*: Polybutadiene binder, ammonium perchlorate and aluminum.
- Nitramite* G: Elastomeric binder, plasticized with a mixture of liquid nitric esters, and filled with RDX or HMX and possibly some ammonium perchlorate. The letter G indicates that

* Trade marks of SNPE.

the manufacturing process is the slurry cast (global) process.

- Nitralane*: Elastomeric binder plasticized with a liquid nitric ester, and filled with HMX, ammonium perchlorate and aluminum.

Besides the main ingredients, propellants may contain several other ingredients, generally in small amounts, used as stabilizers, afterburning suppressants, combustion instabilities suppressants and burning rate modifiers. One of the important tasks of propellant designers is to find a practical way (filler, particle size, burning rate modifier, etc.) to control burning rate, which is a key factor in designing solid propellant grains.

2.2.2. *Propellant selection*

Selection of a propellant for designing a given grain is based on numerous criteria and, here again, there is no strict procedure for selecting a given composition. The type of architecture (case-bonded or free-standing), energy and burning rate criteria, structural integrity considerations, smokelessness and safety considerations, may lead toward a given propellant family. Each of the propellant families covers a certain range of properties, and it is necessary that the properties of the selected propellant allow design and manufacture of a grain satisfying all the requirements. Table 3 summarizes some properties of the main propellant families. The information presented is very succinct and would need more thorough development. However, it allows, in combination with Tables 1 and 2, a first approach in the selection of the couple configuration/propellant which is detailed in Section 4.3.

3. Solid Propellant Grain Requirements

This section addresses technical requirements that propellant grains must meet. That requirements are settled as the consequence of an agreement between the rocket motor designer and the propellant grain designer. They must be clear, complete and consistent, so that the propellant grain designer may precisely define the grain and eventually build the corresponding engineering development program.

Requirements are divided into those related to functional specifications, those related to operational specifications and interface requirements. They are detailed below.

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TABLE 3 *Main characteristics of common propellants*

Propellant	Maximum delivered specific impulse in standard conditions (70/1)	Maximum density (kg/dm^3)	Range of burning rates at 7 MPa (or at the plateau) (mm/s)	Pressure exponent	Temperature coefficient	Architecture	Smoke	Hazard classification*	Card Gap Test (number of cards)	Sensitivity to electrostatic discharge	Manufacturing cost	Ingredients cost
Extruded double-base EDB	225 s	1.65	5–40 (plateau)	≈ 0	very low	free standing	no	1.3	110	no	low	low
Cast double-base CDB	215 s	1.60	4–22 (plateau)	≈ 0	very low	free standing	no	1.3	100	no	high	low
Cast composite modified double-base Nitramite E	230 s	1.70	3–28 (plateau)	0–0.2	low	free standing and case-bonded	no	1.3	150	no	high	moderate (RDX)
Non aluminized composite polybutadiene propellant Butalite	240 s	1.73	4–60	0.3–0.5	low to moderate	free standing and case-bonded	secondary smoke	1.3	0	no	moderate	low
Aluminized composite polybutadiene propellant Butalane	245 s	1.86	5.5–80	0.2–0.5	low to moderate	free standing and case-bonded	primary and secondary smoke	1.3	0	often	moderate	low
Non aluminized cross-linked double base Nitramite G	245 s (with AP) 235 s (without AP)	1.79 1.75	10–25 5–10	0.45–0.6	moderate	case bonded	no (secondary smoke with AP)	1.3	180	no	moderate	moderate (RDX) fairly high (HMX)
Aluminized cross-linked double base NEPE Nitralane	254 s	1.86	9–25	0.5–0.7	moderate	case-bonded smoke	primary and secondary smoke	1.3	180	no	moderate	fairly high (HMX)

* According to French regulations.
Notes: AP: ammonium perchlorate; Card Gap Test is French Gap Test.

3.1. REQUIREMENTS RELATED TO FUNCTIONAL SPECIFICATIONS

3.1.1. Main internal ballistics requirements

Average, minimum and peak values of chamber pressure, thrust, total impulse and burning times must be specified within the full operating temperature range. Envelopes of thrust versus time or mass flow rate versus time curves may also be specified.

3.1.2. Special requirements

Other requirements are necessary to the designer in order to define a satisfactory propellant grain:

- Maximum weight of propellant grain.
- Maximum weight of total inert (thermal insulation, liner and restrictor).
- Maximum axial and transverse acceleration undergone by the propellant grain during operation of the rocket motor.
- Rocket spin rate (for instance for unguided rockets).
- Dispersions on pressure, thrust, total impulse and burning time have to be specified. Depending on the corresponding requirements, manufacturing process and control operations may be strongly affected and thus the cost of the grain also.
- Plume characteristics (emission and transmission in the visible, infrared and electromagnetic wavelengths range).

3.2. REQUIREMENTS RELATED TO OPERATIONAL SPECIFICATIONS

Depending on environmental conditions, definition of the propellant grain may be significantly affected. Such conditions must therefore be well defined in order to be correctly taken into account during the grain structural design phase.

3.2.1. Long-term storage

Desired maximum shelf-life, related temperature cycles and storage conditions must be defined. Particular conditions (relative humidity, salty atmospheres, etc.) which could directly affect propellant grain behavior must be specified.

3.2.2. Thermal environmental conditions

The nature and number of thermal cycles undergone by missiles (for instance during operational flights for airborne missiles) must be defined. Generally they are the limiting factors for structural grain design because very low temperatures may be encountered.

3.2.3. Acceleration, handling and transportation

- Acceleration before and during rocket motor operation: longitudinal acceleration undergone by the rocket motor must be specified, as well as radial acceleration due to rocket spin.
- Handling and transportation: dynamic loadings such as shocks and vibrations encountered during handling (drops) and transportation must also be specified.

3.2.4. Reliability

A level of reliability is more and more commonly required. It is essential to define in which conditions it has to be satisfied. The principle of a method of reliability assessment is discussed in Section 5.

3.2.5. Maintainability

The content and the planning of missiles surveillance, inspection, and maintenance must be defined, as far as they may have an effect on rocket motor environmental conditions.

3.2.6. Safety and vulnerability

These requirements are related to safety and survivability of persons and materials. At present they are not often taken directly into account during grain design analysis. They may induce an *a priori* selection of a type of propellant (e.g. a non-detonable propellant or a propellant having a large critical detonation diameter) or, during engineering development, the performance of safety and vulnerability tests.

3.3. INTERFACE SPECIFICATIONS

Close environment has an important effect on grain behavior during its life and operation. It is often prescribed by the rocket motor designer. The grain designer must take special care that its definition is complete.

3.3.1. *Case geometry and properties*

A blueprint of the case, or at least its geometry (length, configuration of head and aft-ends), is mandatory in order to perform grain preliminary design analysis. Physical and mechanical characteristics of the case have a direct effect on structural and ballistic design:

- type of case (metal, filament winding/resin, etc.);
- thermal expansion coefficient;
- hoop and longitudinal strains as function of internal pressure;
- maximum allowable peak pressure (depending on ultimate elastic elongation of case material);
- maximum temperature allowable at case wall at the end of motor firing.

3.3.2. *Thermal insulations*

Nature and geometry of thermal insulations (especially for case-bonded grains) must be known in order to settle grain definition, either from a ballistic point of view (case wall surfaces subjected to high-temperature combustion products), or on a structural point of view (configuration of stress-relieving flaps and boots). Thermal diffusivity, specific heat capacity and mechanical properties data must also be available.

3.3.3. *Support system*

In the case of free-standing grains the support elements ensure that combustion gas may flow between the grain and the case wall during pressurization due to ignition. The support system must be well determined so that prediction of grain operation may be possible at any temperature.

3.3.4. *Nozzle*

The characteristics of the nozzle have a dramatic effect on practical ballistic performance of a rocket motor. The following characteristics are of particular interest to the grain designer:

- number and orientation of the nozzles (the angle between nozzle center line and rocket motor center line must be known);
- degree of nozzle submergence;
- erosion of the nozzle (diameter evolution) versus operation time at throat and exit planes;
- angle of the exit cone (or a dimensioned sketch, in the case of a contoured nozzle);
- failure pressure of the frangible closure disk (this allows definition of ignition system and control of pressurization at ignition);

- dimensions of the blast pipe (between chamber and nozzle), when existing; this affects rocket motor efficiency.

3.3.5. *Ignition system*

The conditions of propellant grain ignition depend on its configuration (location, volume, design). Important characteristics are:

- pressure at the end of ignition,
- pressurization rate (which affects structural integrity during firing).

Minimum and maximum values of delivered pressure and pressurization rate must be accurately known because they are important factors governing grain structural integrity. An envelope of ignition pressure versus time is of interest for this task.

4. Ballistic and Structural Grain Design Methods

4.1. INPUT

In order to design a propellant grain, two types of data are needed:

- Technical specifications: the preceding section gives an almost complete list of these specifications. They are the reduction of functional, operational and interface requirements that must be satisfied in order that the rocket motor fulfill its assigned mission.
- A data bank on propellants, liners, inhibitors and thermal insulations: this allows the grain designer to have at his disposal, quickly and with a low probability of error, chemical, physical, kinetic, mechanical, thermodynamic, etc. characteristics of the various candidate materials which may be used in a rocket motor. The values of these characteristics will be used as input data in analytical and computational design tools.

4.2. PROCEDURE

When performing a solid propellant grain design analysis, two levels of design accuracy have to be distinguished:

4.2.1. *First level*

This is the level of preliminary design analysis. The tools used at this level must be simple and friendly enough to be operated by propellant grain project managers themselves. They are usually small computer codes based on analytical models, or even graphs which give, very simply, the first results.

In any case, the method involves four main stages:

- selection of a propellant/configuration couple;
- definition of grain geometry satisfying internal ballistic and structural integrity (versus temperature cycles related loads) requirements;
- approximate assessment of erosive burning and potential combustion instabilities;
- assessment of grain structural integrity during pressure rise at ignition.

The method is iterative: depending on the results obtained at the third or fourth stage it allows restarting at the second or even the first stage if it appears that the first definition needs strong modifications.

For a few years, grain designers have been requested to quickly provide fairly precise preliminary design analysis for a given project. In order to satisfy this request a computer-aided grain preliminary design analysis method (MIDAP*) has been developed in France. This method is discussed in detail in Section 4.5.

4.2.2. *Second level*

This is the level of final grain design. The tools required for this task are more sophisticated. They are operated by grain design experts, and are mainly finite differences or finite element computer codes based on two- or three-dimensional models of physical phenomena related to internal ballistics, fluid dynamics, continuous media structural analysis, etc. They allow accurate calculations and therefore optimization of the grain final definition.

The principle of the method is parent to the one developed for preliminary design analysis, but it starts from the final result of this analysis; that is to say the geometry and the propellant selected at the end of the preliminary design analysis.

Starting from this geometry, the evolution of grain burning surface area versus web burned is accurately calculated. Taking into account propellant properties, one obtains the evolution of chamber pressure versus time $p(t)$, and thrust versus time $F(t)$. If necessary, the effect of erosive burning has to be taken into consideration at this stage. The results must then be compared with corresponding requirements (maximum pressure, combustion time, total impulse, etc.). Afterwards the structural safety factor (related to thermal cycles and pressure rise loads) must be assessed with the aid of advanced structural analysis computer codes.

If the results are satisfactory and the design is correct, the propellant grain definition is accepted for starting engineering development. If this is not the

* MIDAP: Méthode Informatisée de Définition des Avants-Projets (computer-aided grain preliminary design).

case, grain definition must be modified so as to increase the safety factor in the critical grain area. Additional structural analysis must be performed in order to check the benefits of geometry modification. Evolution of burning area versus web burned, pressure, and thrust versus time must also be checked so that the ballistic requirements remain satisfied. It may happen that, after these modifications, some of the requirements are no longer satisfied. In this event, selection of the couple propellant/geometry has to be changed or, if there is no other possibility, modification of some requirements has to be considered, in cooperation with the rocket motor designer.

4.3. BALLISTIC DESIGN ANALYSIS

4.3.1. *Basic equations*

Basic equations of solid propellant rocket motor internal ballistics are:

(I) $p = \rho S V_c / C_D A_t$

(II) $V_c = f(p)$ (often ap^n)

(III) $F = p C_F A_t$

p = chamber pressure
ρ = propellant mass density
S = propellant grain burning area
V_c = propellant burning rate
C_D = propellant discharge coefficient
A_t = nozzle throat area
a = burning rate coefficient
n = burning rate pressure exponent
F = motor thrust (specific impulse multiplied by propellant weight flow rate)
C_F = nozzle thrust coefficient

A quick examination of the basic solid propulsion equations indicates the effects of various parameters on motor operation and therefore on motor and propellant grain design:

- evolution of burning area versus web burned is directly connected to pressure evolution versus time;
- sensitivity of burning rate to factors such as propellant initial temperature, rocket motor acceleration, chamber pressure, gas flow, will have an effect on motor operation;
- ρ and C_D, which are specific for a propellant, may be considered for propellant selection;
- initial values, and possible evolutions during firing, of A_t and C_F, which are directly related to nozzle definition (and also to propellant nature), must be accurately known.

In the following sections, the series of stages encountered in ballistic design analysis is described.

4.3.2. *Selection of a geometry associated with a propellant*

This important part of design work has been approached in Section 2 but only through a semi-quantitative analysis. In the present section it is quantitatively treated using a simple method which still preserves the designer's judgement.

Selection is performed with the aid of charts and graphs like the one presented in Fig. 9. The example of this figure illustrates the logical method used, which permanently takes into account technical requirements, properties of actual propellants and characteristics of widespread actual grain configurations. The steps are:

- Calculation of propellant mass (M_p), given total impulse (I_{ft}) and standard delivered specific impulse (I_{sms}) (for an expansion ratio of 70/1 and an optimum expansion ratio nozzle) measured for the propellant likely to be selected. This first calculation is iterative, for the value of I_{sms} has to be corrected so as to be representative of the average conditions of motor operation:

  (a) average chamber pressure (P_c) estimated from the specified maximum pressure

  (b) nozzle expansion ratio depending on maximum allowable nozzle exit cone diameter

$$\varepsilon = \frac{A_s}{A_t}$$

  A_s is limited by the specification on maximum diameter of nozzle exit cone, A_t equals $M_p/P_c \cdot C_D \cdot t_c$, where t_c (burn time) is specified.
- Assessment of volumetric loading fraction (C_R) required to obtain specified total impulse, given the mass density of the propellant likely to be selected and the volume available for the propellant grain.
- Selection of grain configurations. For each family of grain configuration an empirical maximum volumetric loading fraction has been determined. Thus, given the volumetric fraction required, one or several configurations can be selected. Other criteria, such as processing practicality, difficulty of structural analysis, propellant web thickness, have also to be taken into consideration.
- Definition of propellant burning rate V_c: $V_c = e_b/t_c$.
- Verification of consistency between specific impulse, density, and burning rate (at the average chamber pressure).

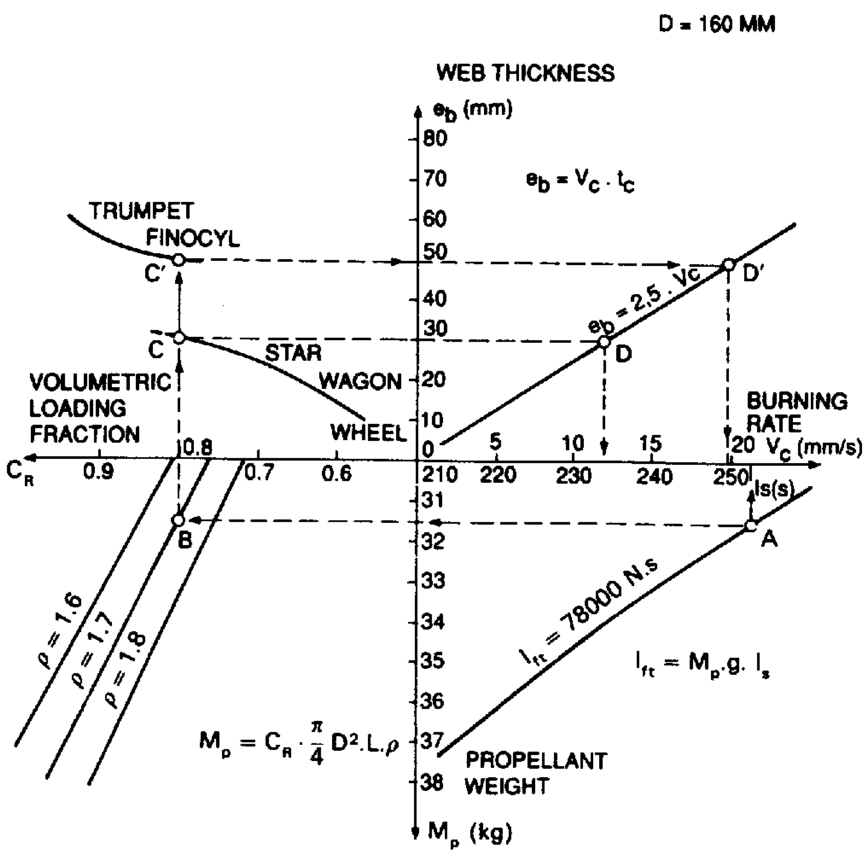

FIG. 2.9. Graph for aiding in critical selection of a couple propellant/geometry.

This approach must be completed by an accurate calculation of nozzle throat diameter generating a maximum pressure lower than that required by the specifications. This step requires a precise definition of grain geometry in order to calculate burning area evolution which is needed for the determination of A_t:

$$A_t = \frac{\rho \cdot S \cdot V_c}{C_D \cdot p_{max}}$$

On Fig. 9 the various steps of the method are represented by the path from A to B, then to C and D, or to C' and D'.

4.3.3. *Calculation of propellant grain burning area*

Accurate prediction of chamber pressure evolution versus time depends on accurate calculation of propellant burning area versus web burned. Computational tools which are commonly used belong to two families: one for "two-dimensional" configurations, the other for "three-dimensional" configurations. Actual grain configurations are three-dimensional, but in numerous cases their geometry is defined by only two coordinates (r, θ) or (r, z); in that case, configurations are said to be two-dimensional.

4.3.3.1. *Two-dimensional geometry computer codes (Fig. 10)*

These programs calculate the evolution of burning area of the following propellant grain configurations:

- grains with a constant port area section;
- axisymmetric grains, presenting a symmetry of revolution with respect to motor center line;
- end-burning grains with axisymmetric slots on the aft-end face.

These various codes require the description (expressed in plane coordinates) of the initial burning area, and of every section of the propellant grain. The computing time can be adjusted according to the level of accuracy desired. Because of the rapidity at which the computations can be done, a visual display of the computed burning areas is possible. As a rule the level of accuracy is excellent. In a more complex case, where the local burning rate of the propellant is not assumed to be independent of the curvilinear abscissa, the evolution of the burning area as a function of time can be computed with the help of a specially designed computerized numerical model [1].

4.3.3.2. *Three-dimensional geometry computer codes*

These codes allow the calculation of burning area of complex configurations, for example finocyl grains having one or several axisymmetric slots. One initial method, limited to the existence of a constant burning rate

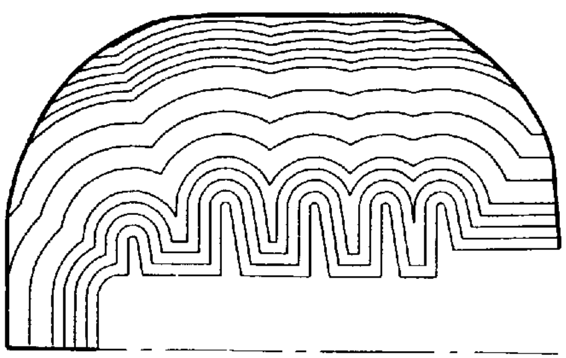

FIG. 2.10. Two dimensional burning area evolution.

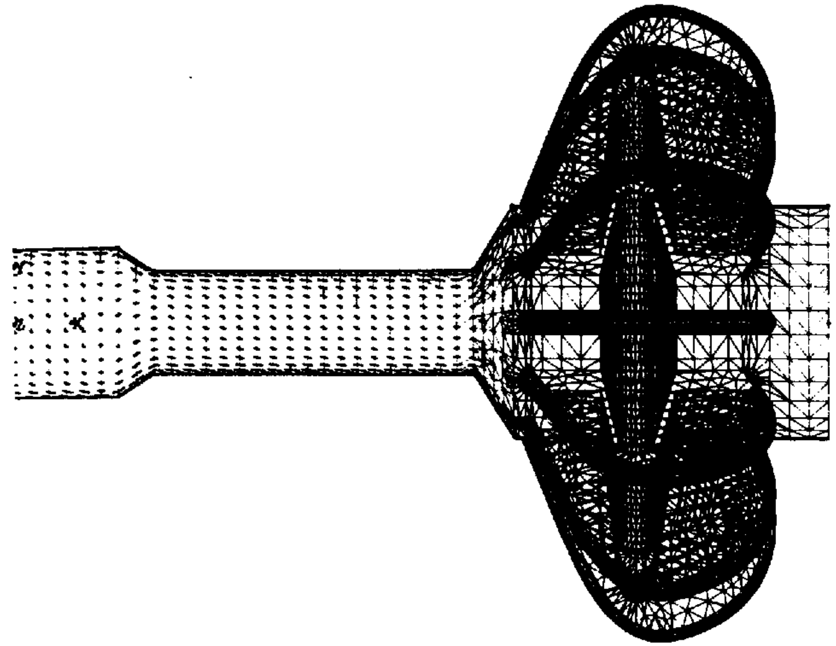

FIG. 2.11. Initial grid of burning area.

throughout the whole grain, uses for each computation step the principle of generation of a surface at a constant distance from the initial surface. This software requires the generation of an initial volumetric grid with a density suitable for the level of accuracy required in the highly three-dimensional zones.

A grid generation can be performed only by an internal ballistics expert, and the analysis of the evolution of the grain burning area, in spite of the existence of grid generation preprocessed data, represents the largest amount of work.

Another method, which allows the burn rate to be a variable function of time and space, uses automatic grid generation and management of the burning area evolution. The assessment of the perpendicular for each triangle of the grid is done utilizing a numerical model using hyperbolic nonlinear differential equations [2]. This method has allowed the development of a very friendly software which requires only the definition of the initial grain geometry (burning area and restricted area—Fig. 11) and parameters that will guide the computation, e.g. level of accuracy of the results, computer time, burning rate versus pressure law, selection of intermediary stages for visualization, etc. (Fig. 12).

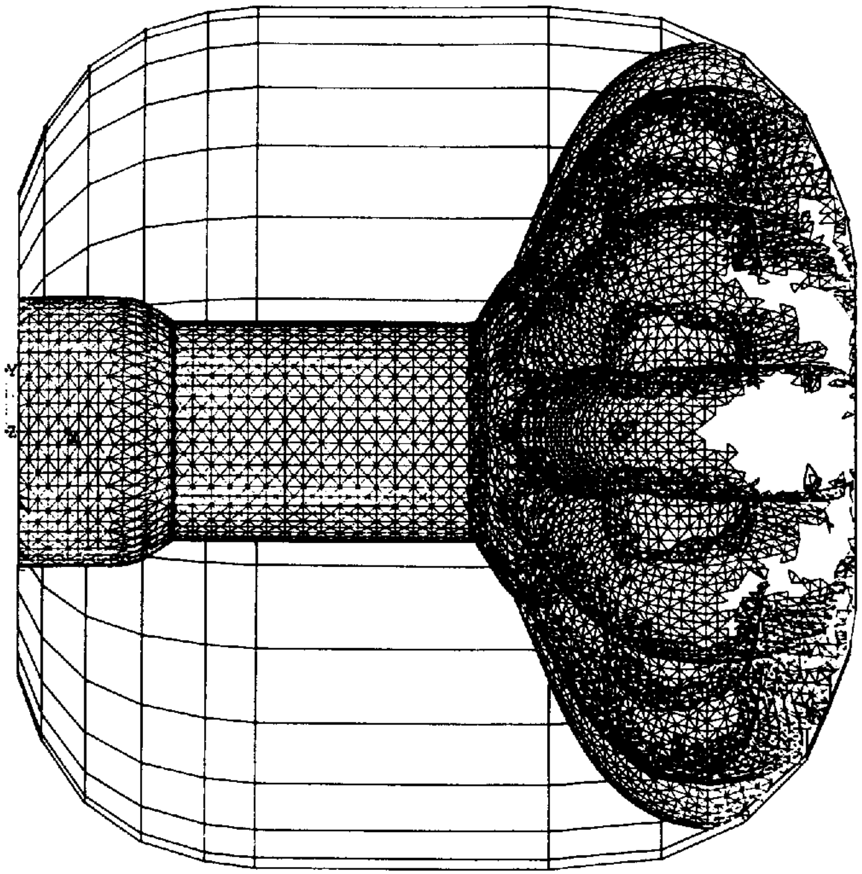

FIG. 2.12. Burning surface evolution, intermediary stage.

4.3.4. Propellants burning rates

Burning rate is one of the major propellant characteristics. It is measured on standard ballistic evaluation motors and it is stored in the data bank mentioned earlier (Section 4.1). It is sensitive to several factors:

- *Pressure.* In the pressure range in which rocket motors operate, a de Saint Robert's burning rate law ($V = ap^n$) is generally preferred. It is also possible to directly use plots of actually measured burning rates versus pressure. The lower the pressure exponent, the more stable the rocket motor internal ballistics.
- *Temperature.* Environmental and use conditions of rocket motors may correspond to a wide temperature range. It is therefore necessary to know burning rate sensitivity to initial propellant tempeature. It is generally

expressed at a given burning surface to throat area ratio, K, as a coefficient defined by:

$$\pi_K = \frac{1}{V}\left(\frac{\partial V}{\partial \theta}\right)_K$$

where θ is the propellant temperature.

- *Acceleration.* Propellant burning rate is sensitive to acceleration, but it is taken into account only when it is more than $10g$.
- *Manufacturing process.* "Hump" effect is the result of change in burning rate as a function of web burned (enhancement of burning rate in radially burning grains in the zone between central port and motor case walls). It is related to manufacturing process. Empirical correlations, drawn from experience, are generally applied to take account of this phenomenon in ballistic design.
- *Internal flow.* Combustion products interact with propellant combustion phenomena and may locally change the burning rate law, which is no longer the one expected. Because of the significant effect of this phenomenon, it is discussed in more detail in the following section.

Burning rate laws, evolution of burning surface versus web burned, and basic internal ballistics equations provide pressure versus time and thrust versus time evolutions. In the simple case where internal flows do not significantly interact with burning rate, eqns (I) and (II) of Section 4.3.1, combined with $V_c = de/dt$, lead to a differential equation which is numerically solved and which provides web burned versus time $e(t)$, burning area versus time $S(t)$, pressure versus time $P(t)$ and thrust versus time $F(t)$.

4.3.5. Effect of internal flows

It is often assumed that flow velocity in the central port exit plane is low enough that it can be neglected in internal ballistics analysis. It is then assumed that flow is accelerated only in the convergence zone of the nozzle so that it reaches sonic velocity at the nozzle throat. In fact this assumption is not satisfactory, because flow calculations demonstrate that velocities of the order of 100–150 m/s are observed in the port exit plane after complete ignition and pressurization. Depending on grain configuration and on propellant properties, two types of phenomenon may be generated:

- a pressure drop between forward and aft-end of the central port,
- a local increase of propellant burning rate due to erosive burning.

4.3.5.1. Criteria for occurrence of non-desired phenomena

When performing a ballistic design analysis one has to quickly assess the magnitude of the phenomena connected with internal flow. Table 4 sum-

TABLE 4 *Intensity of phenomena due to internal flow*

J	K	Erosive burning	Pressure drop
< 0.2	< 50 50 to 100 100 to 150 > 150	no yes when $v < 10$ mm/s yes when $v < 20$ mm/s yes; very important when $v < 10$ mm/s	Low $< 5\%$ P forward end
0.2 to 0.35	< 50 50 to 100 100 to 150 > 150	no yes when $v < 10$ mm/s yes when $v < 20$ mm/s yes; very important when $v < 10$ mm/s	Approximately 10% P forward end when $J = 0.3$
0.35 to 0.5	< 50 50 to 150 > 150	yes when $v < 10$ mm/s yes when $v < 20$ mm/s yes; very important when $v < 10$ mm/s	Approximately 10% P forward end when $J = 0.4$
0.5 to 0.8	< 50 and 50 to 150 > 150	yes; very important when $v < 20$ mm/s yes; very important when $v < 10$ mm/s	40% of$P$ forward may be observed
1	any value	yes; (a) very important when $v < 20$ mm/s (b) low when $v < 30$ mm/s	The pressure in the sonic section is $P \simeq 0.56$ P forward

marizes the knowledge empirically acquired in this field as the result of numerous solid propellant grain design analyses. This table involves a factor J, which is defined as:

$$J = \frac{K_p}{K}$$

$K_p = S'/A_c$
$K = S/A_t$
A_c = area of a given cross-section of central port
S' = propellant burning area upstream of the above cross-section
S = propellant grain burning area
A_t = nozzle throat area

4.3.5.2. *Pressure drop*

Pressure drop is related to a decrease of pressure from grain head-end to grain aft-end. It induces an increase of head-end pressure at the first phase of motor firing, and therefore maximum pressure generally increases. Pressure drops are generally due to:

- energy losses inside the flow, and to phenomena occurring at the interface of flow and propellant surface or to sharp changes of port section or of flow direction,
- side injections from burning propellant walls.

One of the critical steps in rocket motor operation therefore occurs just after ignition when port sections (through which combustion gas must flow) are minimum. Average pressure drop values encountered are of the order of 0.1 MPa between head- and aft-end. In some cases, for special configurations, pressure drops of more than 1 MPa have been observed.

A gaseous flow is fully characterized by the knowledge of local velocities and pressures. Computer codes have been developed in order to determine such characteristics; they are named PROCNE 2 and PROCNE 3 (depending on whether geometry is respectively two- or three-dimensional). They allow:

- description of unsteady phases during pressure rise at ignition,
- calculation of steady flow just after ignition, in the whole cavity and in the nozzle convergence section.

In order to use these codes one has to generate a grid of the combustion chamber. Order "n" symmetry (when existing) is taken into account so as to reduce the analysis to a sector of $2\pi/n$ (n = symmetry number). Figure 13 presents an example of a grid created inside the cavity of a finocyl propellant grain having a symmetry number of 32. Results may be presented either as gas velocity or pressure field (Fig. 14) or as curves representing, for instance, gas velocity as a function of radial distance to the central axis in central port cross-section.

4.3.5.3. *Erosive burning*

Enhancement of propellant burning rate due to tangential gas flow (compared to propellant burning rate without tangential flow) is known as erosive burning. It occurs when the propellant burning surface is subjected to

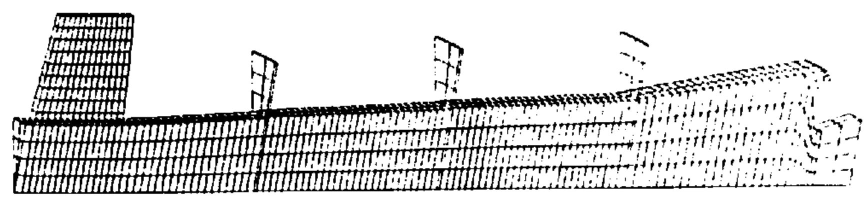

FIG. 2.13. Three dimensional flow inside rocket motor, grid of central cavity.

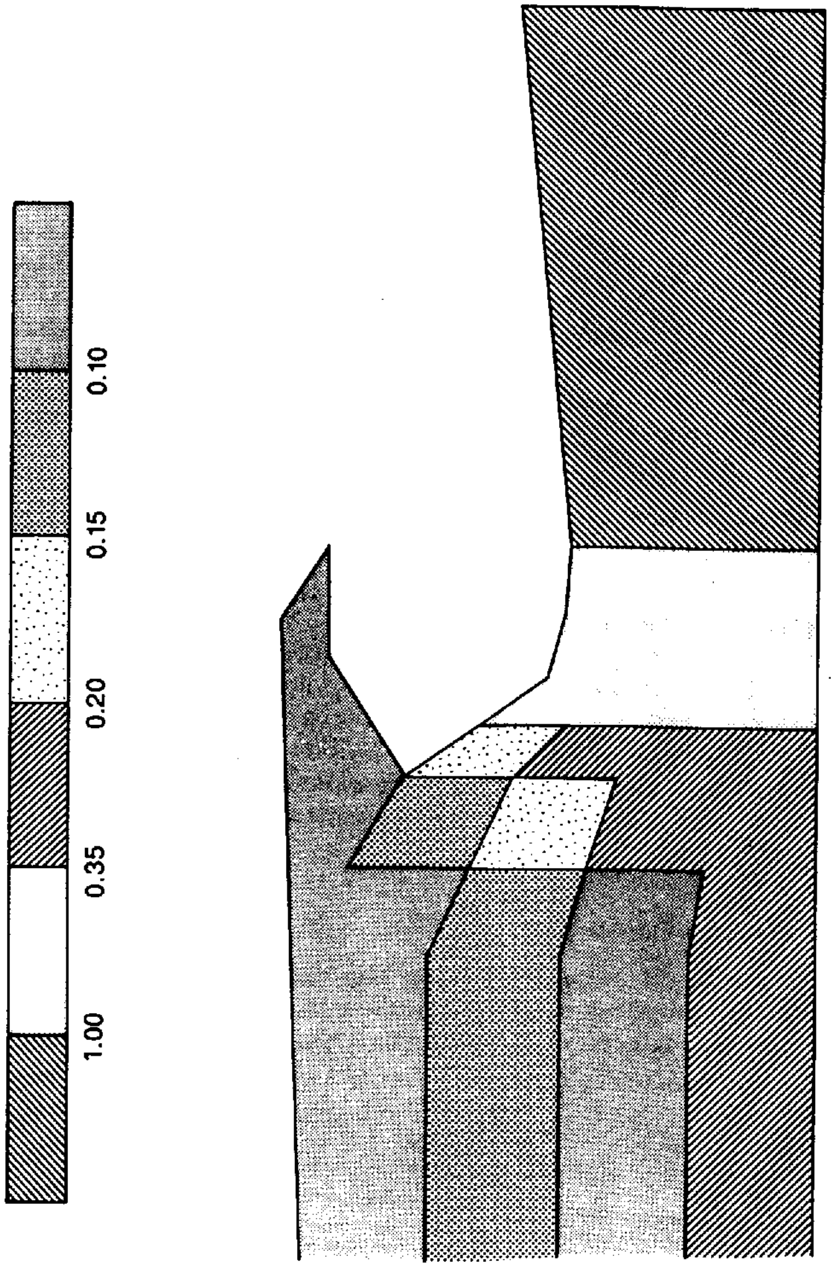

FIG. 2.14. Three dimensional flow inside a rocket motor: velocity field (Mach number of the flow).

a high-velocity combustion gas flow parallel to it. The phenomenon is due to an increase of heat transfer from the flame zone to the propellant surface. There are numerous physical models to explain and to quantify this phenomenon [3]. Practically, a simple computer code (COMBEROS), based on a monodimensional flow model, allows the calculation of the head-end and aft-end pressure evolution in a grain experiencing erosive burning. The erosive burning law selected for the model is:

$$V_e = V_0[1 + \alpha(G - G_0)]$$

V_e = burning rate with erosive burning;
V_0 = burning rate without erosive burning;
G = mass flow rate unit in the given port cross-section;
G_0 = mass flow rate unit threshold (beyond which erosive burning occurs).

Both α and G_0 are obtained empirically.

The COMBEROS code is used systematically in preliminary ballistic design analysis. It implies that grain geometry can be described by the cross-section contour perimeter evolution along the grain axis. Erosive burning is calculated in several cross-sections of the central port according to local flow characteristics (static pressure P and local mass flow rate G) and to the above erosive burning law. The ignition phase is simulated as an unsteady phenomenon; time steps range from 1 to 5 ms. A complete motor firing may be simulated, using a steady-state model and time steps generally ranging from 0.05 to 0.1 s (Fig. 15). A more comprehensive investigation of erosive burning

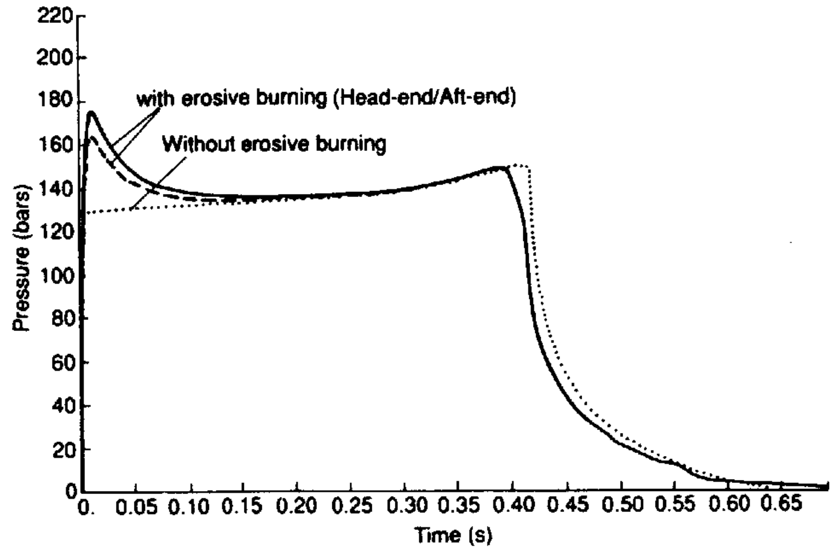

FIG. 2.15. Pressure versus time with, and without, erosive burning.

in propellant grains, though keeping a one-dimensional geometry assumption, may be performed with a more sophisticated code [4].

4.3.6. *Combustion instabilities*

Grain design must incorporate an assessment of combustion stability during motor firing. The phenomenon of combustion instability may occur when perturbations excite oscillation modes of the chamber cavity. Interaction with combustion, flow, particles, nozzle, etc., may induce either an increase or a decrease of the phenomenon. When it increases, pressure vibrations and pressure increase may consequently be driven to an unacceptable level. In order to assess combustion stability, a two-step procedure is followed [5].

Pressure inside the combustion chamber cavity is assumed to be:

$$\frac{p'}{p_0} = \sum_{i=1}^{n} e^{\alpha_i t} e^{j\omega_i t} \Psi_i(M)$$

p_0 = average chamber pressure;
p' = instantaneous pressure at point M;
ω_i = pulsation of mode of rank i and of frequency f_i;
Ψ_i = spatial form of mode of rank i;
M = point in grain cavity;
α_i = damping coefficient (when $\alpha_i < 0$), or gain factor (when $\alpha_i > 0$) of the mode of rank i.

The first step of the analysis consists in calculating the various acoustic modes specific to the grain cavity. A finite-element two-dimensional computer code, VASAX, is used. An example of a two-dimensional grid and the corresponding results are presented respectively in Figs 16 and 17 (the rank of the mode is 3).

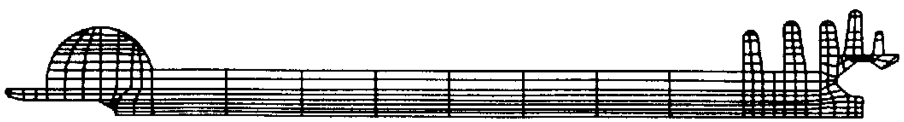

FIG. 2.16. Combustion instabilities: grid of a motor cavity for calculation of acoustical modes.

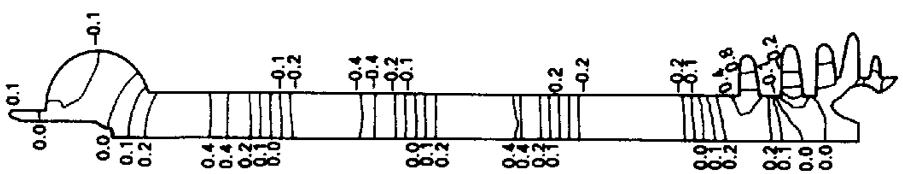

FIG. 2.17. Combustion instabilities: pressure contour lines.

The second step of the analysis consists of calculating the value of α. These calculations need not only the results of the first step, but also data describing propellant response to pressure, effect of condensed particles, etc. The computer code AVER is used.

Depending on the value of α (equal to the algebraic sum of the various gain and damping factors), it is possible to evaluate the grain propensity to experience combustion instabilities: for a mode of frequency f_i, a value of α_i, larger than 0.1 f_i indicates that there is a significant probability that combustion instability may occur. The grain configuration (or propellant) has to be modified.

4.4. STRUCTURAL DESIGN ANALYSIS

4.4.1. *Principles of structural design analysis**

Various loads are imposed on propellant grains throughout their lifetime, from their manufacture until motor firing. These loads depend not only on the rocket motor's own characteristics but also on manufacturing temperature, environmental and operational conditions. Various factors affect loads imposed on a grain (especially a case-bonded grain):

- curing temperature;
- acceleration of gravity;
- type and number of thermal cycles undergone during storage and transportation (for instance captive flights for airborne missiles);
- acceleration during boost phase;
- pressurization during grain ignition.

The goal of structural analysis is to calculate a safety factor defined as:

$$K = \frac{C}{S}$$

where C is the propellant (or bond) structural capability (allowable), and S is a function related to stress/strain induced in the propellant grain region undergoing the more severe loads (margin of safety may be defined as $C - S$ or $C/S - 1$). In order to compare them, C and S must be of the same physical nature.

The safety factor must be higher than 1 during the rocket motor lifetime, including motor firing. According to this definition it is assumed that grain cracking or propellant/liner debonding induce significant modifications to rocket motor internal ballistics having consequences ranging from failure of

* This section may use notions developed in Chapter 6.

missile mission to rocket motor explosion. It is assumed that failure at the most stressed (strained) point does not depend on the stress (strain) gradient in the surrounding region.

If the safety factor calculated for a given propellant grain and given imposed loads is lower than the required value, the propellant grain system has to be redesigned until a satisfactory safety factor is obtained.

Assessment of capability variations (due to manufacturing process, to material reproducibility, to mechanical testing, to aging, etc.) and of induced stress/strain variations (due to uncertainties of boundary conditions, imposed loads and stress/strain determination methods) allows, as a result of a probabilistic analysis, estimation of reliability of a series of propellant grains of a given definition. This subject is discussed in Section 5.

The procedure followed in order to predict safety factors comprises two major aspects: it must define how to assess propellant and propellant-liner bond structural capabilities on one hand, and how to determine induced stress/strain in various loading conditions encountered by the grain, on the other hand. This procedure is schematically presented in Fig. 1 of Chapter 6 of this book.

Propellant and propellant-liner bond capabilities are determined by performing various mechanical tests and require a failure criterion which is defined as the critical value (at failure) of a function related to the state of stress (or strain) of propellant or bond.

Determination of induced stress/strain involves a structural analysis requiring input data such as geometry, boundary conditions (e.g. case displacement), and propellant and bond mechanical behavior.

Results are expressed using the same function selected for failure criterion so that they may be directly compared to propellant and bond capabilities. Experimental validation of the procedure has to be performed, either on the propellant grain itself or on subscale analogs, whenever new elements — such as uncommon grain configurations, new propellants or new bonding systems—have to be considered in safety factor assessment.

4.4.2. Assessment of structural capabilities and of mechanical behavior

Propellant or propellant-liner bond capability is the maximum mechanical loading which can be imposed on the propellant or the bond before failure occurs. Capability is determined by performing tensile (and other) testing on various specimens. The main parameters affecting propellant and propellant-liner bond capability are:

- loading rates (which are very different when thermal cooling or pressurization at ignition have to be simulated);
- temperature;
- surrounding pressure (when simulating ignition pressurization).

As a whole, the experimental work performed on this subject has led to the conclusion [6-8] that propellant behavior is:

- viscoelastic, as evidenced by relaxation tests;
- nonlinear, although considered linear for small deformations;
- Incompressible (Poisson's ratio is very close to 0.5), until dewetting is significant enough to cause volume variations during tensile testing.

The function which expresses propellant capabilities is described in Section 4.4.4.

4.4.3. Determination of induced stress/strain fields

The determination of induced stress/strain fields in the propellant grain requires a knowledge of:

- geometry on which loads are imposed;
- boundary conditions which describe imposed loads;
- propellant and propellant-liner bond behavior.

In most cases the geometry is three-dimensional, loads are static, dynamic or thermally induced and propellant behavior is viscoelastic and nonlinear. Loads which are the limiting factors in structural grain design are generally:

- thermally induced, in the case of grains for tactical missiles (low-temperature cycling);
- pressurization-induced, in the case of grains for large ballistic missiles (stored in almost isothermal conditions).

At the preliminary design phase the expected maximum value of stress/strain induced in the grain is quickly assessed using analytical expressions. For instance, in the case of fairly simple internally perforated grains, the following expressions are commonly used:

$$\varepsilon = 2\alpha \cdot \Delta T \cdot K_t \cdot C \cdot (b/a)^2$$

for a thermally induced strain.

- ε is the equivalent strain at the grain inner bore surface;
- α is the propellant thermal expansion coefficient (assumed to be at least an order of magnitude higher than the case material thermal expansion coefficient);
- ΔT is the difference between stress free temperature and temperature at which induced strain has to be estimated (ΔT may be as large as 100°C);
- K_t and C are corrective coefficients taking into account respectively central port exact geometry and end effects;
- b and a are respectively grain outer and inner radii.

In the case of a pressurization-induced strain,

$$\varepsilon = k \cdot \varepsilon_{\theta s} \cdot \beta \cdot K_t \cdot C \cdot (b/a)^2$$

$\varepsilon_{\theta s}$ is hoop strain of the empty case submitted to ignition maximum pressure;
β takes into account case stiffness increase due to propellant grain;
k is an empirical coefficient.

These values of ε are input data for a first assessment of the grain safety factor.

The final design phase involves extensive use of computational methods based on finite-element techniques applied to grain stress/strain field analysis. The procedure comprises three stages:

4.4.3.1. *The determination of the induced stress/strain field*

This assumes a linear behavior for the material. The stress/strain field is governed by the incompressible behavior of practically all of the propellant grain.

The mechanical load establishing the boundary conditions is expressed either as prescribed displacement, or as prescribed forces at the nodal points of surface elements. Several different computer analysis programs, either two- or three-dimensional, may be used for this phase.

A typical program will have the following characteristics:

- Finite-element method.
- Quadratic elements with 20 nodal points.
- Quadratic surface elements with eight nodal points to allow accurate assessment of stress/strain at the surface of the grain. The use of surface elements increases the accuracy by dramatically reducing the uncertainties caused by the fairly loose extrapolations necessary to calculate maximum stress/strain when there are no skin elements.
- HERRMAN reformulation on incompressibility.

The level of accuracy of the results is a function of the precision of the grid generated to represent the geometries. The number of nodal points must be limited because of computer capacity and CPU time. A typical grid will include 7000 nodal points and 1000 elements.

Figure 18 shows an example of a two-dimensional grid.

4.4.3.2. *Post-processing analysis*

The assumption is made that the propellant behavior is linear and incompressible. The regions of the grain where the stress/strain is the greatest are identified. Figure 19 gives a three-dimensional grid example with stress contour lines for equal stress. The maximum stress occurs, in this case, at the forward slot bore junction.

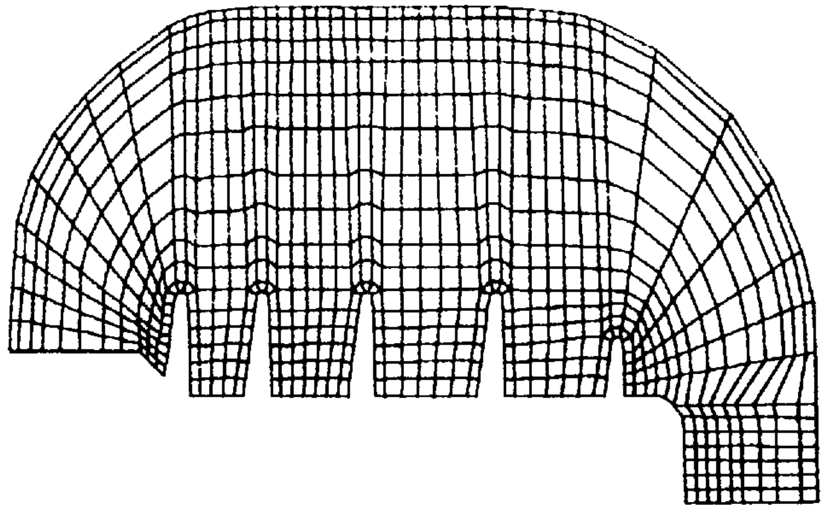

FIG. 2.18. Two dimensional grid network.

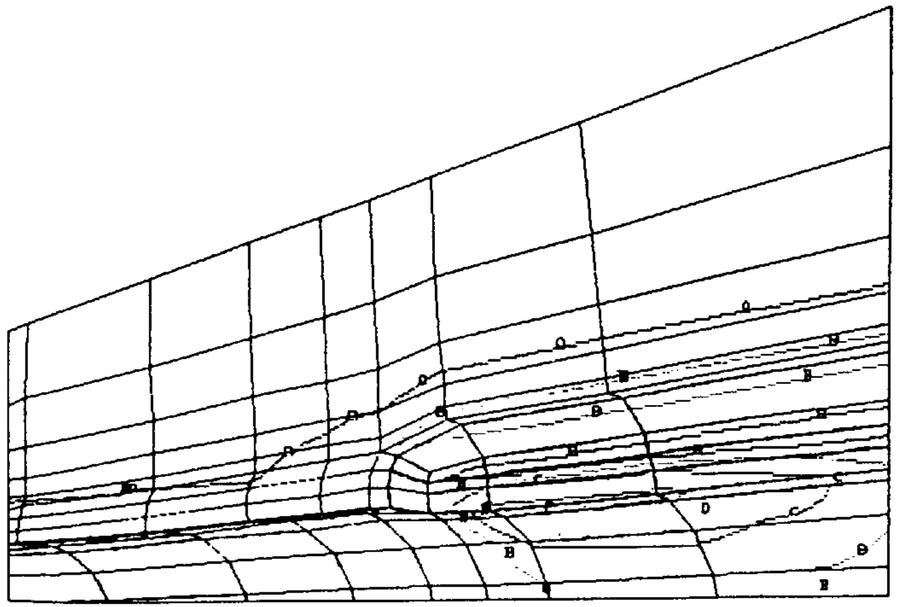

FIG. 2.19. Three dimensional grid network.

4.4.3.3 Determination of stress/strain in the regions experiencing the greatest induced load

Starting with the above results, the determination of strain/stress in the most highly loaded regions is refined by introducing a viscoelastic nonlinear model for thermally induced strain/stress, and an elastic nonlinear model for pressure-induced stress at ignition.

(a) Thermally induced load

The structural model used is a viscoelastic nonlinear model. It provides, at any moment of an imposed thermal cycle, the values of the principal stresses in the propellant ($\sigma_{1th}, \sigma_{2th}, \sigma_{3th}$). The numerical method used is incremental with respect to the time, the principles consists in calculating stresses at a given time t from the values known at time $t - \Delta t$. For that purpose, the effect of the thermal cycles are handled as successive stresses with simultaneous relaxation of the stresses observed at the preceding time [9].

The results of this program have been compared many times with the results of tests performed on propellant grains. The program itself is continuously being improved.

(b) Pressurization-induced stress at ignition

This structural model requires propellant master curves and data characteristics of the pressurization (pressure rise time, final pressure, temperature). It provides the values of the main stresses ($\sigma_{1p}, \sigma_{2p}, \sigma_{3p}$) corresponding to the maximum pressure.

When performing a structural analysis of a propellant grain at the time of firing, which occurs after the effect of a thermal cycle, the maximum stresses will be determined by adding the principal stresses resulting from the thermally induced stresses and pressure-induced stresses, provided that the principal directions for both stresses are identical. This is true for external surfaces, where the most stressed areas are frequently located.

4.4.4. Determination of structural safety factor

At the stage of preliminary design analysis, simple analytical formulas provide the magnitude of strain either due to thermal loading or due to pressure rise (see Section 4.4.3.). In both cases, propellant capability is obtained from the master maximum strain curves at t/a_T corresponding to the loading conditions. So a first assessment of the safety factor is:

$$K = \frac{\varepsilon \text{ (due to thermal loading or pressure rise loading)}}{e_m \text{ (at } t/a_T \text{ corresponding to the loading conditions)}}$$

Propellant capability (Section 4.4.2.) is related to uniaxial tensile tests; it is represented by maximum stress (σ_m) or maximum strain (e_m). Induced stress (strain) (Section 4.4.3.) is the result of a stress (strain) analysis; it is expressed as principal stresses ($\sigma_1, \sigma_2, \sigma_3$) (strains, $\varepsilon_1, \varepsilon_2, \varepsilon_3$) in the most severely stressed (strained) region of the grain.

In order to be able to directly compare capability and induced stress (strain), failure criteria are needed [10]. They are based on an equivalence between principal stresses and an equivalent uniaxial stress defined by:

Von Mises criterion

$$\sigma_0 = [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2}/a^{1/2}$$

or
Stassi criterion:

$$\sigma_0 = [(\sigma_1 + \sigma_2 + \sigma_3) + [(\sigma_1 + \sigma_2 + \sigma_3)^2 + b[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]]^{1/2}]/c$$

a, b, c are coefficients which generally depend on the propellant, but do not depend on strain rate and temperature.

According to the magnitudes of ($\sigma_1, \sigma_2, \sigma_3$), it is either the Stassi criterion or the Von Mises criterion that is used. The Stassi criterion is used mainly in the case of thermally induced stresses, and the Von Mises criterion is used mainly for pressure-induced stress at propellant grain ignition (ignition at 7 Mpa). The parameters of the induced pressure on the propellant grain to determine these criteria are obtained experimentally by performing tensile tests under atmospheric and various other pressures, at various temperatures and stress rates.

The propellant grain safety factor is then defined as the ratio of the maximum stress (obtained in a uniaxial tensile test performed at the strain rate and temperature equivalent to those applied to the grain) to the principal maximum uniaxial stress (obtained from the failure criterion, either from Stassi or from Von Mises criteria, depending on the type of stresses encountered in the most stressed region of the grain), equivalent to the maximum three-dimensional state of stress calculated in the propellant grain:

$$K_\sigma = \frac{S_m(t/a_T)}{\sigma_0(S, VM)}$$

The safety factor may also be defined as:

$$K_\varepsilon = \frac{e_m(t/a_T)}{\varepsilon_0(S, VM)}$$

Where ε_0 is the ratio of equivalent uniaxial stress to the modulus.

There are other methods to predict safety factors; these are discussed in Chapter 6, Section 6. In addition, an analysis of most of these methods was recently published [11].

In the case of propellant liner bonds the problem is a different one because of the presence at all points of the interface of two different materials—the propellant and the liner. The tensors representing stress/strain on both sides, propellant grain and liner, are different. Only the force applied to the interface is continuous. Its components are: a perpendicular strength, σ_n, and a shear strength, τ. The safety factor is determined by comparing the modulus of interface strength (components σ_n and τ) to the modulus of the interface force at the time of failure, obtained under identical conditions on a propellant liner bond specimen.

In most cases the propellant liner bonds are designed for failure to occur in a propellant grain area close to the interface. Furthermore, should the propellant in this area have the same properties as the bulk of the propellant, the safety factor will be calculated the same way, and:

$$K_{\text{bond}} = \min.[K_{\text{strength at bond}}, K_{\text{propellant}}]$$

4.5. COMPUTER-AIDED PRELIMINARY DESIGN OF PROPELLANT GRAINS

4.5.1. *General description*

As mentioned in Section 4.2., there is an increasing pressure to have, as early as the preliminary design phase, quick and relatively accurate results defining the propellant grain. Moreover, further changes in technical requirements need to be easily taken into account. A computer code satisfying these needs is now on service. It is named MIDAP [12] and it involves, today, around 20,000 statements in its newest version. Figure 20 presents the general architecture of the code. Each type of grain configuration (star-shaped, slotted tube, axisymmetric, finocyl, etc.) is individually treated inside the code. The procedure for any of these configurations is the one which is generally followed to perform propellant grain preliminary design analysis (it is described in Section 4.2.). The architecture of the code is modular so that any addition of a new module, or any improvement of an existing module, may be very simply worked out.

Runs are controlled by the user from the graphic terminal. CPU time is negligible as compared to time spent by the user performing the design analysis.

The process is iterative and, besides the input of technical specifications, the user has only to answer yes or no to the option proposed on the screen. Results are presented either as tables or as curves. The block diagram

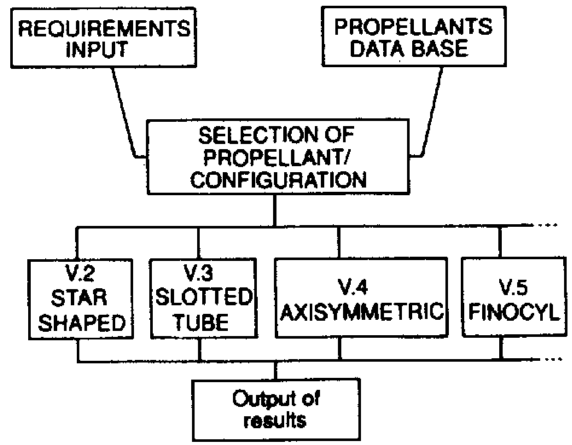

FIG. 2.20. General structure of preliminary design analysis computer code.

presented in Fig. 20 emphasizes the role of propellant/configuration selection, which provides several possibilities, ranked according to a given set of criteria. The selection of propellant/configuration depends:

- on the one hand, on technical requirements (total impulse, burning time, etc.);
- on the other hand, on semi-quantitative requirements related, for instance, to manufacturing process practicality, industrial and economical aspects, etc.

Due to the dual nature of the criteria, an expert system was selected and implemented for this critical stage of preliminary design analysis.

4.5.2. *Description of the code*

All the branches of the code have the same basic structure. The slotted tube branch is detailed below. On the flow chart of Fig. 21 the main stages of the analysis appear. Two possibilities are provided to the user:

- design of a case-bonded (or free-standing) grain meeting technical requirements;
- for a given rocket motor, calculation of motor operation (pressure versus time, thrust versus time, etc.).

In the case of slotted tube configurations, geometrical characteristics which are taken into account for design analysis are:

- cylindrical motor case, presenting possibly thermal insulation overthickness in the aft-end zone (slotted zone);

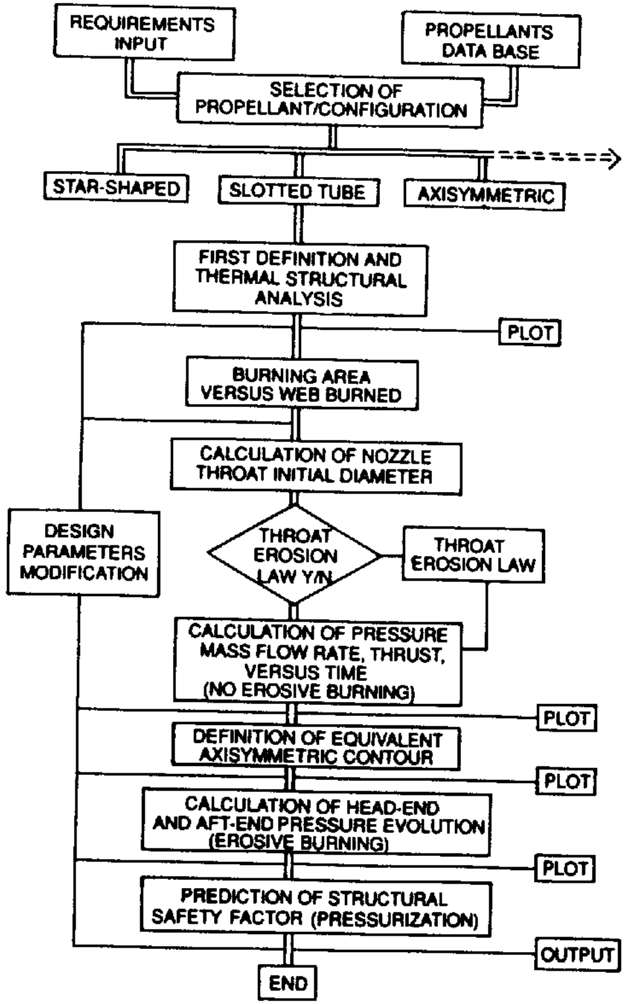

FIG. 2.21.

- plane grain aft-end zone (slotted zone);
- cylindrical central port;
- slot walls may be parallel or not (star-shaped);
- possibility of a tapered port in grain aft zone (in order to limit erosive burning effects).

The successive steps of the analysis are:

- First definition of the grain and thermal structural analysis. This definition meets various requirements, the priorities of which are ranked as follows:
  (1) Structural integrity (for thermally induced strains) corresponding to a safety factor higher than 2.

  (2) Maximum operating pressure.
  (3) Total impulse.
  (4) Evolution of pressure versus time.
  (5) Burning time.

  The structural analysis is based on data obtained from regression analysis of stress/strain field determination results, performed with the aid of three-dimensional computer codes on various selected slotted tube geometries.
- Burning area evolution. This task is performed by dividing the grain into three parts as described in Section 4.3.3.
- Determination of nozzle throat initial diameter. This is the minimum throat diameter value consistent with the specification on maximum acceptable pressure.
- Determination of pressure, mass flow rate and thrust evolutions versus time. In a first approach, erosive burning is not taken into account. The calculations provide expansion ratio, and thus nozzle optimum expansion ratio and exit diameter. Nozzle exit diameter is then compared to corresponding requirement. Afterwards, ratios K (burning area to nozzle throat area) and J (burning area to central port cross-section) are calculated. If needed, a tapered zone is designed in the grain slots region so as to meet a criterion on J (maximum permitted value). Burning area versus time is then calculated again.
- Definition of equivalent axisymmetric longitudinal port contour. This is based on equal flow rates in any port cross-section for actual (three-dimensional) and equivalent (two-dimensional axisymmetric) contours. It allows a simplified analysis of erosive burning which is taken into account at the following stage.
- Calculation of head-end and aft-end pressure evolution. At this stage, erosive burning is taken into account. The module provides pressure evolution inside grain central port, as well as peak pressure at ignition and pressure rise time.
- Prediction of safety factor related to ignition pressurization. Preliminary structural design analysis is performed, as described in Section 4.3.3.

5. Propellant Grain Reliability

Reliability is the probability that a system will fulfill a required mission in given conditions and during a given period of time. Reliability must be considered:

- at the design phase—the system must be designed so that its reliability will meet the requirement;
- at the realization phase—it must be demonstrated that the reliability requirement has been met.

Reliability of a solid propellant rocket motor results from the reliabilities of constitutive elements, such as case, thermal insulation, igniter, nozzle, propellant grain, etc. Grain reliability has several components; but main components are ballistic and structural reliability. In the case of case-bonded grains, past experience and analysis performed according to FMECA (failure modes, effects, and criticality analysis) have demonstrated that structural reliability is the most important component of overall grain reliability.

There are two possible approaches in assessing propellant grain reliability: an analytical approach and an experimental approach; both of them are complementary.

5.1. ANALYTICAL APPROACH

This is performed according to the FMECA method [13]. It consists of:

- listing the functions the propellant grain must fulfill;
- describing failure modes;
- assessing probability of failure occurrence for each mode;
- validating assessments by comparing with overtests and analog experimental results.

As mentioned above, this method places emphasis on the structural component of case-bonded grain reliability. This aspect is therefore discussed below. A comprehensive description of the methods used in propellant grain structural reliability assessment would need extensive discussion because of the complexity of phenomena and analytical tools involved. Consequently, the following section provides only an idea of the principles governing structural reliability assessment.

Safety factors are considered in Section 4 as having known values: propellant grain failure occurs when $K = C/S = 1$. K is the structural safety factor, C is capability and S is induced stress/strain. In fact, most of the parameters involved in safety factor prediction are randomly distributed and their statistical distribution law is not always well known. These parameters are related to:

- grain geometry
- boundary conditions
- propellant and bond behavior

} which define induced stress/strain field,

- capability
- failure criterion

} which may take aging into account,

When designing a case-bonded grain the distribution law of the parameters defining grains and imposed loads must be known, so that the distribution law of C and S can be known. It is then possible to determine the minimum safety factor ensuring required reliability. In a second stage, taking into account variations due to manufacture (and possibly to aging), it is possible

to define a mean safety factor (higher than the preceding one) that must be the objective at the design phase. Grains designed so as to meet this requirement on K have the desired reliability at a high confidence level.

For a given propellant grain, variations of capability and induced stresses (strains) are due to:

- errors in test measurements of propellant and bond mechanical properties,
- the probabilistic nature of loads imposed to the grain before, and during firing,
- uncertainties related to structural models and to failure criteria determination.

Let C and S be the mean values respectively of capability and of induced equivalent stress (strain) and CV_c and CV_s corresponding deviation factors (which are assumed to be independent of the mean value), then $K = C/S$, and it can be demonstrated that the probability that grain failure does not occur is:

$$\text{Prob}(C > S) = \Phi\left[\frac{K - 1}{(K^2 CV_c^2 + CV_s^2)^{1/2}}\right]$$

where Φ is the repartition function of normal distribution law. C and S are generally assumed not to be correlated (which is not correct but acceptable). It is possible, however, to take a correlation into account if it is clearly demonstrated.

Minimum safety factor, K_{min}, ensuring required reliability F, is then obtained by determining the value of K_{min} which satisfies the relationship:

$$\left[\frac{K_{min} - 1}{(K_{min}^2 CV_c^2 + CV_s^2)^{1/2}}\right] = F$$

Taking into account variations due to manufacture, deviation of the safety factor is assessed. It is then possible to calculate a value of safety factor which is the objective of structural design analysis: it ensures that grains accordingly designed have a given probability of meeting the reliability requirement.

5.2. EXPERIMENTAL APPROACH

Safety margin is $C - S$. There is a discrepancy between actual margin of safety $(C - S)_R$ and predicted margin of safety $(C - S)_c$, due to the use of an approximate model. It is possible to write $(C - S)_R = (C - S)_c + \xi$, where ξ is assumed to obey a normal distribution law. The ξ mean value, m_ξ, represents the shift of the model. Deviation σ_ξ represents variations of this shift; m_ξ and σ_ξ must be assessed by performing significant experimental tests. There are two possibilities: either overtests or tests on grain analogs.

5.2.1. *Overtest*

The first method is to assume that ξ obeys a given normal distribution law and to use overtest results in order to refine this distribution law: this is the Bayesian method [14]. Grain overtests are tests which have a moderate probability (much higher than in a normal motor firing or temperature cycling) that failure does occur. Much information is thus obtained on grain reliability. Overtests are defined by changing thermal cycles applied to the grain (colder temperature, larger cycles number) or firing conditions (reduced nozzle throat diameter) compared to normal conditions.

The most interesting information is obtained when overtest performance does not induce propellant or bond failure: a more accurate definition of distribution law may thus be proposed.

5.2.2. *Tests on grain analogs*

A second method to quantify the shift of the model consists in performing loading tests on analogs [15]. This analog grain has the following characteristics:

- configuration is simple enough so that analogs may be easily manufactured at low cost;
- two-dimensional geometry induces low-cost computational structural analysis;
- the ratio of maximum stress (strain) to mean stress (strain) induced in the analog is of the same order of magnitude as the one encountered in actual grains;
- maximum induced stress (strain) can be easily adjusted by simply modifying analog manufacture tooling.

The main drawback is that the analog is not... the grain itself, which means that propellant, liner and bond are not exactly the same, and are not in the same surrounding conditions as those constitutive of the actual grain.

Mechanical testing consists in loading a given number of analogs in identical conditions until failures occur. Analysis of failure results and deviations yields the shift ξ between actual margin of safety and predicted margin of safety. It is then assumed that the shift observed on the analogs is equal to the shift existing in actual grains.

This set of complementary theoretical and experimental methods allows the assessment of structural reliability of case-bonded solid propellant grains.

6. Special Cases

Probably over 90% of cases encountered in practice are included in the preceding discussions. There are, however, some special applications that do

require special configurations or designs, such as: (1) segmented propellant grains; (2) nozzleless grains; and (3) wired, end-burning grains. Finally, an additional special grain, designed to reduce the base drag of shells, is discussed in Chapter 8 and is the special topic of integral boosters in Chapter 12.

6.1. SEGMENTED PROPELLANT GRAINS FOR SPACE LAUNCHERS

The need to launch objects with ever-increasing weight revealed the necessity for rocket motors capable of very high levels of thrust and total impulse at the beginning of the launching operation, to provide the energy needed for lift-off, and to traverse the thick layers of the atmosphere under very precise conditions of acceleration. This has led to the design of special rocket motors. During the first 2 minutes of flight they may deliver a thrust 10 times greater than the thrust of the central rocket motor. The required thrust levels are found within the operating ranges of solid propellant grains, so that solid propellant rocket motors are a basic complement to the classic liquid fuel of these launchers [16].

These rocket motors are positioned on the periphery of the central liquid fuel motors, requiring a very high ratio of length to diameter, that can be as high as 10; the propellant grain may weight over several hundred tonnes.

It is therefore very difficult to manufacture a rocket motor of this type in one single monolithic assembly in classic manufacturing facilities designed usually for the production of propulsion stages for ballistic missiles. The manufacture of these rocket motors turned toward assembling several sections, called segments. Each of these segments consists of several tens of tons of propellant grain case-bonded in a section of metallic case. The segments are then assembled to reproduce the classic configuration of a case-bonded propellant grain [16,17]. Figure 22 shows a four-segment configuration. As a rule, one segment has a star-shaped configuration providing a greater impulse at lift-off, for approximately 20 seconds.

The particular characteristics of these types of propellant grains are the burning areas on the end faces of the segments, resulting in high gas flow rate in the proximity of the joints.

The thrust curve is controlled by working on the star-shaped configuration, the taper coefficient of the central port and the restriction of head-end surfaces by inhibitors.

The general criteria used for the geometry design, and other methods discussed in the preceding sections, are for the most part applicable. A certain number of specific problems must, however, be resolved:

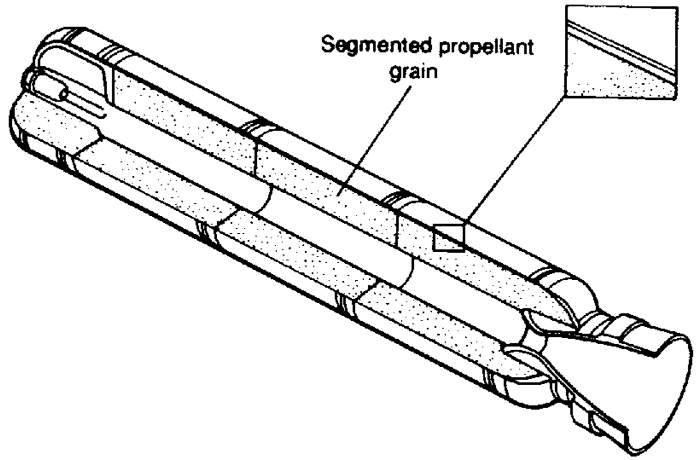

FIG. 2.22. Example of a segmented propellant grain.

6.1.1. *Acoustics*

The head-end burning areas, at the segments' interface, can produce radial flows disturbing the central gas flow. These disturbances may cause instabilities in the gas flow. This type of situation is similar to axisymmetric propellant grains with radial slots. In addition, the exposure of the inhibitors during the burning process may cause pulsations in the gas flow and be the source of additional acoustic energy [18].

6.1.2. *Segment assembly*

The segments are connected to each other, and the motor cases are assembled together through a system of clamps and pins.

Sealing of junctions is ensured by flexible seals. These junctions must be protected against the presence of combustion gas in the intersegments during the ignition phase. These junctions are one of the weak points of the rocket motor. The difficulty comes from the stress/strain imposed on the junction during pressure rise at ignition, and from the difficulty of inspecting this area after assembly.

6.1.3. *Rocket motor pairs*

There are at least two boosters located on the outside of the launcher. Consequently, closely matched thrusts from each specimen are necessary to allow good control of the flight, and particularly to avoid any troubles at separation at burn-out.

Thrust imbalance between boosters is usually the result of variations in the burning rate of the propellant grain, but may also be caused by other factors linked to the nozzle (erosion, etc.).

Control of thrust imbalance is obtained through adjustment of the ingredients, the manufacture process, and control method [1].

6.2. NOZZLELESS BOOSTERS

The nozzleless booster is an early design that resulted from an analysis of a classic rocket motor, demonstrating that the nozzle accounted for a significant portion of the cost, weight and size of a rocket motor.

The propellant grain of a nozzleless booster is usually case-bonded with a generally cylindrical central port, connected at the aft-end to an exit cone tailored in the propellant.

In the absence of a nozzle throat, the operating conditions of the nozzleless rocket motor are governed by an aerodynamic constriction of the gas flow. The exit cone section at the aft-end of the grain is designed to allow gas expansion.

The development of nozzleless rocket motors is linked to composite propellants with high burning rates, because this type of propellant grain requires a minimum flow rate to ensure a stable performance, in turn requiring prohibitive length-to-diameter ratios when other propellant compositions are used.

The main advantages of nozzleless rocket motors are:

- simplicity of the propellant grain geometry,
- greater loading ratio than in a conventional rocket motor,
- weight reduction, due to absence of mechanical parts at the aft-end and absence of nozzle,
- significant decrease of the cost of the rocket motor,
- performance improvement for a given size,
- elimination of a nozzle which otherwise would have to be ejected at the end of the acceleration phase, making this concept very interesting for use as an integral booster for rocket ramjet systems [4].

There are some drawbacks as the price to pay for these advantages:

- loss of approximately 20% of specific impulse in comparison to a conventional system [19];
- the possibility of occurrence of combustion instabilities at low pressure;
- the need to have a thorough knowledge of the normal burning rate and erosive combustion for a very large range of pressures, to enable performance predictions [20];
- the propellant grain mechanical deformation directly reflected in the ballistics of the motor [21].

A highly regressive combustion pressure curve is a characteristic of a nozzleless rocket motor operation, reaching very low pressure (Fig. 23). Because of these particular operational characteristics, the design analysis of a nozzleless rocket motor can be performed only with very specific computer models.

All of these models are monodimensional, a typical code using a single-phase, quasi-stationary description of the flow. The precision of the calculations depends on a very exact knowledge of propellant grain combustion laws, as well as an exact description of the geometry of the structure and the grain's central port, including structural strains.

Before undertaking the computer analysis, and in order to limit computer time, the following method may be used to perform a preliminary design analysis:

- limit the minimum central port diameter for structural integrity reasons;
- in the case of a given diameter, determine the required length of the propellant grain by using I_s of approximately 215 seconds for Butalane and 200 seconds for Butalite.
- determine the diameter of the central port, for a given maximum pressure, using the following simplified formula [22]:

$$D = \frac{4\rho L C^*}{0.8} \cdot \frac{v(p)}{p}$$

p = maximum pressure;
$v(p)$ = burning rate at maximum p;
ρ = density;
C^* = characteristic burning rate;
L = estimated length of the propellant grain.

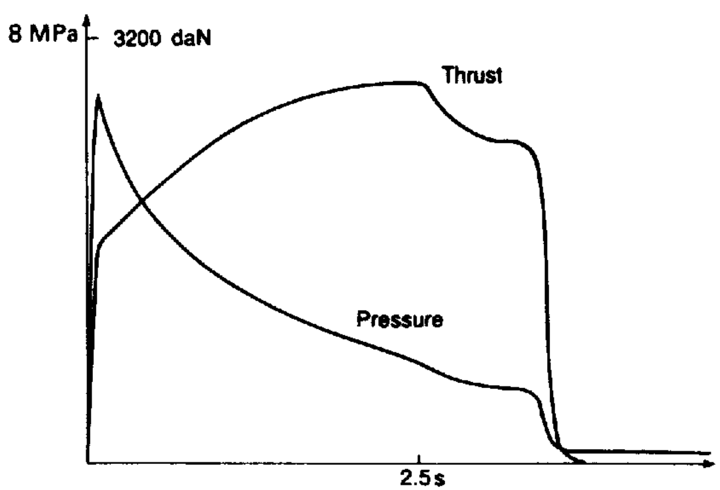

FIG. 2.23. Typical thrust-pressure curves of a nozzleless propellant grain.

These preliminary values for the diameter and the length are used in the computer analyses to optimize the propellant grain.

6.3. WIRE END-BURNING GRAIN

The concept of the wire end-burning free-standing propellant grain first appeared at the beginning of the 1960s. Wire end-burning is one of the means available to increase the effective burning rate of a propellant grain and, in addition, it is particularly well suited for a high loading ratio of the rocket motor. It is based on a simple observation: the burning rate along a wire embedded in the propellant grain is faster than inside the propellant grain itself [24].

Consequently, the burning surface is modified by the formation of a cone whose vertex travels along the wire. Its half-angle at the vertex is solely a function of the ratio between the propellant grain's specific burning rate and the burning rate along the wire. The overburning rate coefficient N is calculated as follows:

$$N = \frac{1}{\sin\alpha} = \frac{V_{wire}}{V}$$

V_{wire} = burning rate along the wire;
V = propellant burning rate;
α = half-angle of the cone.

This phenomenon is linked to a modification of the thermal field next to the wire, and is a function of the nature and diameter of the wire as well as the nature of the propellant. The principle is used to increase the gas flow rate of front-end burning grains by placing continuous, straight wires, perpendicular to the initial burning surface.

The preliminary analysis of the performance of this type of propellant grain involves two distinct steps that cannot be handled using traditional means:

- The determination of the overburning rate coefficient of a given wire/propellant combination. Currently, this determination is mostly experimental. Theoretical approaches are being developed, requiring the generation of a complete data base for thermal data on wires and on propellant, e.g. diffusivity, conductibility, thermal capacity, etc.
- Determination of the evolution of the wire grain burning surface. When dealing with front-end-burning propellant grains the computational programs are relatively easy to generate, since the evolution of the grain's burning surface involves nothing but cones. The difficulties increase with the number of wires embedded in the propellant grain.

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