Factors of $10^n - 1, 10^n + 1, 2^n - 1$ and $2^n + 1$.

In 1838 Westerberg produced a table of factors of $10^n \pm 1$ for n less than 16. The number $10^n - 1$ in decimal is a sequence of n nines; for instance $10^3 - 1 = 999$, so $(10^n - 1)/9$ is a sequence of n ones. In 1869 Landry published a table of factors of $2^n - 1$, for n less than 64, with a few gaps.

n	prime factors of $(10^n - 1)/9$	prime factors of $10^n + 1$	prime factors of $2^n - 1$	prime factors of $2^n + 1$
2	11	101	3	5
3	3, 37	7, 11, 13	7	3
4	11, 101	73, 137	3, 5	17
5	41, 271	11, 9091	31	3, 11
6	3, 7, 11, 13, 37	101, 9901	3, 7	5, 13
7	239, 4649	11, 909091	127	3, 43
8	11, 73, 101, 137	17, 5882353	3, 5, 17	257
9	3, 37, 333667	7, 11, 13, 19, 52579	7, 73	3, 19
10	11, 41, 271, 9091	101, 3541, 27961	3, 11, 31	5, 41
11	21649, 513239	11, 23, 4093, 8779	23, 89	3, 683
12	3, 7, 11, 13, 37, 101, 9901	73, 137, 99990001	3, 5, 7, 13	17, 241
13	53, 79, 265371653	11, 859, 1058313049	8191	3, 2731
14	11, 239, 4649, 909091	29, 101, 281, 121499449	3, 43, 127	5, 29, 113
15	3, 31, 37, 41, 271, 2906161	7, 11, 13, 211, 241, 2161, 9091	7, 31, 151	3, 11, 331
16	11, 17, 73, 101, 137, 5882353	353, 449, 641, 1409, 69857	3, 5, 17, 257	65537
17	2071723, 5363222357	11, 103, 4013, 21993833369	131071	3, 43691
18	3, 7, 11, 13, 19, 37, 52579, 333667	101, 9901, 999999000001	3, 7, 19, 73	5, 13, 37, 109
19	1111111111111111111	11, 909090909090909091	524287	3, 174763
20	11, 41, 101, 271, 3541, 9091, 27961	73, 137, 1676321, 5964848081	3, 5, 11, 31, 41	17, 61681

Notice that $(10^{19} - 1)/9$ is the 19 digit prime 1111111111111111111 and that $(10^{17} - 1)/9$ is the product of two quite large primes. It has been known for some time that $(10^{71} - 1)/9$ is composite. However to find its factors is not at all straightforward. It is the product of a 30 and a 41 digit prime which were discovered in 1984 by Davis & Holdridge.

Some things to think about.

(1) Find the prime factors of $3^n + 1$ and $3^n - 1$ as n varies from 1 to 20.
(2) If the positive integer N is not prime, why must it have a prime factor p with $1 < p \leq \sqrt{N}$?
(3) What is the reason that the prime factors of $10^7 + 1$ are also prime factors of $10^{14} - 1$?
(4) What are the prime factors of $n! + 1$ for small n?
(5) How many different prime factors does 10^{100} have?
(6) How many different positive factors does $239^2 \times 79^3$ have?

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