

(3)

4-3. An astronomer finds a new absorption line with $\lambda = 164.1 \text{ nm}$ in the ultraviolet region of the sun's continuous spectrum. He attributes the line to hydrogen's Lyman series. Is he right? Justify your answer.

4-3. $\frac{1}{\lambda_{mn}} = R \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$ where $m = 1$ for Lyman series (Equation 4-2)

$$\frac{1}{164.1 \text{ nm}} \approx \frac{1.097 \times 10^7 \text{ m}^{-1}}{10^9 \text{ nm/m}} \left(1 - \frac{1}{n^2} \right)$$

$$\Rightarrow \frac{1}{n^2} = 1 - \frac{10^9 \text{ nm/m}}{164.1 \text{ nm} (1.097 \times 10^7 \text{ m}^{-1})} = 1 - 0.5555 = 0.4445$$

$$n = (1/0.4445)^{1/2} \approx 1.5$$

No, this is not a hydrogen Lyman series transition since n is not an integer.

(13)

4-13. The radius of the $n = 1$ orbit in the hydrogen atom is $a_0 = 0.053 \text{ nm}$. (a) Compute the radius of the $n = 6$ orbit. (b) Compute the radius of the $n = 6$ orbit in singly ionized helium (He^+), which is hydrogen-like, i.e., it has only a single electron outside the nucleus.

Use equation 4-18, $r_n = \frac{n^2 a_0}{Z}$.

(a) For hydrogen, $Z = 1$, so $r_6 = 6^2 a_0 = 36 a_0 \approx 36 (0.0529 \text{ nm}) \approx \boxed{1.9 \text{ nm}}$.

(b) For singly ionized helium, $Z = 2$, so $r_6 = \frac{36 a_0}{2} = 18 a_0 \approx \boxed{0.95 \text{ nm}}$.

(19)

4-19. It is possible for a muon to be captured by a proton to form a muonic atom. A muon is identical to an electron except for its mass, which is $105.7 \text{ MeV}/c^2$. (a) Calculate the radius of the first Bohr orbit of a muonic atom. (b) Calculate the magnitude of the lowest energy state. (c) What is the shortest wavelength in the Lyman series for this atom?

reduced mass: $\mu = \frac{m_{\text{muon}} \cdot m_{\text{proton}}}{m_{\text{muon}} + m_{\text{proton}}} \approx \frac{(105.66)(938.27)}{105.66 + 938.27} \frac{\text{MeV}}{c^2} \Rightarrow \mu \approx 94.966 \frac{\text{MeV}}{c^2}$

(a) Eqn 4-19: $a_0 = \frac{\hbar^2}{\mu k e^2} = \frac{m}{\mu} \left(\frac{\hbar^2}{m k e^2} \right) \approx \frac{m}{\mu} a_0 \approx \frac{0.511}{94.966} (0.0529 \text{ nm}) \approx \boxed{0.000285 \text{ nm}}$

(b) Eqn 4-24: $E_0 = \frac{\mu k^2 e^4}{2 \hbar^2} = \frac{\mu}{m} \left(\frac{m k^2 e^4}{2 \hbar^2} \right) \approx \frac{94.966}{0.511} (13.6 \text{ eV}) \approx 2.53 \times 10^3 \text{ eV} = \boxed{2.53 \text{ keV}}$

(c) Shortest wavelength Lyman line: $n = \infty$ to $n = 1$, so its energy is E_0 . Then

$$E_0 = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E_0} \approx \frac{1.24 \text{ keV} \cdot \text{nm}}{2.53 \text{ keV}} \Rightarrow \boxed{\lambda \approx 0.49 \text{ nm}}$$

(20)

4-20. In the lithium atom ($Z = 3$) two electrons are in the $n = 1$ orbit and the third is in the $n = 2$ orbit. (Only two are allowed in the $n = 1$ orbit because of the exclusion principle, which will be discussed in Chapter 7.) The interaction of the inner electrons with the outer one can be approximated by writing the energy of the outer electron as

$$E = -Z'^2 \frac{E_1}{n^2}$$

where $E_1 = 13.6 \text{ eV}$, $n = 2$, and Z' is the effective nuclear charge, which is less than 3 because of the screening effect of the two inner electrons. Using the measured ionization energy of 5.39 eV , calculate Z' .

$$4-20. \quad E = -(Z')^2 E_1 / n^2 \Rightarrow Z' = \left[\frac{-n^2 E}{E_1} \right]^{\frac{1}{2}} = \left[\frac{-2^2 (-5.39 \text{ eV})}{13.6 \text{ eV}} \right]^{\frac{1}{2}} = 1.26$$

(24)

4-24. The electron-positron pair that was discussed in Chapter 2 can form a hydrogenlike system called *positronium*. Calculate (a) the energies of the three lowest states and (b) the wavelength of the Lyman α and β lines. (Detection of those lines is a "signature" of positronium formation.)

4-24. (a) The reduced mass correction to the Rydberg constant is important in this case.

$$R = R_\infty \left(\frac{1}{1 + m/M} \right) = R_\infty \left(\frac{1}{2} \right) = 5.4869 \times 10^6 \text{ m}^{-1} \quad (\text{from Equation 4-26 and 4-27})$$

$$E_n = -hcR/n^2 \quad (\text{from Equation 4-23 and 4-24})$$

$$E_1 = -(1240 \text{ eV} \cdot \text{nm}) (5.4869 \times 10^6 \text{ m}^{-1}) (10^{-9} \text{ m/nm}) / (1)^2 = -6.804 \text{ eV}$$

$$\text{Similarly, } E_2 = -1.701 \text{ eV} \quad \text{and} \quad E_3 = -0.756 \text{ eV}$$

(b) Lyman α is the $n = 2 \rightarrow n = 1$ transition.

$$\frac{hc}{\lambda_\alpha} = E_2 - E_1 \rightarrow \lambda_\alpha = \frac{hc}{E_2 - E_1} = \frac{1240 \text{ eV} \cdot \text{nm}}{-1.701 \text{ eV} - (-6.804 \text{ eV})} = 243 \text{ nm}$$

Lyman β is the $n = 3 \rightarrow n = 1$ transition.

$$\lambda_\beta = \frac{hc}{E_3 - E_1} = \frac{1240 \text{ eV} \cdot \text{nm}}{-0.756 \text{ eV} - (-6.804 \text{ eV})} = 205 \text{ nm}$$

(27)

4-27. The wavelength of the K_α x-ray line for an element is measured to be 0.0794 nm . What is the element?

$$\text{Using eqn 4-37 with } n=2, \text{ we have } \lambda = \frac{1}{R_\infty (Z-1)^2 (1 - \frac{1}{4})} = \frac{4}{3R_\infty (Z-1)^2}$$

$$\Rightarrow (Z-1)^2 = \frac{4}{3R_\infty \lambda} \Rightarrow Z-1 = \frac{2}{\sqrt{3R_\infty \lambda}}, \text{ or } Z = 1 + \frac{2}{\sqrt{3R_\infty \lambda}}, \text{ so}$$

$$Z \approx 1 + \frac{2}{\sqrt{3(1.097 \times 10^7 \text{ m}^{-1})(0.0794 \times 10^{-9} \text{ m})}} \approx 40, \text{ so the element is } \boxed{\text{Zirconium}}.$$

(32)

4-32. (a) Compute the energy of an electron in the $n = 1$ (K shell) of tungsten, using $Z - 1$ for the effective nuclear charge. (b) The experimental result for this energy is 69.5 keV. Assume that the effective nuclear charge is $Z - \sigma$, where σ is called the screening constant, and calculate σ from the experimental result.

(a) Using eqn 4-20, with $Z = 74$ for tungsten, the magnitude of the energy is

$$E_1 = (Z-1)^2 \left(\frac{mk^2 e^4}{2\hbar^2} \right) = (Z-1)^2 (13.6 \text{ eV}) = 73^2 (13.6 \text{ eV}) \approx \boxed{72.5 \text{ keV}}.$$

$$(b) (Z-\sigma)^2 = \frac{E_1}{13.6 \text{ eV}} \Rightarrow Z-\sigma = \sqrt{\frac{E_1}{13.6 \text{ eV}}} \Rightarrow \sigma \approx Z - \sqrt{\frac{E_1}{13.6 \text{ eV}}}, \text{ or}$$

$$\sigma \approx 74 - \sqrt{\frac{69.5 \text{ keV}}{0.0136 \text{ keV}}} \approx 74 - 71.456, \text{ so } \sigma \approx \boxed{2.51}.$$

(36)

4-36. The transition from the first excited state to the ground state in potassium results in the emission of a photon with $\lambda = 770 \text{ nm}$. If potassium vapor is used in a Franck-Hertz experiment, at what voltage would you expect to see the first decrease in current?

$$4-36. \Delta E = \frac{hc}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{770 \text{ nm}} = 1.610 \text{ eV} \text{ The first decrease in current will occur when the}$$

voltage reaches 1.61 eV.

(37)

4-37. If we could somehow fill a Franck-Hertz tube with positronium, what cathode-grid voltage would be needed to reach the second current decrease in the positronium equivalent of Figure 4-23? (See Problem 4-24.)

4-37. Using the results from Problem 4-24, the energy of the positronium Lyman α line is

$$\Delta E = E_2 - E_1 = -1.701 \text{ eV} - (-6.804 \text{ eV}) = 5.10 \text{ eV}. \text{ The first Franck-Hertz current}$$

decrease would occur at 5.10 V, the second at 10.2 V.

(43)

4-43. Show that a small change in the reduced mass of the electron produces a small change in a spectral line given by $\Delta\lambda/\lambda = \Delta\mu/\mu$. Use this to calculate the difference $\Delta\lambda$ in the Balmer red line $\lambda = 656.3 \text{ nm}$ between hydrogen and deuterium, which has a nucleus with twice the mass of hydrogen.

In this problem, $\Delta\lambda$ and $\Delta\mu$ are taken to be the magnitudes of corresponding small changes in the wavelength λ and the reduced mass μ . That is, both $\Delta\lambda$ and $\Delta\mu$ are (P.T.O.)

(43) (cont'd)

considered to be positive, even though an increase in μ , for instance, leads to a decrease in λ .

From equation (4-22), $\lambda = \left[Z^2 R \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \right]^{-1}$, where we use the exact expression for R , with the electron mass m in equation (4-23) replaced by the reduced mass μ of the electron-nucleus system: $R = \frac{\mu k^2 e^4}{4\pi c \hbar^3}$.

From calculus, we know that $\Delta\lambda \approx \left| \frac{d\lambda}{d\mu} \right| \Delta\mu$. Using the equation for λ from above, we have

$$\frac{d\lambda}{d\mu} = \left[Z^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \right]^{-1} \frac{d(R^{-1})}{d\lambda} = \left[Z^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \right]^{-1} (-R^{-2}) \frac{dR}{d\mu}.$$

But from the equation for R from above, we see

that, since $R \propto \mu$, $\frac{dR}{d\mu} = \frac{R}{\mu}$. Therefore the equation above

$$\text{gives } \left| \frac{d\lambda}{d\mu} \right| = \left[Z^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \right]^{-1} \left(\frac{1}{R^2} \right) \left(\frac{R}{\mu} \right) = \frac{1}{\mu} \left[Z^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) R \right]^{-1} = \frac{\lambda}{\mu},$$

$$\text{so } \Delta\lambda \approx \left| \frac{d\lambda}{d\mu} \right| \Delta\mu = \frac{\lambda}{\mu} \Delta\mu \Rightarrow \boxed{\frac{\Delta\lambda}{\lambda} = \frac{\Delta\mu}{\mu}}. \quad \text{Q.E.D.}$$

$$\mu_H = \frac{m_e m_p}{m_e + m_p} \quad \text{and} \quad \mu_D = \frac{m_e m_D}{m_e + m_D}.$$

$$\begin{aligned} \therefore \frac{\Delta\mu}{\mu} &= \frac{\mu_D - \mu_H}{\mu_H} = \frac{\mu_D}{\mu_H} - 1 = \frac{m_e m_D (m_e + m_p)}{m_e m_p (m_e + m_D)} - 1 \\ &= \frac{m_D (m_e + m_p) - m_p (m_e + m_D)}{m_p (m_e + m_D)} = \frac{m_e (m_D - m_p)}{m_p (m_e + m_D)}. \end{aligned}$$

If we approximate $m_D = 2m_p$ and $m_e \ll m_D$, this becomes

$$\frac{\Delta\mu}{\mu} \approx \frac{m_e (2m_p - m_p)}{m_p (0 + 2m_p)} = \frac{m_e}{2m_p}. \quad \text{Then } \frac{\Delta\lambda}{\lambda} = \frac{\Delta\mu}{\mu} \text{ implies}$$

(P.T.O.)

(43) (cont'd)

that $\Delta\lambda = \lambda \left(\frac{\Delta\mu}{\mu} \right) \approx \lambda \left(\frac{m_e}{2m_p} \right) \approx \underbrace{(656.3 \text{ nm})}_{\text{(for hydrogen)}} \left(\frac{0.511 \text{ MeV}/c^2}{2 \times 938.28 \text{ MeV}/c^2} \right)$

$\Rightarrow \boxed{\Delta\lambda \approx 0.179 \text{ nm}}$

Note: Taking into account the signs of the changes, as well as their magnitudes, we have $\frac{\Delta\lambda}{\lambda} = -\frac{\Delta\mu}{\mu}$, so $\Delta\lambda = -0.179 \text{ nm} = \lambda_D - \lambda_H$ in the case above. The corresponding Balmer red line for deuterium therefore has the wavelength $\lambda_D = \lambda_H + \Delta\lambda = 656.3 \text{ nm} - 0.179 \text{ nm} \approx 656.12 \text{ nm}$.

(45)

4-45. The Li^{2+} ion is essentially identical to the H atom in Bohr's theory, aside from the effect of the different nuclear charges and masses. (a) What transitions in Li^{2+} will yield emission lines whose wavelengths are very nearly equal to the first two lines of the Lyman series in hydrogen? (b) Calculate the difference between the wavelength of the Lyman α line of hydrogen and the emission line from Li^{2+} that has very nearly the same wavelength.

4-45. (a) $E_n = -E_0 Z^2/n^2$ (Equation 4-20)

For Li^{++} , $Z = 3$ and $E_n = -13.6 \text{ eV}(9)/n^2 = -122.4/n^2 \text{ eV}$

The first three Li^{++} levels that have the same (nearly) energy as H are:

$n = 3, E_3 = -13.6 \text{ eV}$ $n = 6, E_6 = -3.4 \text{ eV}$ $n = 9, E_9 = -1.51 \text{ eV}$

Lyman α corresponds to the $n = 6 \rightarrow n = 3 \text{ Li}^{++}$ transition. Lyman β corresponds to the $n = 9 \rightarrow n = 3 \text{ Li}^{++}$ transition.

That is

Li^{++}	has same energy as	H
$n=3$		$n=1$
$n=6$		$n=2$
$n=9$		$n=3$
	etc.	

(b) $R(H) = R_\infty (1/(1 + 0.511 \text{ MeV}/938.3 \text{ MeV})) = 1.096776 \times 10^7 \text{ m}^{-1}$

$R(\text{Li}) = R_\infty (1/(1 + 0.511 \text{ MeV}/6535 \text{ MeV})) = 1.097287 \times 10^7 \text{ m}^{-1}$

For Lyman α :

$\frac{1}{\lambda} = R(H) \left(1 - \frac{1}{2^2} \right) = 1.096776 \times 10^7 \text{ m}^{-1} (10^{-9} \text{ m/nm}) (3/4) \rightarrow \lambda = 121.568 \text{ nm}$

For Li^{++} equivalent:

$\frac{1}{\lambda} = R(\text{Li}) \left(\frac{1}{3^2} - \frac{1}{6^2} \right) Z^2 = 1.097287 \times 10^7 \text{ m}^{-1} (10^{-9} \text{ m/nm}) \left(\frac{1}{9} - \frac{1}{36} \right) (3)^2$

$\lambda = 121.512 \text{ nm}$

$\Delta\lambda = 0.056 \text{ nm}$

} using equation 4-26

rest energy of Li nucleus = $7.016003 \text{ (931.49432 MeV)}$ $\frac{1}{c^2}$ [u... unified mass unit]
see Appendix A

(48)

4-48, Figure 3-15b shows the K_α and K_β characteristic x rays emitted by a molybdenum (Mo) target in an x-ray tube whose accelerating potential is 35 kV. The wavelengths are $K_\alpha = 0.071 \text{ nm}$ and $K_\beta = 0.063 \text{ nm}$. (a) Compute the corresponding energies of these photons. (b) Suppose we wish to prepare a beam consisting primarily of K_α x rays by passing the molybdenum x rays through a material that absorbs K_β x rays more strongly than K_α x rays by photoelectric effect on K-shell electrons of the material. Which of the materials listed in the accompanying table with their K-shell binding energies would you choose? Explain your answer.

Element	Zr	Nb	Mo	Tc	Ru
Z	40	41	42	43	44
E_K (keV)	18.00	18.99	20.00	21.04	22.12

4-48. (a) $E_\alpha = hc/\lambda = 1240 \text{ eV}\cdot\text{nm} / 0.071 \text{ nm} = 17.465 \text{ keV}$

$E_\beta = hc/\lambda = 1240 \text{ eV}\cdot\text{nm} / 0.063 \text{ nm} = 19.683 \text{ keV}$

(b) Select Nb ($Z=41$)

The K_β Mo X rays have enough energy to eject photoelectrons, producing 0.693 keV electrons. The K_α Mo X rays could not produce photoelectrons in Nb.

(Note: $\underbrace{19.683 \text{ keV}}_{E_\beta} - \underbrace{18.99 \text{ keV}}_{E_K \text{ for Nb}} = \underbrace{0.693 \text{ keV}}_{\text{K.E. of emitted photoelectron}}$)

(52)

4-52. In this problem you are to obtain the Bohr results for the energy levels in hydrogen without using the quantization condition of Equation 4-17. In order to relate Equation 4-14 to the Balmer-Ritz formula, assume that the radii of allowed orbits are given by $r_n = n^2 r_0$, where n is an integer and r_0 is a constant to be determined. (a) Show that the frequency of radiation for a transition from $n_f = n - 1$ is given by $f \approx kZe^2/hr_0 n^3$ for large n . (b) Show that the frequency of revolution is given by

$$f_{\text{rev}}^2 = \frac{kZe^2}{4\pi^2 m r_0^3 n^6}$$

(c) Use the correspondence principle to determine r_0 and compare with Equation 4-19.

(a) Using eqns 4-14 and 4-15 with $r_i = r_n = n^2 r_0$ and

$r_f = r_{n-1} = (n-1)^2 r_0$, we have

$$f = -\frac{1}{2} \frac{kZe^2}{h} \left(\frac{1}{n^2 r_0} - \frac{1}{(n-1)^2 r_0} \right) = \frac{kZe^2}{2hr_0} \left(\frac{1}{(n-1)^2} - \frac{1}{n^2} \right)$$

$$= \frac{kZe^2}{2hr_0} \left[\frac{n^2 - (n-1)^2}{n^2 (n-1)^2} \right] = \frac{kZe^2}{2hr_0} \left[\frac{n^2 - (n^2 - 2n + 1)}{n^2 (n-1)^2} \right]$$

$$\Rightarrow f = \frac{kZe^2}{2hr_0} \left[\frac{2n-1}{n^2 (n-1)^2} \right] \approx \frac{kZe^2}{2hr_0} \left[\frac{2n}{n^4} \right] \text{ for } n \gg 1.$$

we find

$$f \approx \frac{kZe^2}{hr_0 n^3} \text{ for large } n.$$

Q.E.D.

(52) (cont'd)

(b) The period of revolution is given by $T_{\text{rev}} = \frac{\text{distance travelled}}{\text{speed}} = \frac{2\pi r}{v}$,
 so $f_{\text{rev}} = \frac{1}{T_{\text{rev}}} = \frac{v}{2\pi r}$, and

$$f_{\text{rev}}^2 = \frac{v^2}{4\pi^2 r^2} = \frac{1}{4\pi^2 m r} \cdot \frac{mv^2}{r} \stackrel{\text{(by eqn. for centripetal force, p 159)}}{=} \frac{1}{4\pi^2 m r} \cdot \frac{kZe^2}{r^2} = \frac{kZe^2}{4\pi^2 m r^3}.$$

But $r = n^2 r_0$, so $f_{\text{rev}}^2 = \frac{kZe^2}{4\pi^2 m r_0^3 n^6}$. Q.E.D.

(c) According to the correspondence principle, in the limit of large orbits (i.e. in the limit of large quantum number n , since $r = n^2 r_0$) we should recover the classical result that frequency of revolution equals frequency of radiation. We therefore equate our results for f^2 from (a) and f_{rev}^2 from (b):

$$f^2 = f_{\text{rev}}^2 \Rightarrow \frac{k^2 Z^2 e^4}{h^2 r_0^2 n^6} = \frac{kZe^2}{4\pi^2 m r_0^3 n^6} \Rightarrow r_0 = \frac{h^2}{4\pi^2 m k Z e^2}$$

$$\Rightarrow \boxed{r_0 = \frac{h^2}{m Z k e^2}}.$$

This reduces to the result of equation (4-19): $a_0 = \frac{h^2}{m k e^2}$,
 for the case of hydrogen ($Z = 1$). Using $r = n^2 r_0$, with the value of r_0 found above, in eqn. (4-14) gives the energy levels of equation (4-20). Q.E.D.