



**The Abdus Salam  
International Centre for Theoretical Physics**



**2332-26**

**School on Synchrotron and FEL Based Methods and their Multi-Disciplinary  
Applications**

*19 - 30 March 2012*

**History and fundamentals of coherent diffraction imaging**

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# History and fundamentals of coherent diffraction imaging

Janos Kirz  
ALS Berkeley

- Motivation
- Basic ideas
- Coherence
- The phase problem
- Solutions - holography
- Solutions - Diffraction microscopy
- Prior knowledge
- The apparatus
- First experiments
- Challenges

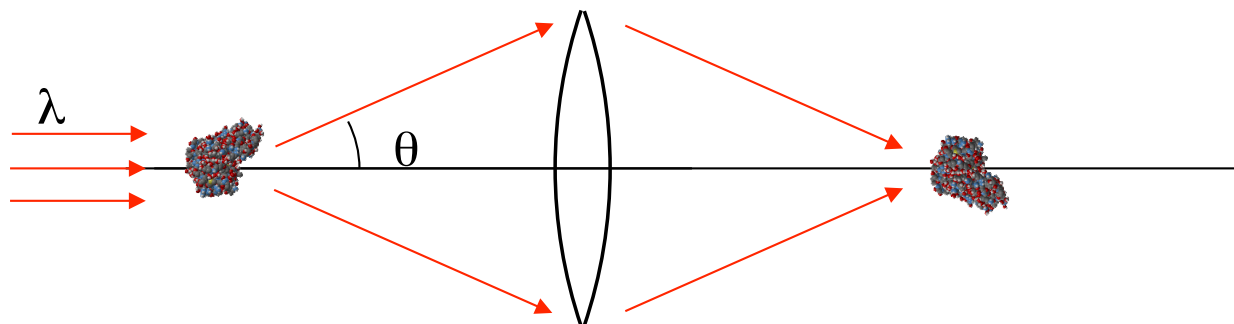
- A technique for 3D imaging of 0.5 - 20  $\mu\text{m}$  isolated objects
- Too thick for EM (0.5  $\mu\text{m}$  is practical upper limit)
- Too thick for tomographic X-ray microscopy (depth of focus  $< 1 \mu\text{m}$  at 10 nm resolution for soft X-rays even if lenses become available)
- Flash imaging: (Chapman lectures this afternoon)

## Goals @ synchrotrons

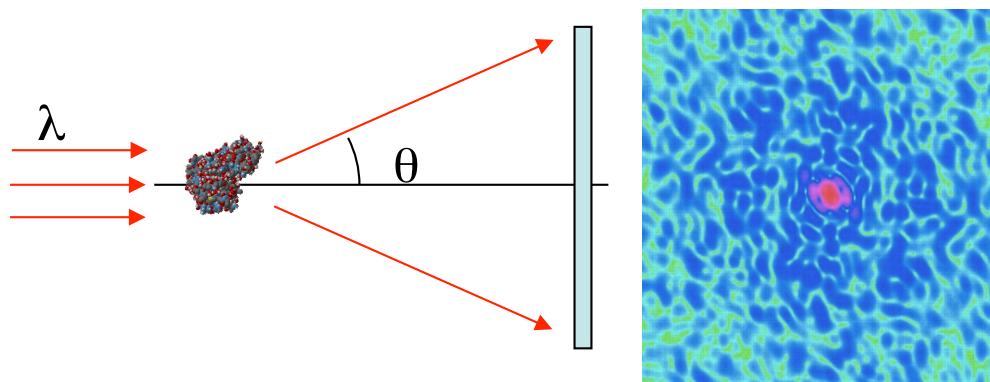
- 10 nm resolution (3D) in 1 - 10  $\mu\text{m}$  size biological specimens  
(small frozen hydrated cell, organelle; see macromolecular aggregates)  
Limitation: radiation damage!
- $< 4$  nm resolution in less sensitive nanostructures  
(Inclusions, porosity, clusters, composite nanostructures, aerosols...)  
eg: molecular sieves, catalysts, crack propagation



A lens recombines scattered rays with correct phases to form the image



Lenses have limitations. Do we really need them?



If you record the diffraction pattern, you lose the phase

Resolution:  $\delta = \lambda / \sin \theta$

Image  $\rightarrow$  Fourier transform  $\rightarrow$  zero magnitude or phase  $\rightarrow$  inverse Fourier transform



Malcolm Howells at  
La Clusaz

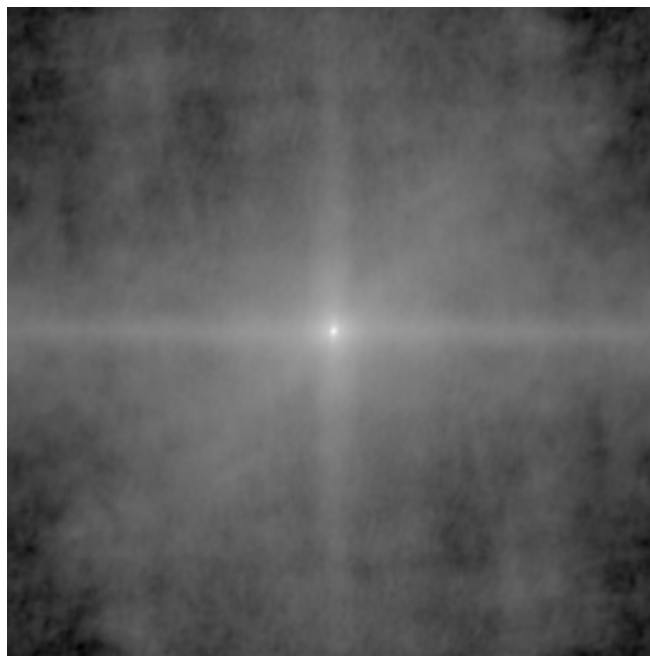


Image using only  
Fourier magnitudes



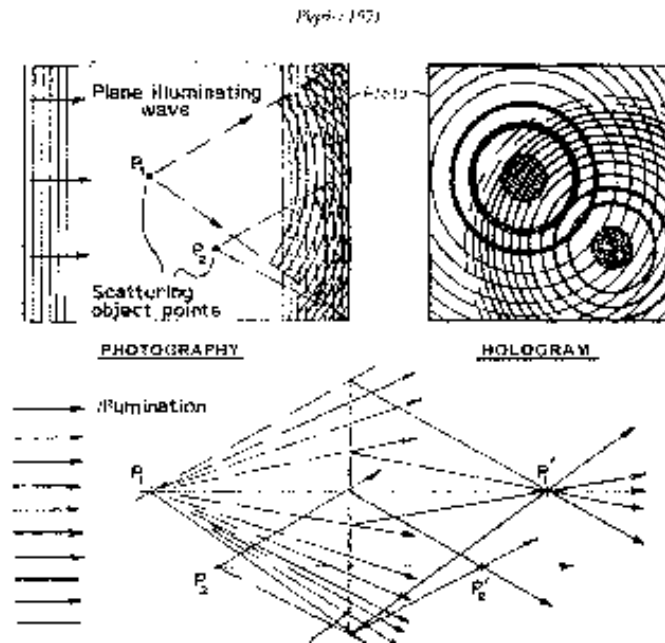
Image using only  
Fourier phases

C. Jacobsen

# Image reconstruction from the diffraction pattern

- Lenses do it, mirrors do it
  - but they use the full complex amplitude!
- Recording the diffraction *intensity* leads to the “phase problem”!
- Holographers do it - but they mix in a reference wave, need very high resolution detector or similar precision apparatus
- Crystallographers do it - but they use MAD, isomorphous replacement, or other tricks (plus the amplification of many repeats)

- Gabor Nobel lecture 1971
- Gabor in-line holography



# First holography experiment with synchrotron radiation: Aoki, Ichihara & Kikuta, 1972



Fig. 2. An X-ray hologram.

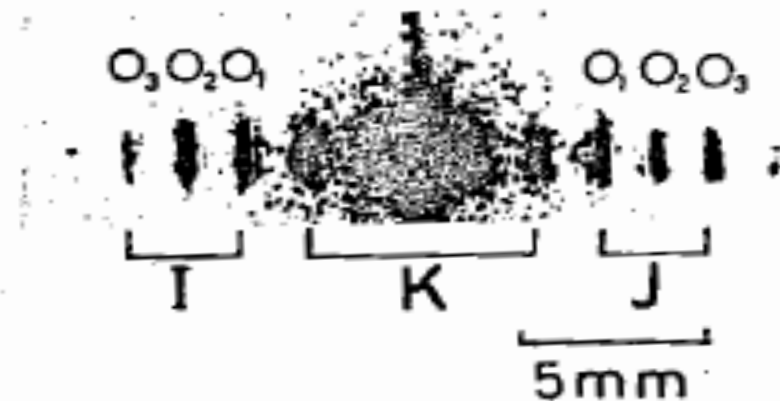
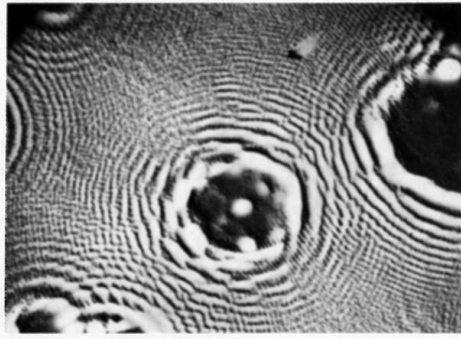
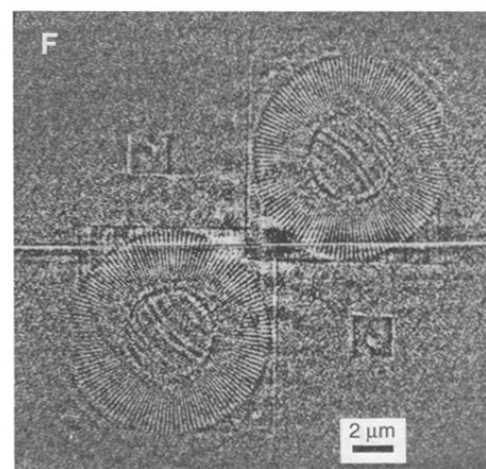
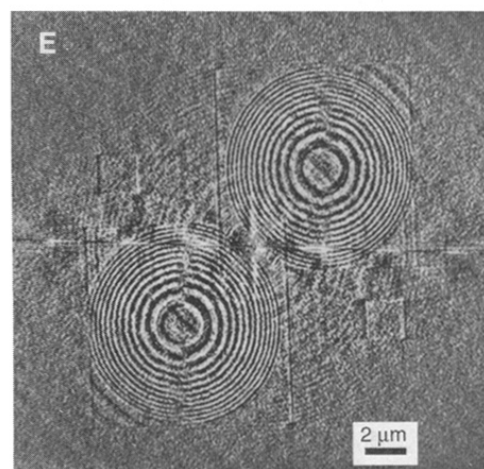
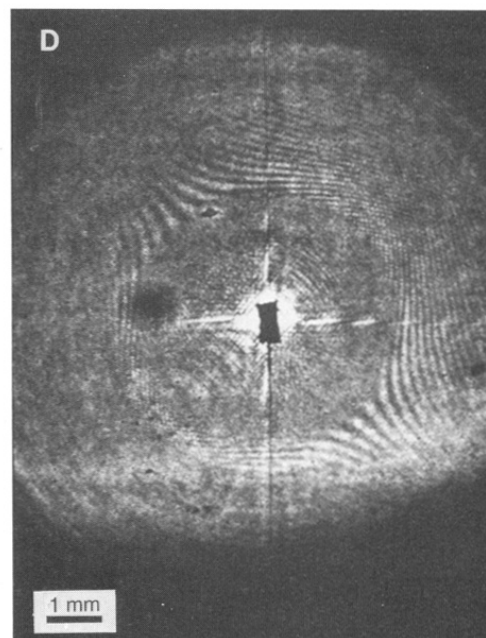


Fig. 3. Reconstructed images I and J, and the noise image K.

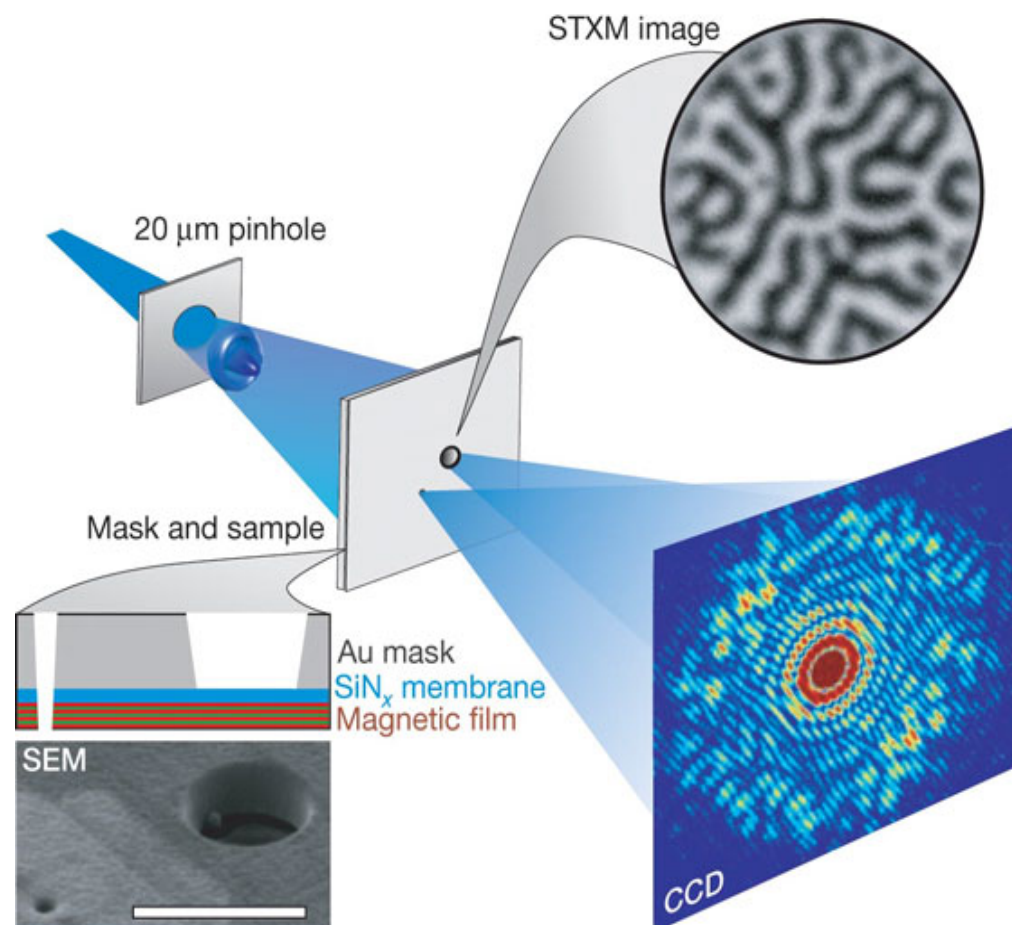


- Gabor holography
  - Encodes phase in fringes/speckles
  - Mimic reconstruction by computer
  - Requires high resolution detector
  - Aoki, Ichihara & Kikuta JJAP 11, 1847 (1972)
  - Howells, et al., Science 238, 514, (1987)
  - Not used much for high resolution imaging
- Fourier transform holography
  - Spherical reference wave spreads speckles
  - Simple reconstruction by inverse FT
  - How to get spherical reference?
  - McNulty et al., Science 256, 1009 (1992)





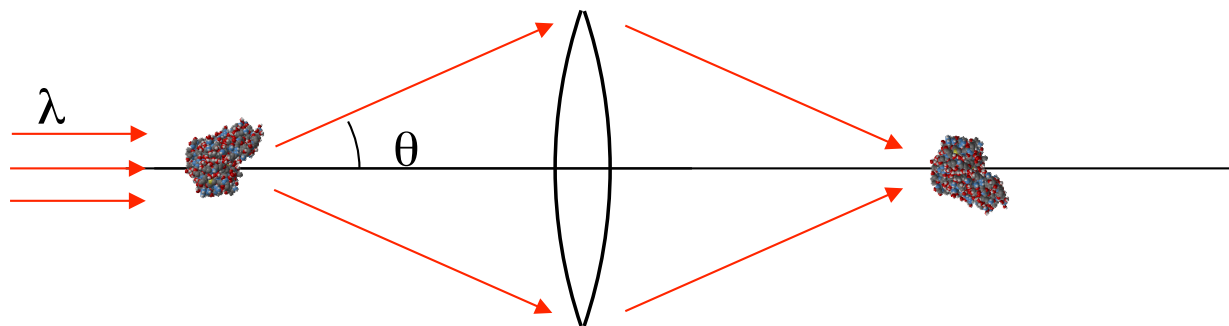
S. Eisebitt, J. Lüning, W. F. Schlotter, M. Lörngen, O. Hellwig, W. Eberhardt and J. Stöhr *Nature* 432, 885-888(2004)



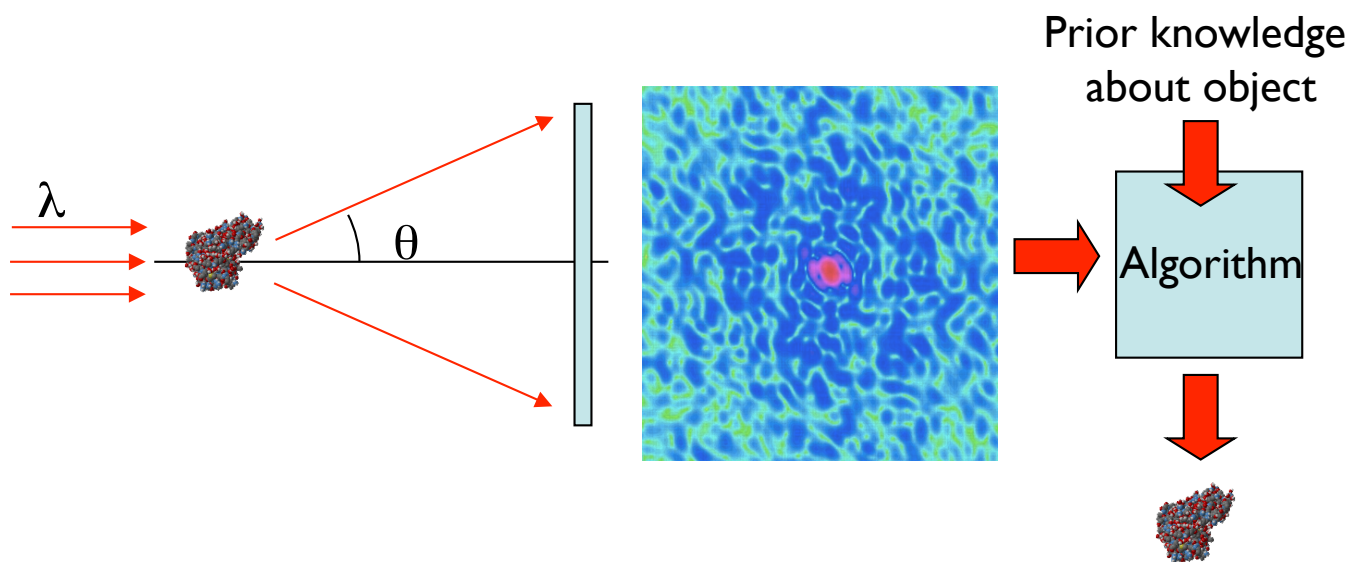


- Size of pinhole sets resolution
- How to get enough photons through?
- Do we really need a reference wave?

Use a computer to phase the scattered light, rather than a lens



A lens recombines the scattered rays with correct phases to give the image



An algorithm finds the phases that are consistent with measurements and prior knowledge

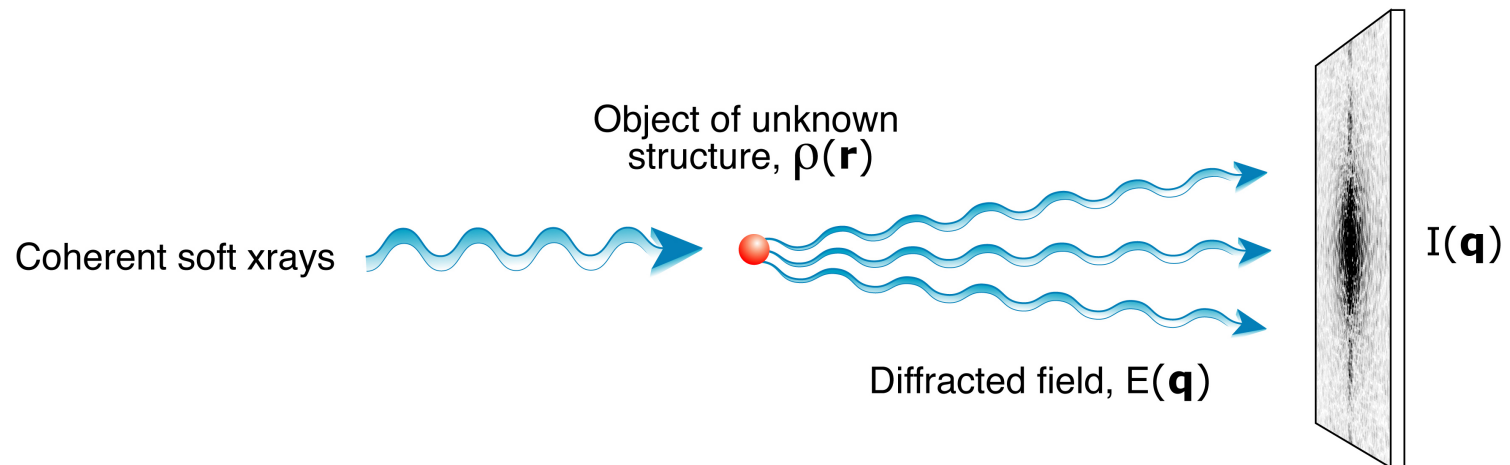
Resolution:  $\delta = \lambda / \sin \theta$

Idea of David Sayre

- Single object, plane wave incident, scattered amplitude is Fourier transform of (complex) electron density  $f(\mathbf{r})$

$$F(\mathbf{k}) = \int f(\mathbf{r}) e^{-2\pi i \mathbf{k} \cdot \mathbf{r}} d\mathbf{r}$$

- Assume: Born Approximation
- Assume coherent illumination

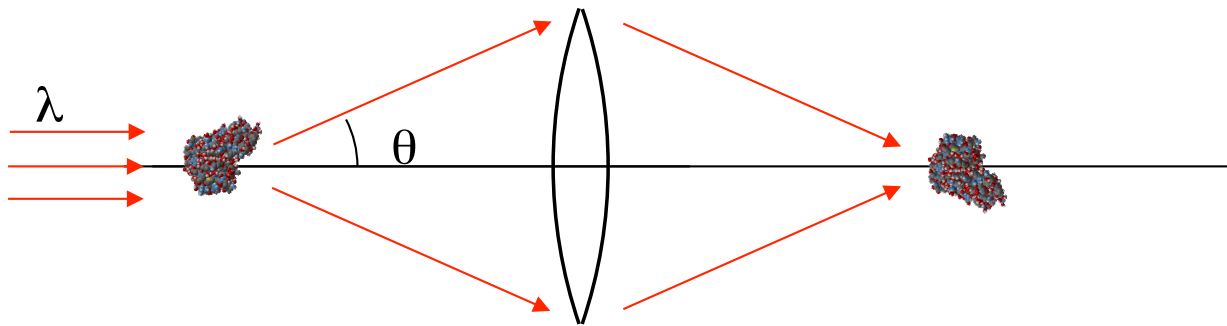


- Life before lasers
- Temporal coherence
  - spectral lines or grating monochromators
  - measure of temporal coherence:  $\lambda/\Delta\lambda$
- Spatial coherence (plane or spherical waves)
  - slits or pinholes “spatial filter”
  - Impose  $\Delta x \cdot \Delta\theta < \lambda$  in each dimension
  - As gets  $\lambda$  shorter, acceptance becomes smaller

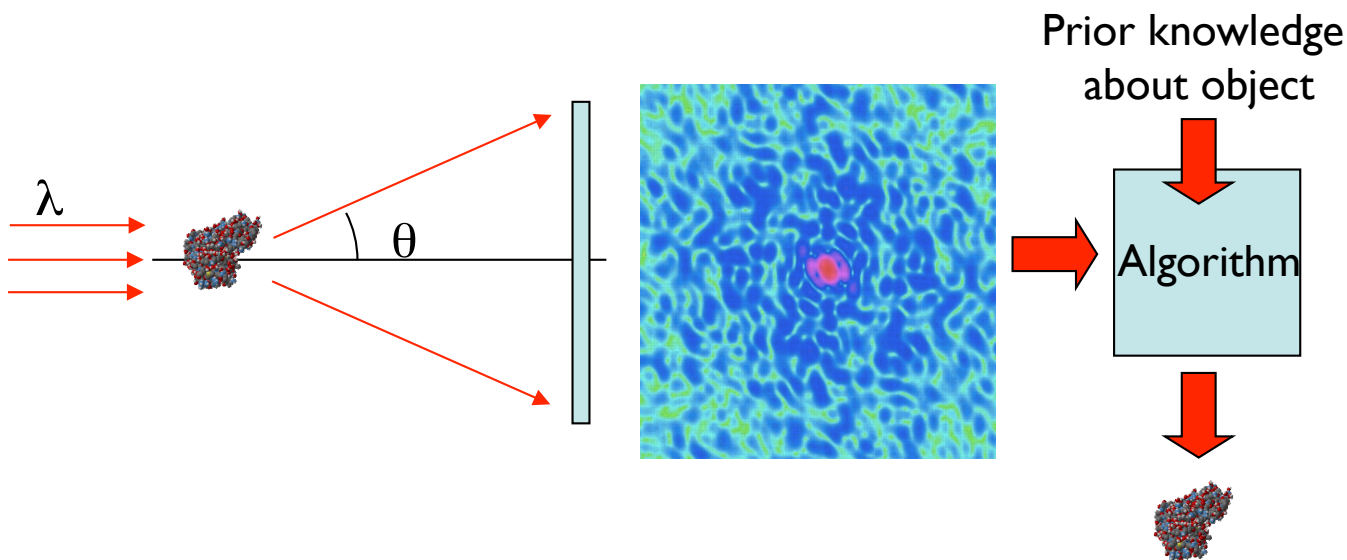
- X-ray tubes
  - electron bombardment of solid target
- Synchrotron light sources
  - bending magnets
  - wigglers
  - undulators
- High harmonic generation
- Free electron lasers

	$\Delta x \cdot \Delta y$	$\Delta \Omega$
• X-ray tubes	$0.1\text{mm}^2 \cdot 4\pi$	$\sim 10^{14}\lambda^2$
• Synchrotron light sources (bend magn)	$0.01\text{mm}^2 \cdot 10^{-6}$	$\sim 10^6\lambda^2$
• Undulators	$0.01\text{mm}^2 \cdot 10^{-8}$	$\sim 10^4\lambda^2$
• FELs		Can be mostly coherent

Use a computer to phase the scattered light, rather than a lens



A lens recombines the scattered rays with correct phases to give the image



An algorithm finds the phases that are consistent with measurements and prior knowledge

Resolution:  $\delta = \lambda / \sin \theta$

Idea of David Sayre

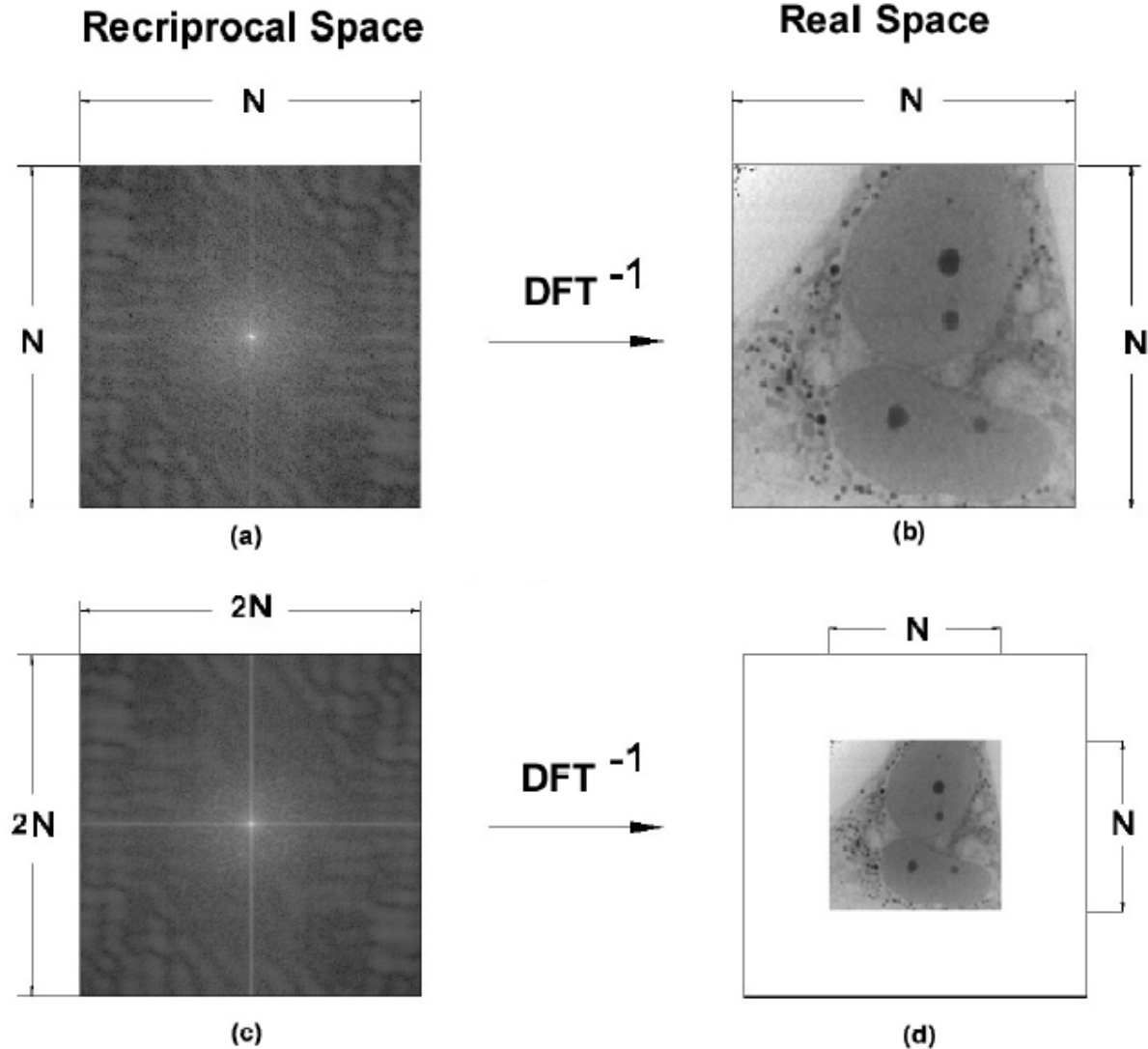
“Oversampling”:

Non-crystals:  
pattern continuous,  
can do finer sampling  
of intensity

Finer sampling;  
larger array;  
smaller transform;  
“finite support”

(area around specimen  
must be clear!)

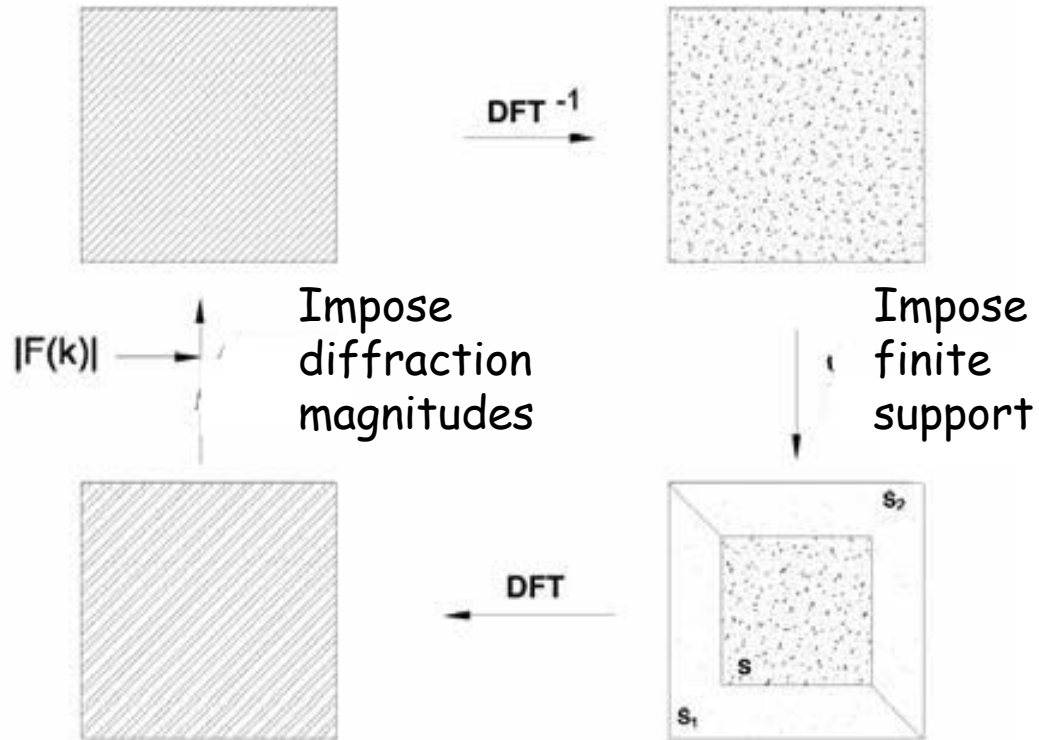
Miao thesis





Equations can still not be solved analytically

Fienuip iterative algorithm  
Reciprocal space                      Real space



•Positivity of electron density helps!

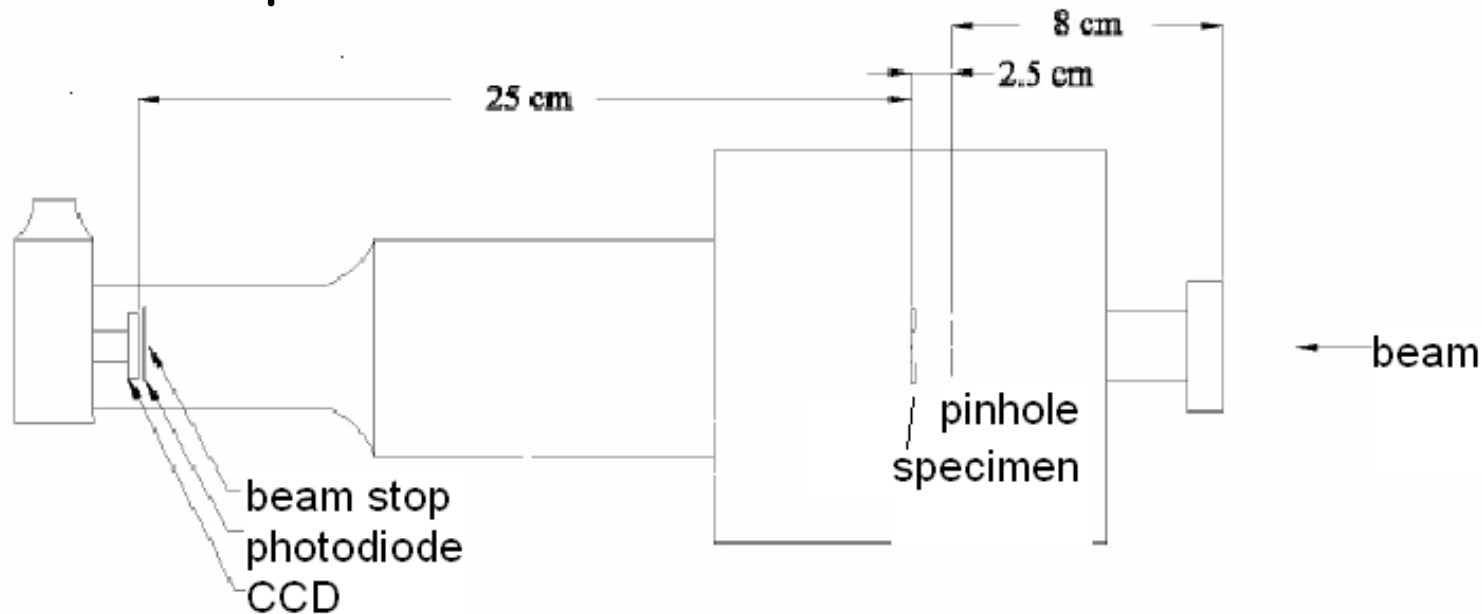
Miao thesis

3/28/12

20

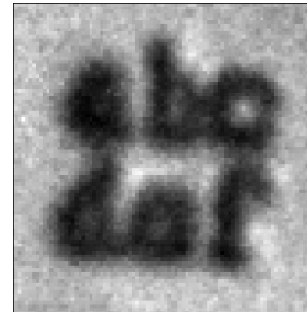
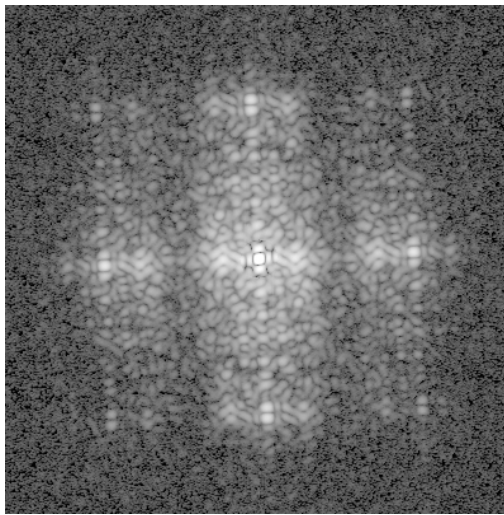
- Sayre 1952: Shannon sampling theorem in crystallography
- Gerchberg & Saxton, 1971: iterative phase retrieval algorithm in EM
- Sayre 1980: pattern stronger with soft X-rays; use SR to work without xtals!
- Fienup 1982: Hybrid Input-Output, support
- Bates 1982: 2x Bragg sampling gives unique answer for  $\geq 2$  dimensions
- Yun, Kirz & Sayre 1984-87: first experimental attempts

- 1998: Sayre, Chapman, Miao: oversampling & Fienup algorithm for X-rays
- 1999: first experimental demonstration in 2D



Data collected at NSLS beamline X1B

$\lambda=1.8$  nm  
soft x-ray  
diffraction  
pattern



Low angle data  
From optical  
micrograph

Scanning  
electron  
micrograph  
of object

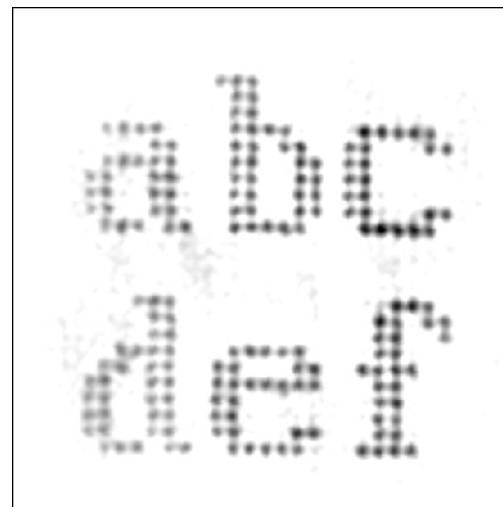
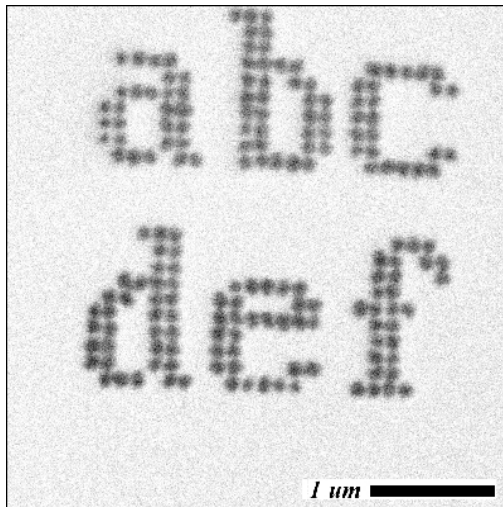
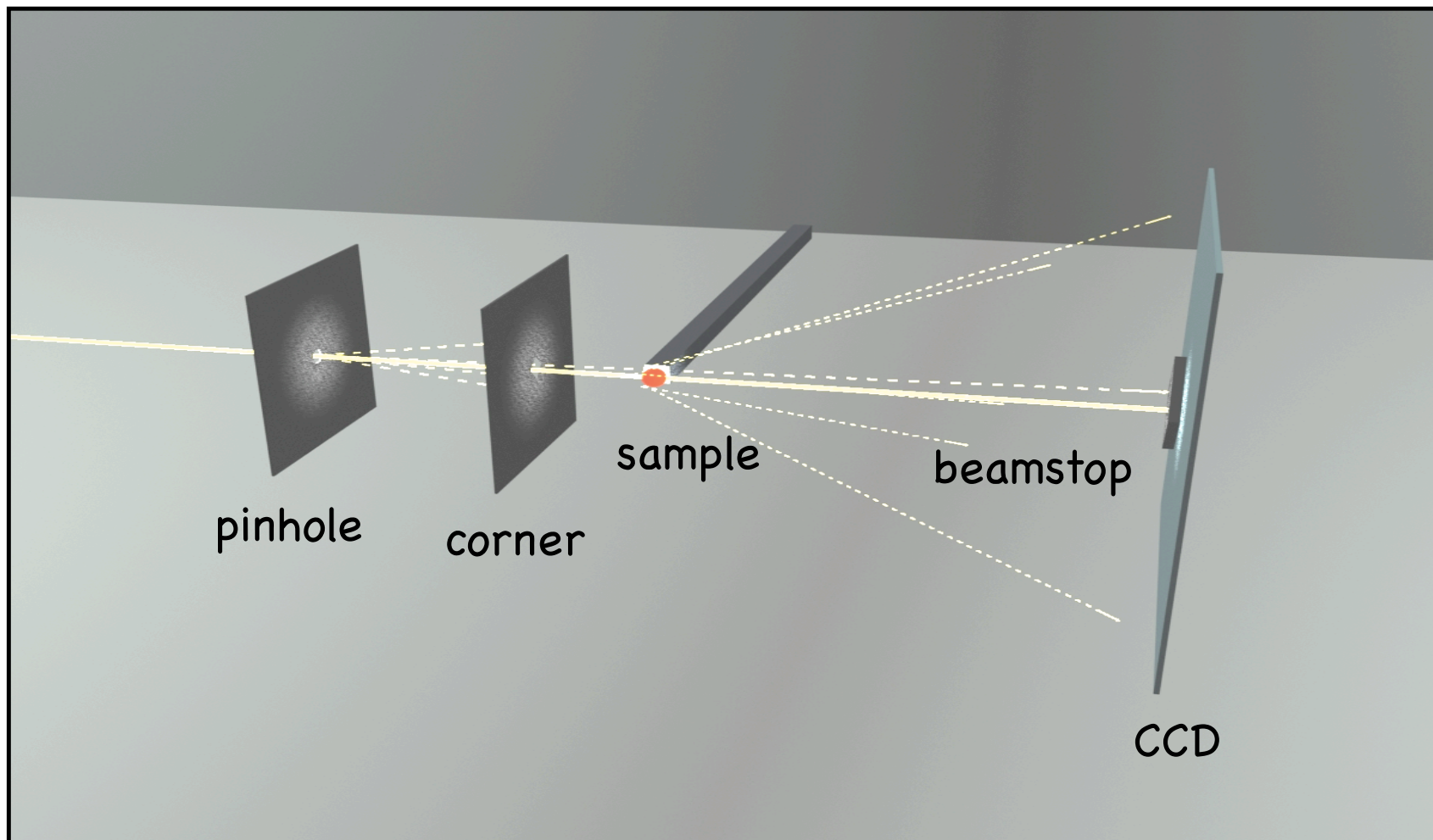


Image  
reconstructed from  
diffraction pattern  
( $\theta_{\max}$  corresponds to  
80 nm). Assumed  
positivity

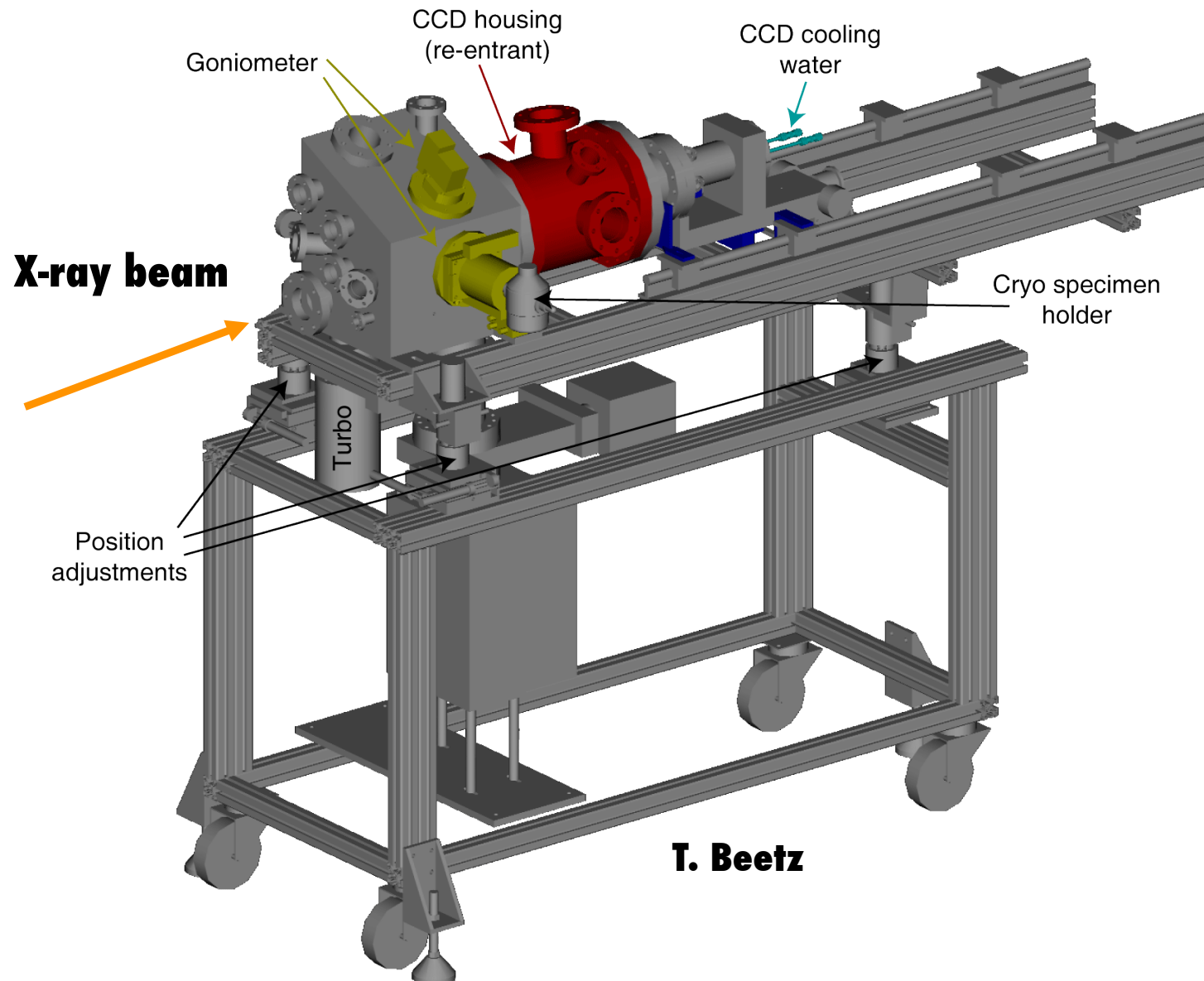
- Collect a high resolution 3D data set in an hour or two
- Reconstruct reliably in a comparable amount of time

- Beamline to supply sufficient coherent photons
  - Eliminate higher orders: aperiodic undulator?
- Shielding detector from all but diffracted signal
- Aligning specimen with small beam-spot,
  - Keeping it aligned as specimen is rotated
- Minimizing missing data
  - (beam stop, large rotation angles, etc.)
- Dynamic range of detector
- Automation of data collection





# Diffraction Microscope by Stony Brook and NSLS





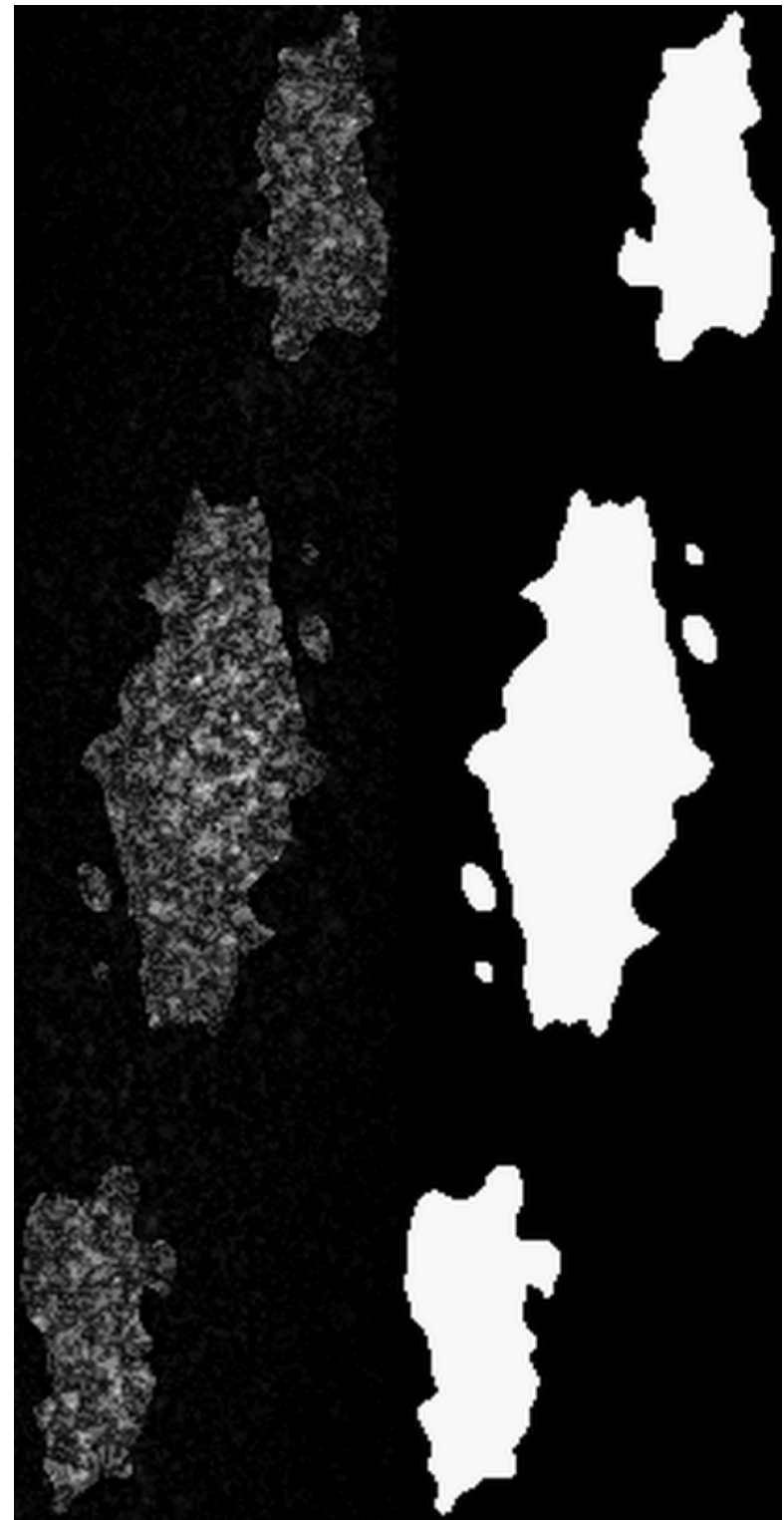
# Gatan 630 cryo holder



- How to avoid stagnation; local minima?
  - The enantiomorph problem
- How to tell whether algorithm converged?
  - (easy when object known...)
  - Multiple random starts
- How to make best use of the data?
  - Of prior knowledge? (Fienup, Elser, Szöke)
- How to optimize use of computer resources?
  - Want many  $1024^3$  DFT
- Much work remains to be done!

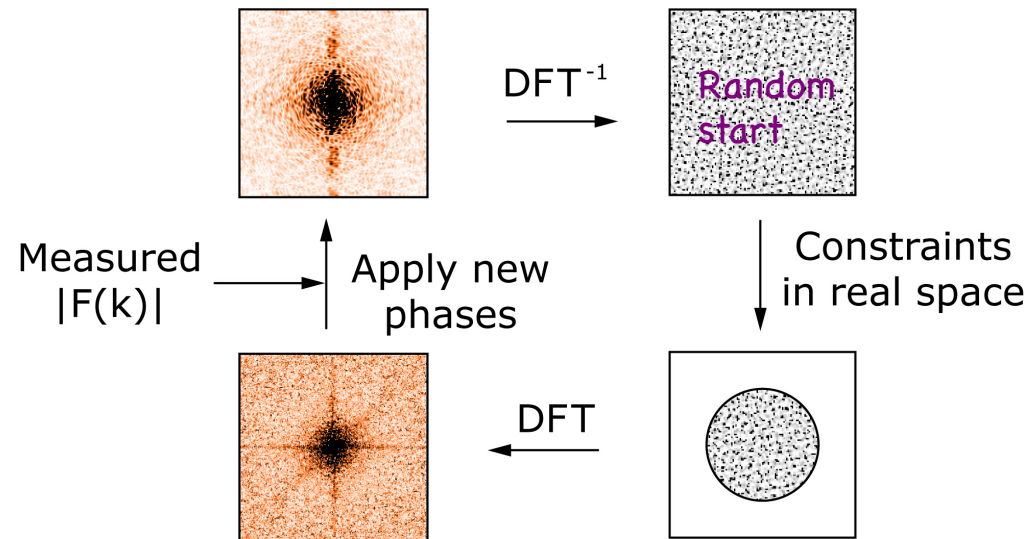
**When rough support is  
not available, it can be  
found from “Shrink-wrap”**

**Marchesini *et al.*, *Phys. Rev. B*  
68, 140101 (2003)**



- **Algorithm starts with an image (random)**
- **Apply projections**
- **Iteratively modify image until converge**

## hybrid input-output



(Fienup, *Appl. Opt.* 21, 2759 (1982))

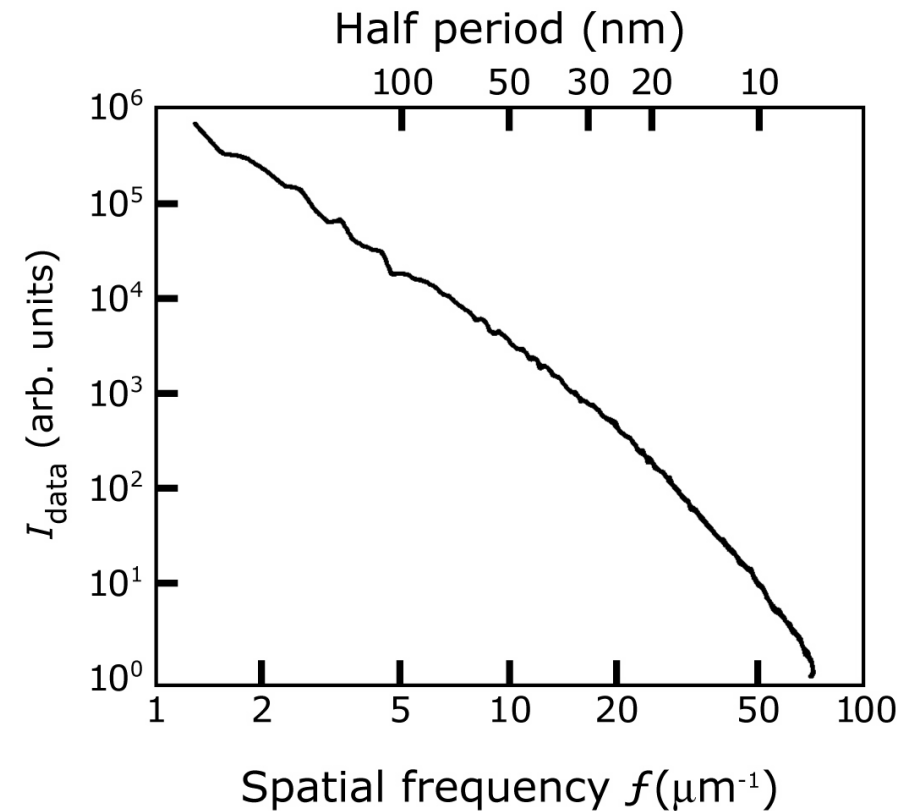
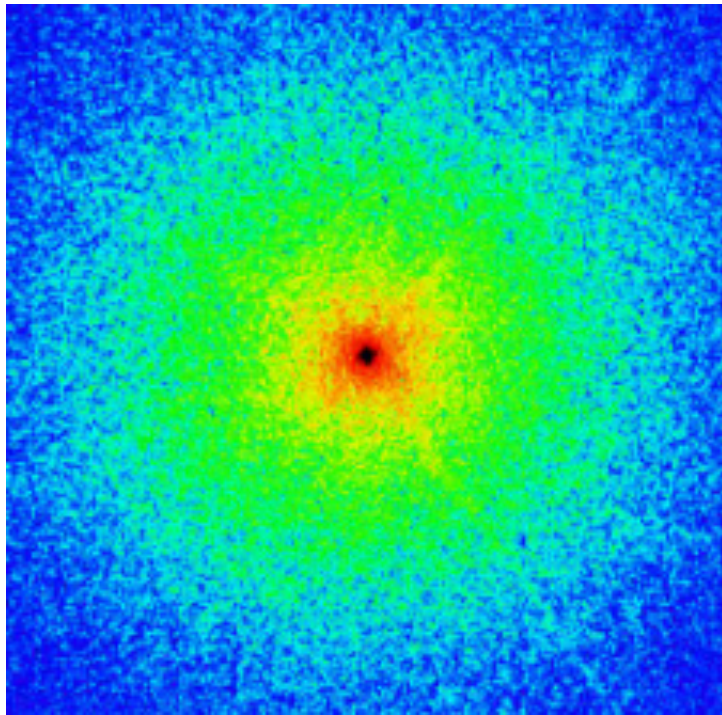
**difference map: Elser, *J. Opt. Soc. Am. A* 4, 118 (2002)**

**by adding the difference of two projections**

- Works perfectly for perfect, complete data
- Algorithm often requires thousands of iterations, stagnates sometimes
  - (Enantiomorph problem)
- Works even better for 3D!
- Real data are rarely perfect, or complete

# Diffraction data and its reconstruction of freeze-dried yeast cell

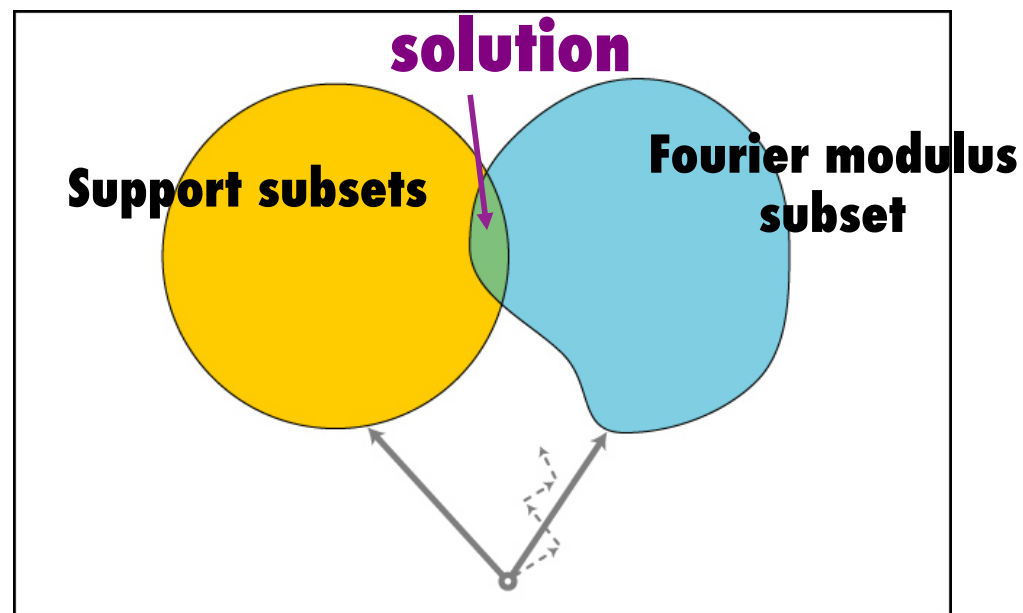
**Yeast cell: 2.5 micron thick, unstained freeze-dried, at 750 eV**  
**Total dose  $\sim 10^8$  Gray (room temperature)**  
**Oversampling is about 5 in each dimension**



**David Shapiro, Stony Brook, now at ALS**

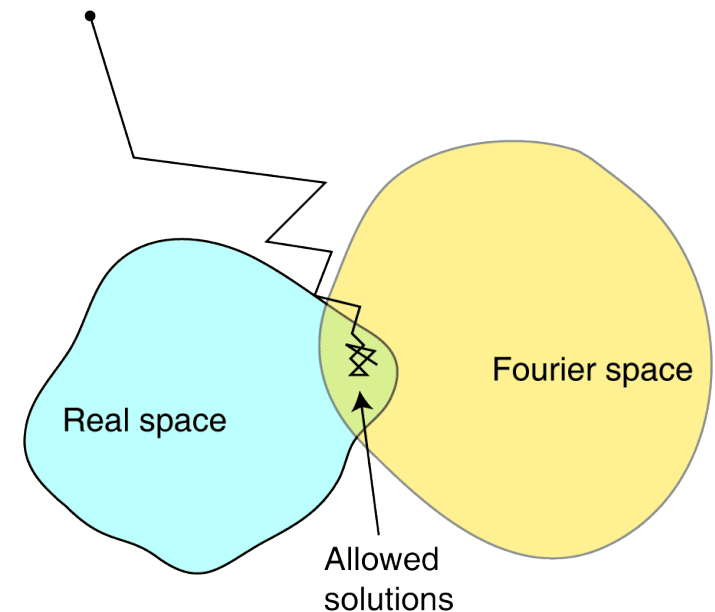
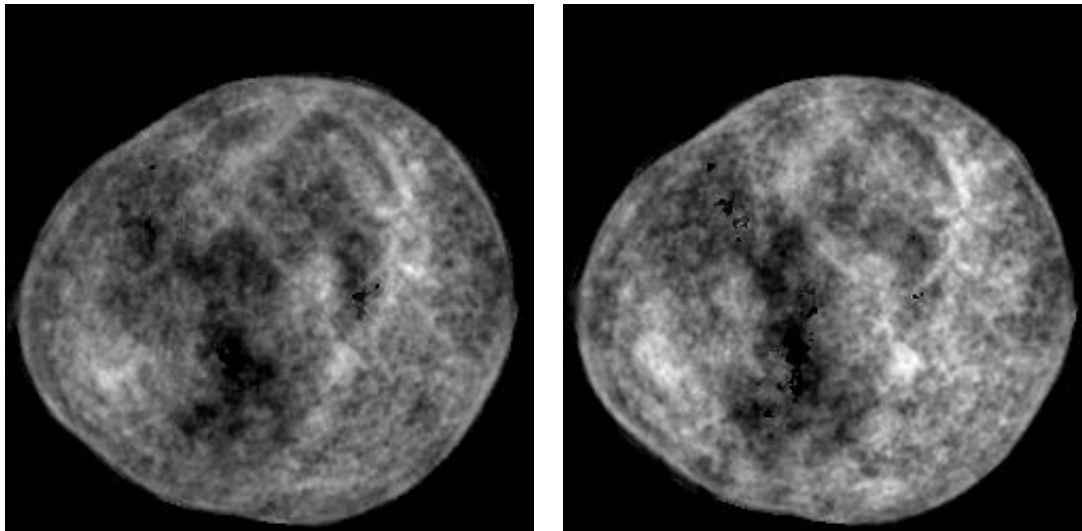
Impose known constraints  
(information about the sample)

1. Impose measured Fourier magnitude
2. Impose sample boundary (support)





**Two images (iterates) separated by 40 iterations**



**Noise in the data gives random fluctuations in the reconstructed image**

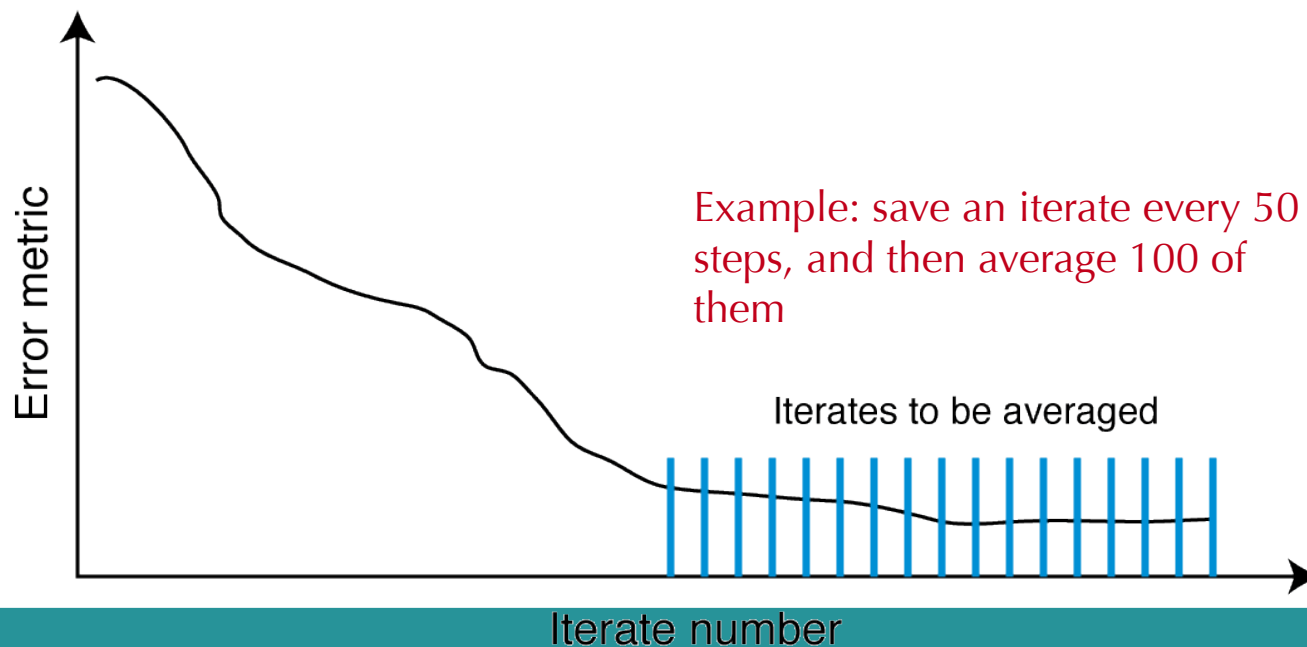
**Averaging many iterates:**

- **reinforce reproducible information**
- **suppress non-reproducible information**

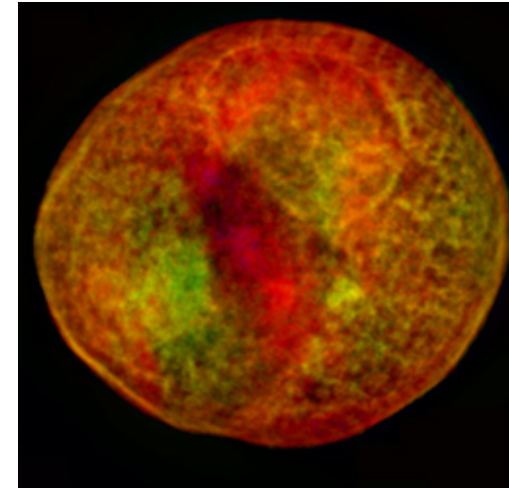
**D. Shapiro et al.,** Biological imaging by soft x-ray diffraction microscopy,  
*PNAS* **102** (43), 15343, (2005)



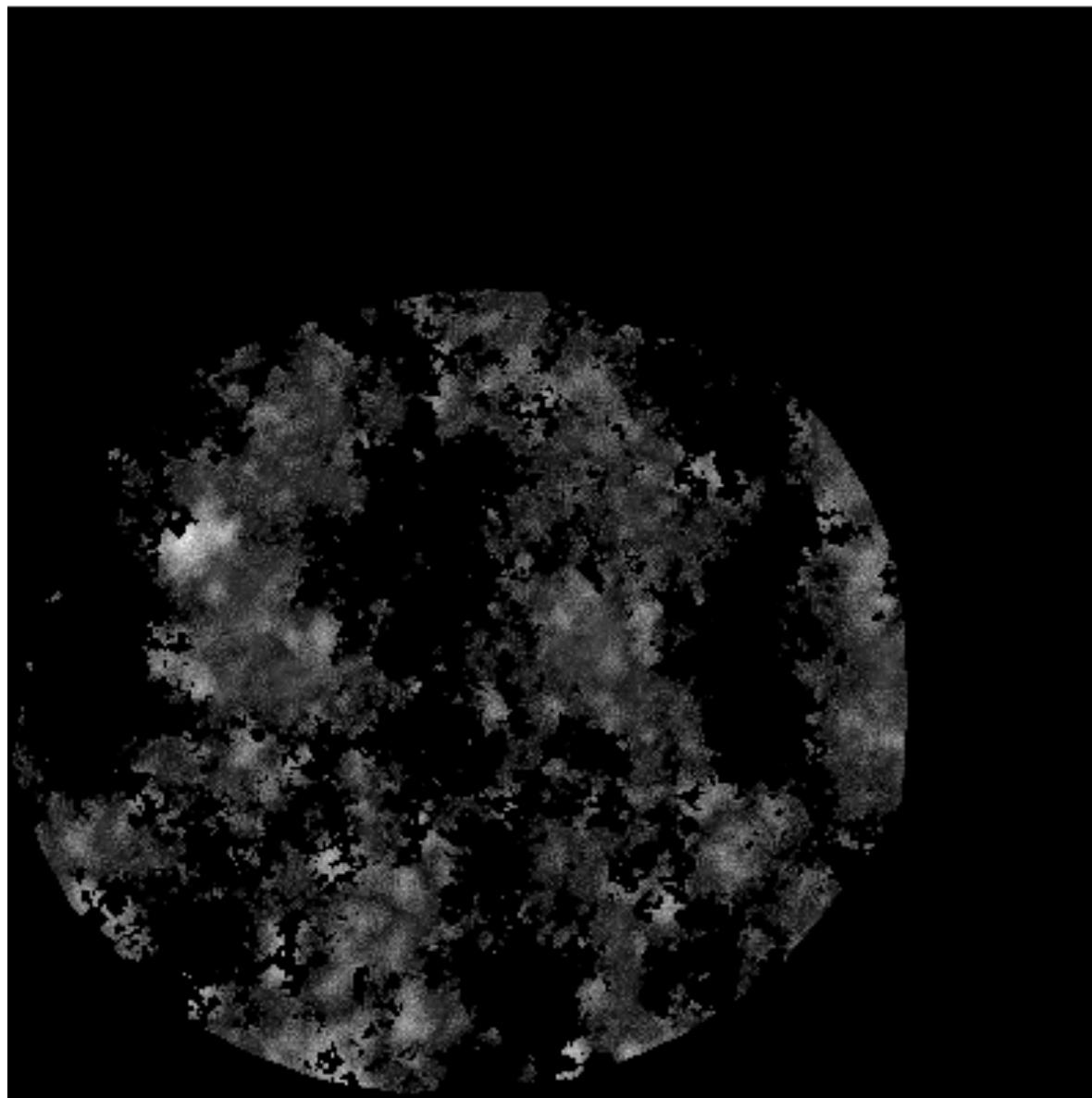
- If the solution fluctuates, let's take many samples and average them!
- Non-reproducible phases get washed out; reproducible phases get reinforced
- Thibault, Elser, Jacobsen, Shapiro, and Sayre, *Acta Crystallographica A* 62, 248 (2006)
- Other approaches: compare results from several different starting random phases (e.g., Miao, Robinson)



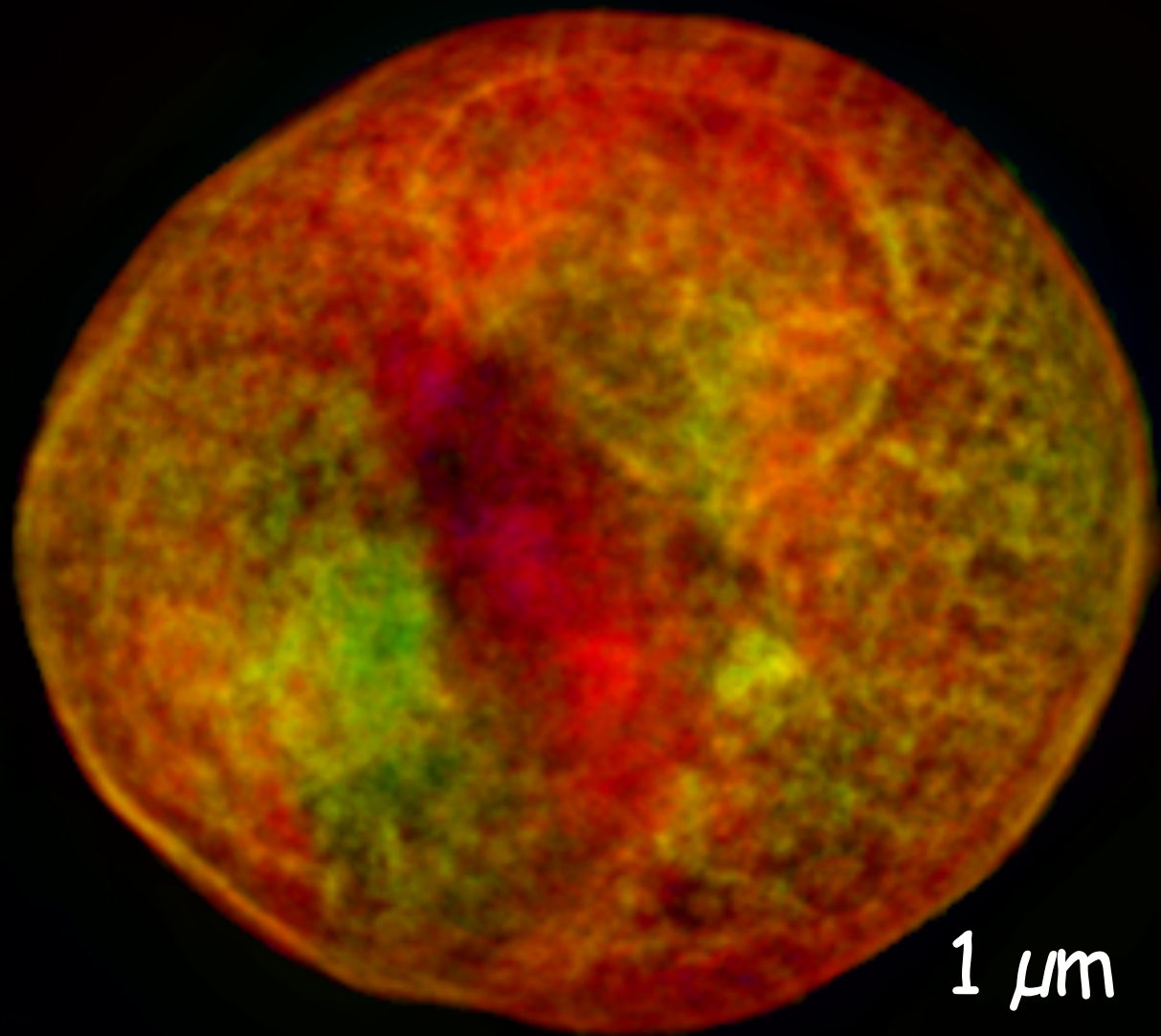
- Final reconstruction was obtained by averaging iterates  
10,000 iterations  
Brightness - amplitude, hue - phase averaged over 100 iterates



# The reconstruction

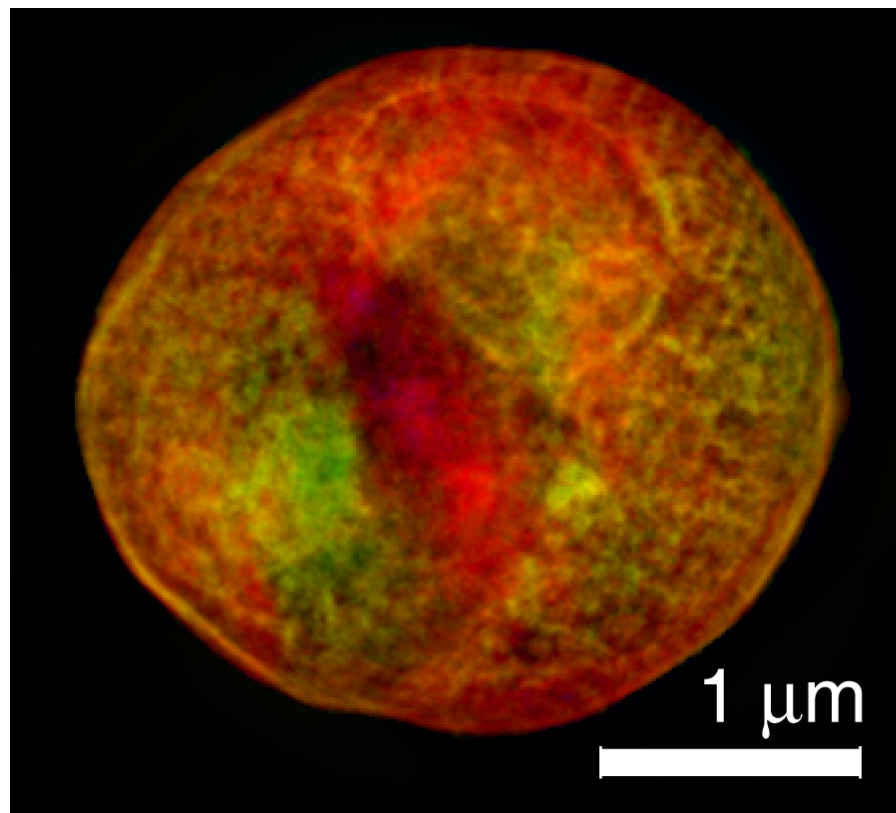


Shapiro *et al.*, *Proc. Nat. Acad. Sci.* **102**, 15343 (2005).

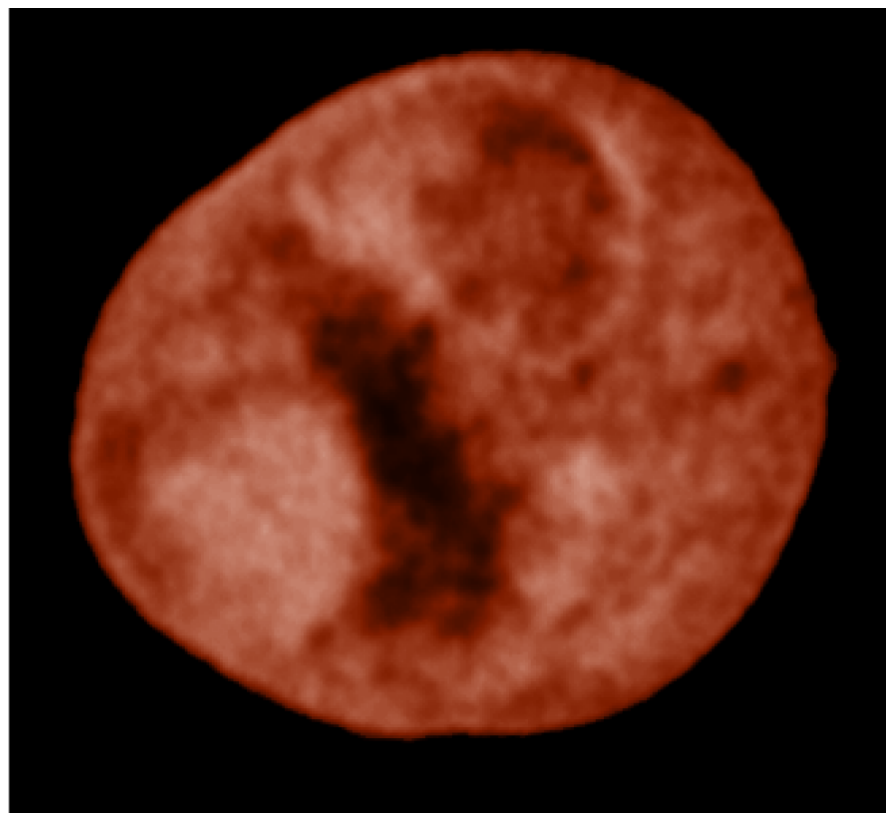


1  $\mu\text{m}$

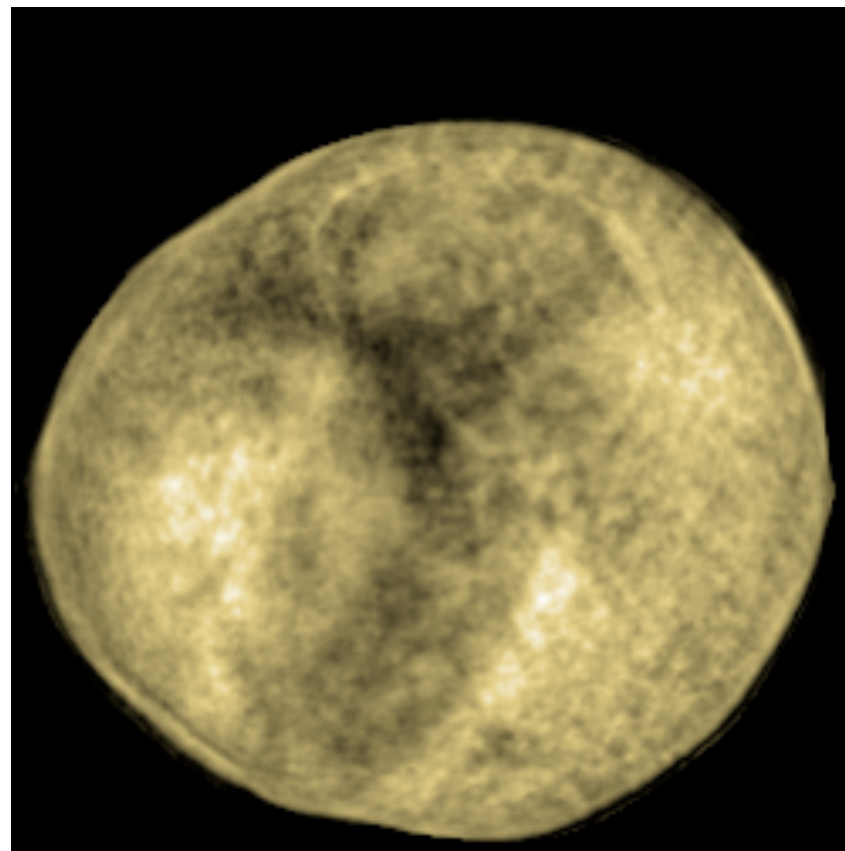
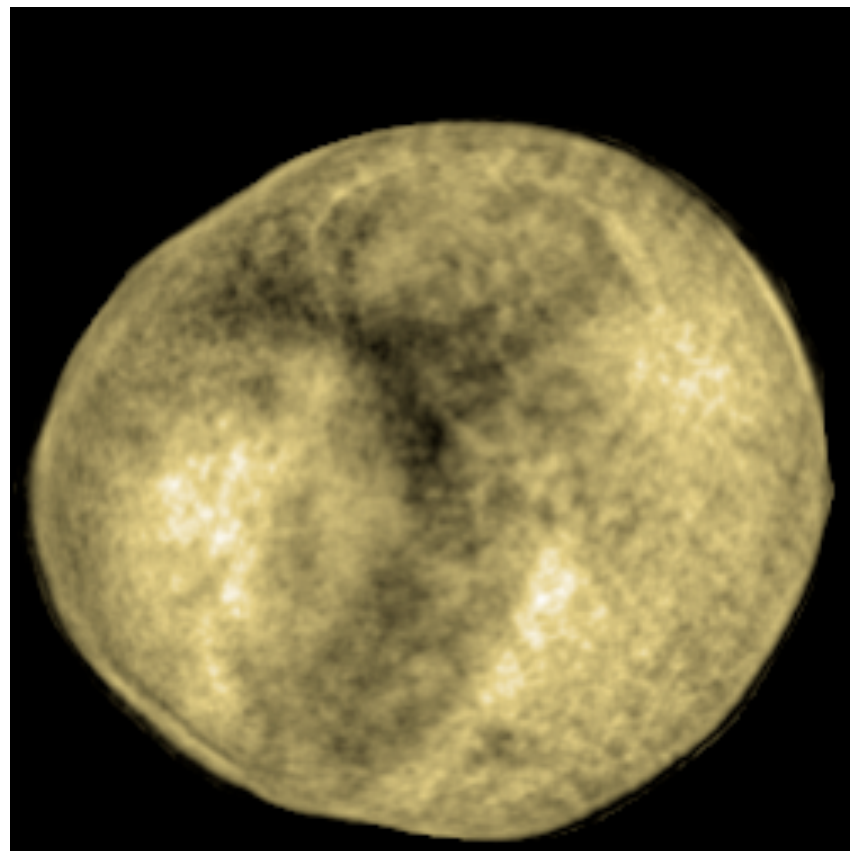
**Is the solution unique and faithful?**



**Diffraction reconstruction  
(data taken at 750 eV;  
absorption as brightness,  
phase as hue).**

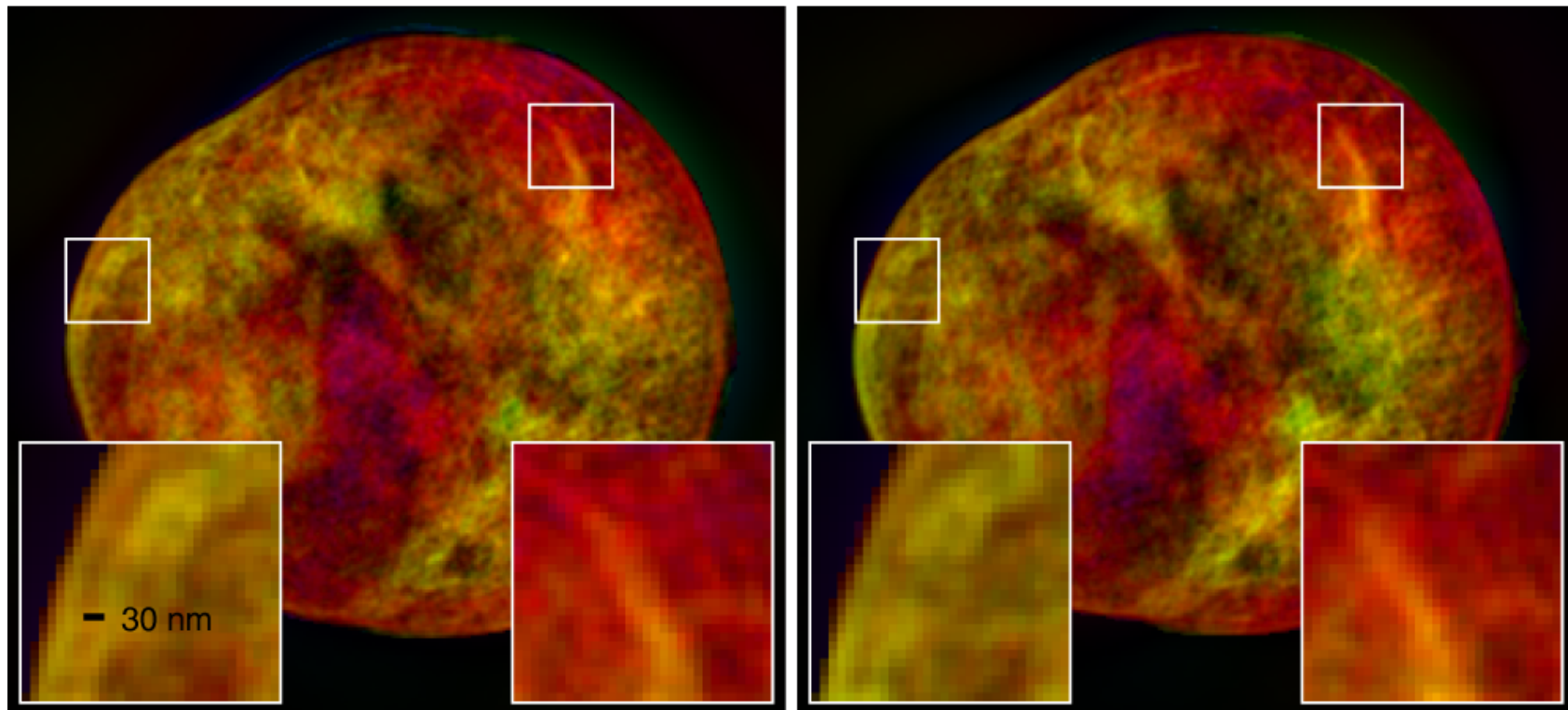


**Stony Brook/NSLS STXM image  
with 45 nm Rayleigh resolution  
zone plate at 520 eV  
(absorption as brightness)**



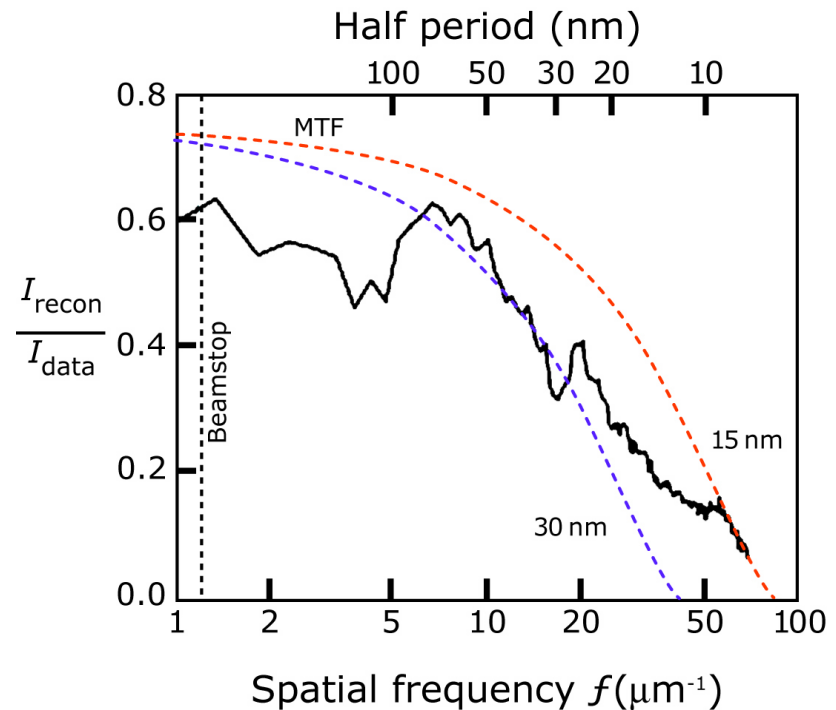
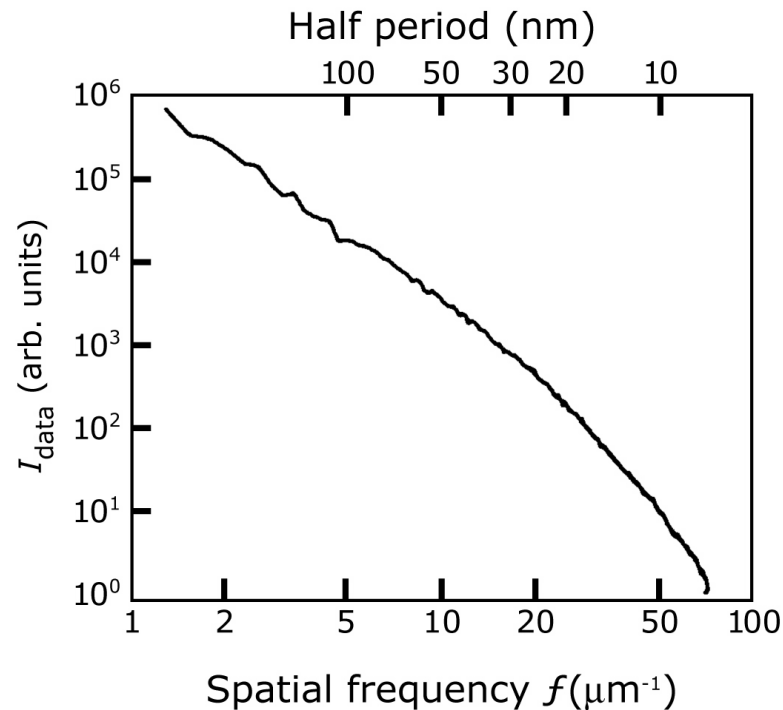
Two separate runs of algorithm with different random starting phases. In both cases, 125 iterates spaced 40 iterations apart were averaged (E. Lima).





# What is the resolution?

- **Data extends to an angle corresponding to 9 nm half-period but is it all equally well phased?**
- **Fourier intensity of reconstructed solution versus raw data**  
→ **analogous to the modulation transfer function**



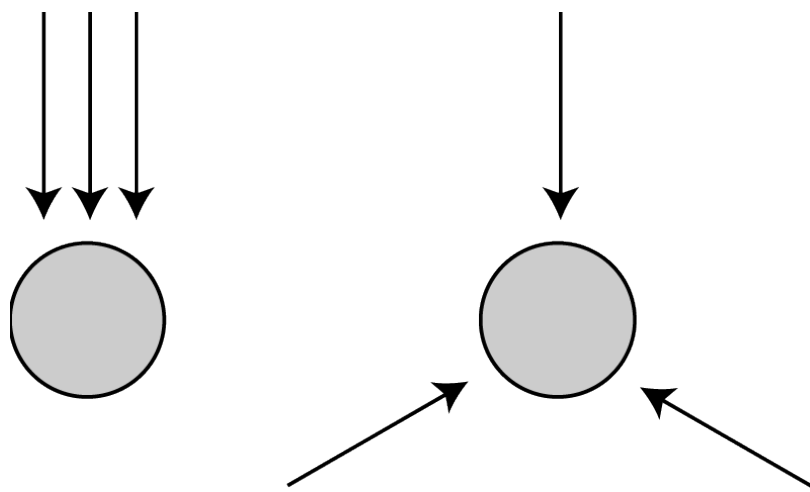
→ **Reconstructed image at 30 nm resolution**

# How can we believe the phasing?

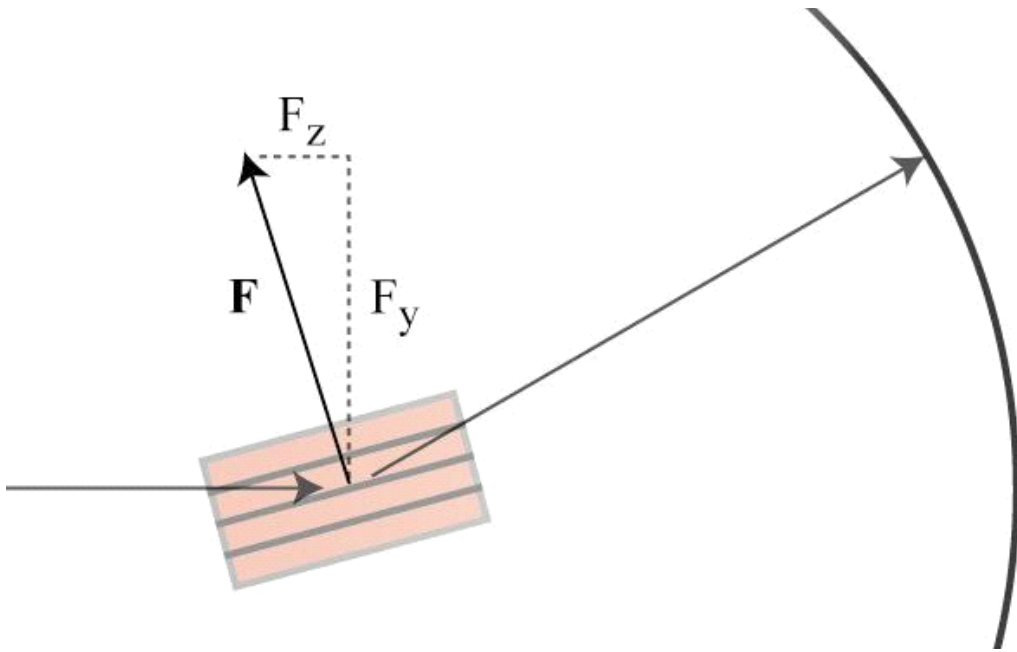
- By understanding the nature of solution finding and averaging iterates (Elser and Thibault).
- By comparing reconstruction with a microscope image.
- By getting similar images from separate data sets from tilts  $1^\circ$  apart.
- By getting similar images from independent runs on the same data with different random starting phases.

- The ultimate limitation for radiation-sensitive materials only
- Dose fractionation  
(Hegerl and Hoppe 1976, McEwen 1995)

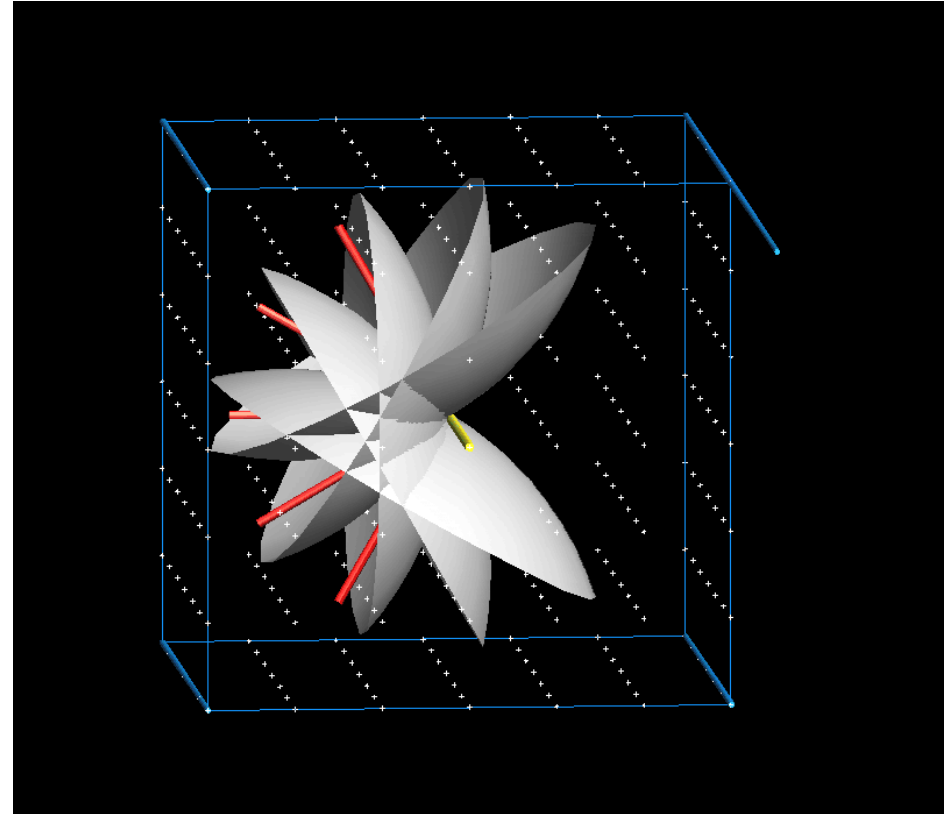
- You can divide the number of photons needed for a good 2D view into 3D views.
- Hegerl and Hoppe, *Z. Naturforschung* **31a**, 1717 (1976); McEwen *et al.*, *Ultramic.* **60**, 357 (1995).



# Diffraction microscopy in 3D

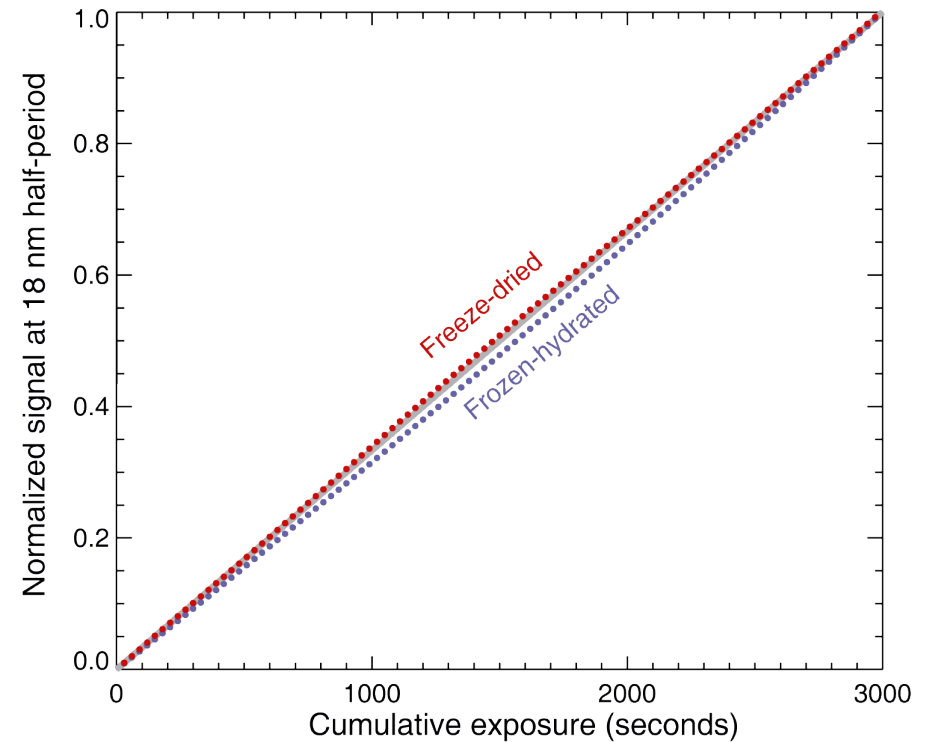
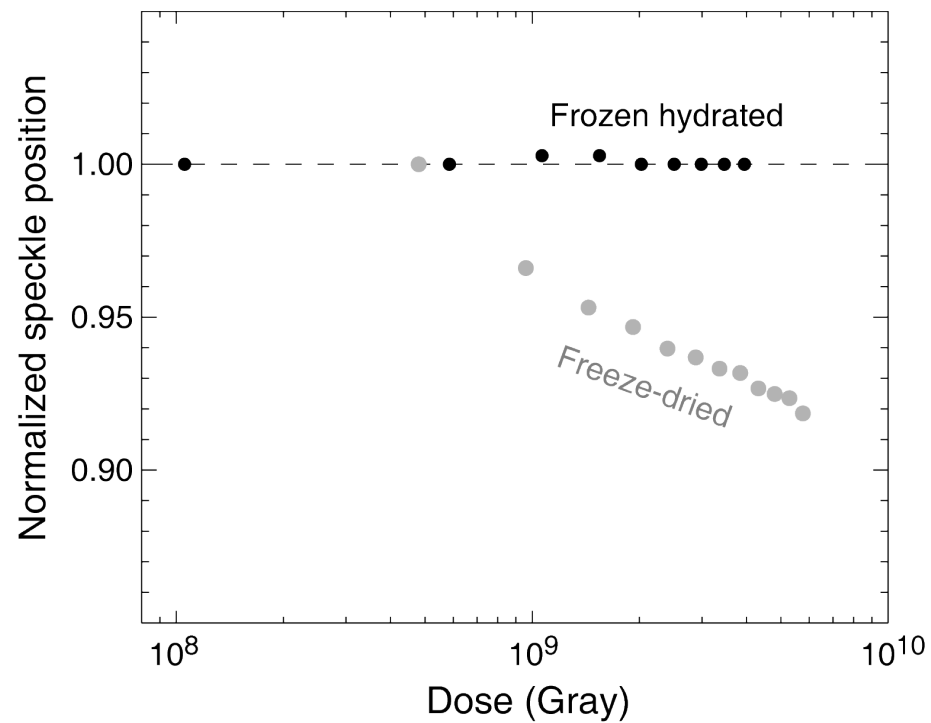


Bragg gratings that diffract to a certain angle represent a specific transverse and longitudinal periodicity (Ewald sphere)



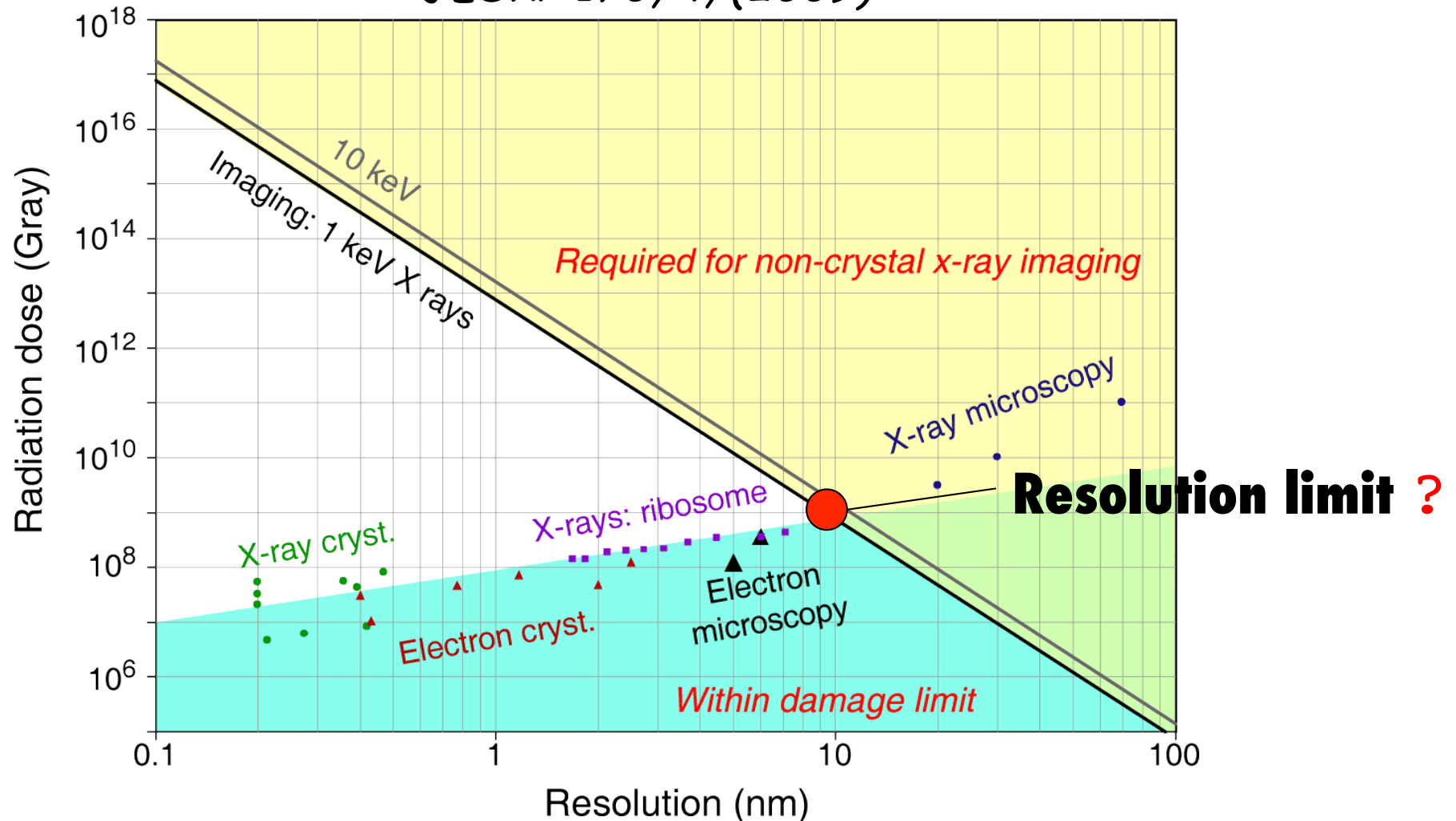
Data collection over a series of rotations about an axis fills in 3D Fourier space for phasing

- D. Shapiro, PhD thesis





Radiation damage in biological samples in XDM:  
Frozen hydrated state of protein by Howells *et al.*  
*JESRP 170, 4, (2009)*



**Inverse fourth power law of dose vs resolution: Dose  $\sim 1/\text{resolution-size}^4$**

- David Sayre
- Wenbing Yun
- Chris Jacobsen, Malcolm Howells
- Henry Chapman
- John Miao
- David Shapiro, Enju Lima, Stefano Marchesini
- Veit Elser & Pierre Thibault
  
- DOE/BES; NIH

- Method of choice for micron-size specimens
- Damage will set limit on resolution for radiation-sensitive specimens