Answering Math Problems

Kohji Suzuki*

kohjisuzuki@yandex.com

Abstract

We provide answers to two questions raised in Math Problems (Twitter: @mathprobs). Pertinent codes are also included.

Problem 337

What's $1 + 2 + 4 + 7 + 11 + 16 + 22 + 29 + \ldots + 301$?

(Rather intuitive) answer

The above seems to be asking us to compute $\displaystyle\sum_{k=1}^{n} a_k$, where $a_k = \frac{k(k-1)}{2} + 1$, n being a certain integer ≥ 10. In fact,

$$a_1 = \frac{1(1-1)}{2} + 1 = 1, \quad a_2 = \frac{2(2-1)}{2} + 1 = 2, \quad a_3 = \frac{3(3-1)}{2} + 1 = 4,$$
$$a_4 = \frac{4(4-1)}{2} + 1 = 7, \quad a_5 = \frac{5(5-1)}{2} + 1 = 11, \quad a_6 = \frac{6(6-1)}{2} + 1 = 16,$$
$$a_7 = \frac{7(7-1)}{2} + 1 = 22, \quad a_8 = \frac{8(8-1)}{2} + 1 = 29 \ ^{1},$$

---

* Protein Science Society of Japan

[1] Incidentally, we can find the integer sequence 1, 2, 4, 7, 11, ... elsewhere . (See also [1].)

1

which prompts us to solve the quadratic equation $\frac{n(n-1)}{2} - 300 = 0$ to get $n = 25$ [2].

Thus, the number 301 seems to amount to a_{25}. Next, we expand $\sum_{k=1}^{25}\left\{\frac{k(k-1)}{2} + 1\right\}$

to get $\frac{1}{2}\cdot\sum_{k=1}^{25}k^2 - \frac{1}{2}\cdot\sum_{k=1}^{25}k + 25$, and using the formulae $\sum_{k=1}^{n}k^2 = \frac{n(n+1)(2n+1)}{6}$ [2]

and $\sum_{k=1}^{n}k = \frac{n(n+1)}{2}$ [3], we get $\frac{1}{2}\cdot\frac{25\cdot(25+1)\cdot(2\cdot25+1)}{6} - \frac{1}{2}\cdot\frac{25\cdot(25+1)}{2} + 25 =$

$\frac{5525}{2} - \frac{325}{2} + 25 = 2625.$

Computational double-checking [3]

Clojure

```
$ clojure
Clojure 1.4.0
user=> (for [a (range 1 9)]
(+ (/ (* a (- a 1)) 2) 1))
(1 2 4 7 11 16 22 29)
user=> (for [a (range 9 25)]
(+ (/ (* a (- a 1)) 2) 1))
(37 46 56 67 79 92 106 121 137 154 172 191 211 232 254 277)
user=> (for [a (range 25 26)]
(+ (/ (* a (- a 1)) 2) 1))
(301)
user=> (Math/round (+ (- (* (/ 1.0 2) (/ (* 25 (+ 25 1)
 (+ (* 2 25) 1)) 6)) (* (/ 1 2) (* (/ 1 2) 25 (+ 25 1)))) 25)
)
2625
```

---

[2] We omit -24, another solution, since it is a negative integer.
[3] Henceforth, we verify our answers on quad-core Intel processors of a Debian GNU/Linux
7.9 (wheezy) machine.

Perl

```
$ which perl
/usr/bin/perl
$ perl -v

This is perl 5, version 14, subversion 2 (v5.14.2) built
for x86_64-linux-gnu-thread-multi
(with 89 registered patches, see perl -V for more detail)

Copyright 1987-2011, Larry Wall

Perl may be copied only under the terms of either the Artistic
License or the GNU General Public License, which may be found
in the Perl 5 source kit.

Complete documentation for Perl, including FAQ lists, should
be found on this system using "man perl" or "perldoc perl".  If you
have access to the Internet, point your browser at http://www.perl.org/,
the Perl Home Page.

$ cat mp_337.pl
#!/usr/bin/perl
my($a,$b,$c);
for($a=1;$a<=8;$a++)
{$b=($a*($a-1))/2+1;
print("$b ");}
print("\n");
for($a=9;$a<=24;$a++)
{$b=($a*($a-1))/2+1;
print("$b ");}
print("\n");
for($a=25;$a<=25;$a++)
{$b=($a*($a-1))/2+1;
$c=((25*(25+1)*(2*25+1))/6-(25*(25+1))/2)/2+25;
print("$b ");}
print("\n\n");
print("Sum of the above terms = $c.\n");
```

3

```
$ perl mp_337.pl
1 2 4 7 11 16 22 29
37 46 56 67 79 92 106 121 137 154 172 191 211 232 254 277
301

Sum of the above terms = 2625.
```

Taken together, our answer is 2625 [4] .

Problem 361

What is the largest 5-digit number that's divisible by 9, 7 and 17?

(Rather time-saving) answer

Since the least common multiple of $(9, 7, 17)$ is $9 \times 7 \times 17 = 1071$, we scan the 'neighborhood' of 99999, the largest 5-digit number, as follows:

$$\vdots$$

$1071 \times 100 = 107100$
$107100 - 1071 = 106029$
$106029 - 1071 = 104958$

$$\vdots$$

$101745 - 1071 = 100674$
$100674 - 1071 = \underline{99603}$
$99603 - 1071 = 98532$

$$\vdots$$

The answer has been underlined.

---

[4] However, we find it difficult to rule out the possibilities that $a_9 \neq 37 = \frac{9(9-1)}{2} + 1, a_{10} \neq 46$ $= \frac{10(10-1)}{2} + 1$, so forth, since the terms between 29 and 301 are not shown in this problem.

Computational double-checking

Perl6

```
$ which perl6
/usr/bin/perl6
$ perl6 --version
This is perl6 version 2012.01 built on parrot 4.0.0 revision 0
$ cat perl6_mp_361.pl
#!/usr/bin/perl6
my $a=98000;
while (($a=$a+1)<=101120)
{if ($a%9==0&&$a%7==0&&$a%17==0)
{print("",$a,"\n");}}
$ perl6 perl6_mp_361.pl
98532
99603
100674
```

Ruby

```
$ which ruby
/usr/bin/ruby
$ ruby -v
ruby 1.9.3p194 (2012-04-20 revision 35410) [x86_64-linux]
$ cat mp_361.rb
#!/usr/bin/ruby
a=98000
while a<=101554
a +=1
if(a%9==0 && a%7==0 && a%17==0)
print("",a,"\n")
end end
$ ruby mp_361.rb
98532
99603
100674
```

Taken together, our answer is 99603.

References

[1] Lovász, L., Pelikán, J., and Vesztergombi, K., "Discrete Mathematics: Elementary and Beyond," Springer-Verlag New York Inc. 2003 p182-183.

[2] Balakrishnan, V. K., "Introductory Discrete Mathematics," Dover Publications 1996 p104.

[3] *Idem*, *ibid* p18.