ALFRED T. KAMAJIAN

By Max Tegmark

Parallel Universes

Not just a staple of science fiction, other universes are a direct implication of cosmological observations

Is there a copy of you

reading this article? A person who is not you but who lives on a planet called Earth, with misty mountains, fertile fields and sprawling cities, in a solar system with eight other planets? The life of this person has been identical to yours in every respect. But perhaps he or she now decides to put down this article without finishing it, while you read on.

The idea of such an alter ego seems strange and implausible, but it looks as if we will just have to live with it, because it is supported by astronomical observations. The simplest and most popular cosmological model today predicts that you have a twin in a galaxy about 10 to the 10^{28} meters from here. This distance is so large that it is beyond astronomical, but that does not make your doppelgänger any less real. The estimate is derived from elementary probability and does not even assume speculative modern physics, merely that space is infinite (or at least sufficiently large) in size and almost uniformly filled with matter, as observations indicate. In infinite space, even the most unlikely events must take place somewhere. There are infinitely many other inhabited planets, including not just one but infinitely many that have people with the same appearance, name and memories as you, who play out every possible permutation of your life choices.

You will probably never see your other selves. The farthest you can observe is the distance that light has been able to travel during the 14 billion years since the big bang expansion began. The most distant visible objects are now about 4×10^{26} meters away—a distance that defines our observable universe, also called our Hubble volume, our horizon volume or simply our universe. Likewise, the universes of your other selves are spheres of the same size centered on their planets. They are the most straightforward example of parallel universes. Each universe is merely a small part of a larger "multiverse."

By this very definition of "universe," one might expect the notion of a multiverse to be forever in the domain of metaphysics. Yet the borderline between physics and metaphysics is defined by whether a theory is experimentally testable, not by whether it is weird or involves unobservable entities. The frontiers of physics have gradually expanded to incorporate ever more abstract (and once metaphysical) concepts such as a round Earth, invisible electromagnetic fields, time slowdown at high speeds, quantum superpositions, curved space, and black holes. Over the past several years the concept of a multiverse has joined this list. It is grounded in well-tested theories such as relativity and quantum mechanics, and it fulfills both of the basic criteria of an empirical science: it makes predictions, and it can be falsified. Scientists have discussed as many as four distinct types of parallel universes. The key question is not whether the multiverse exists but rather how many levels it has.

Level I: Beyond Our Cosmic Horizon

THE PARALLEL UNIVERSES of your alter egos constitute the Level I multiverse. It is the least controversial type. We all accept the existence of things that we cannot see but could see if we moved to a different vantage point or merely waited, like people watching for ships to come over the horizon. Objects beyond the cosmic horizon have a similar status. The observable universe grows by a light-year every year as light from farther away has time to reach us. An infinity lies out there, waiting to be seen. You will probably die long before your alter egos come into view, but in principle, and if cosmic expansion cooperates, your descendants could observe them through a sufficiently powerful telescope.

If anything, the Level I multiverse sounds trivially obvious. How could space *not* be infinite? Is there a sign somewhere saying "Space Ends Here—Mind the Gap"? If so, what lies beyond it? In fact, Einstein's theory of gravity calls this intuition into question. Space could be finite if it has a convex curvature or an unusual topology (that is, interconnectedness). A spherical, doughnut-shaped or pretzel-shaped universe would have a limited volume and no edges. The cosmic microwave background radiation allows sensitive tests of such scenarios [see "Is Space Finite?" by Jean-Pierre Luminet, Glenn D. Starkman and Jeffrey R. Weeks; Scientific American, April 1999]. So far, however, the evidence is against them. Infinite models fit the data, and strong limits have been placed on the alternatives.

Another possibility is that space is infinite but matter is confined to a finite region around us—the historically popular "island universe" model. In a variant on this model, matter thins out on large scales in a fractal pattern. In both cases, almost all universes in the Level I multiverse would be empty and dead. But recent observations of the three-dimensional galaxy distribution and the microwave background have shown that the arrangement of matter gives way to dull uniformity on large scales, with no coherent structures larger than about 10^{24} meters. Assuming that this pattern continues, space beyond our observable universe teems with galaxies, stars and planets.

Observers living in Level I parallel universes experience the same laws of physics as we do but with different initial conditions. According to current theories, processes early in the big bang spread matter around with a degree of randomness, generating all possible arrangements with nonzero probability. Cosmologists assume that our universe, with an almost uniform distribution of matter and initial density fluctuations of one part in 100,000, is a fairly typical one (at least among those that contain observers). That assumption underlies the estimate that your closest identical copy is 10 to the 10^{28} meters away. About 10 to the 10^{92} meters away, there should be a sphere of radius 100 light-years identical to the one centered here, so all perceptions that we have during the next century will be identical to those of our counterparts over there. About 10 to the 10^{118} meters away should be an entire Hubble volume identical to ours.

These are extremely conservative estimates, derived simply by counting all possible quantum states that a Hubble volume can have if it is no hotter than 10^8 kelvins. One way to do the calculation is to ask how many protons could be packed into a Hubble volume at that temperature. The answer is 10^{118} protons. Each of those particles may or may not, in fact, be present, which makes for 2 to the 10^{118} possible arrangements of protons. A box containing that many Hubble volumes exhausts all the possibilities. If you round off the numbers, such a box is about 10 to the 10^{118} meters across. Beyond that box, universes—including ours—must repeat. Roughly the same number could be derived by using thermodynamic or quantum-gravitational estimates of the total information content of the universe.

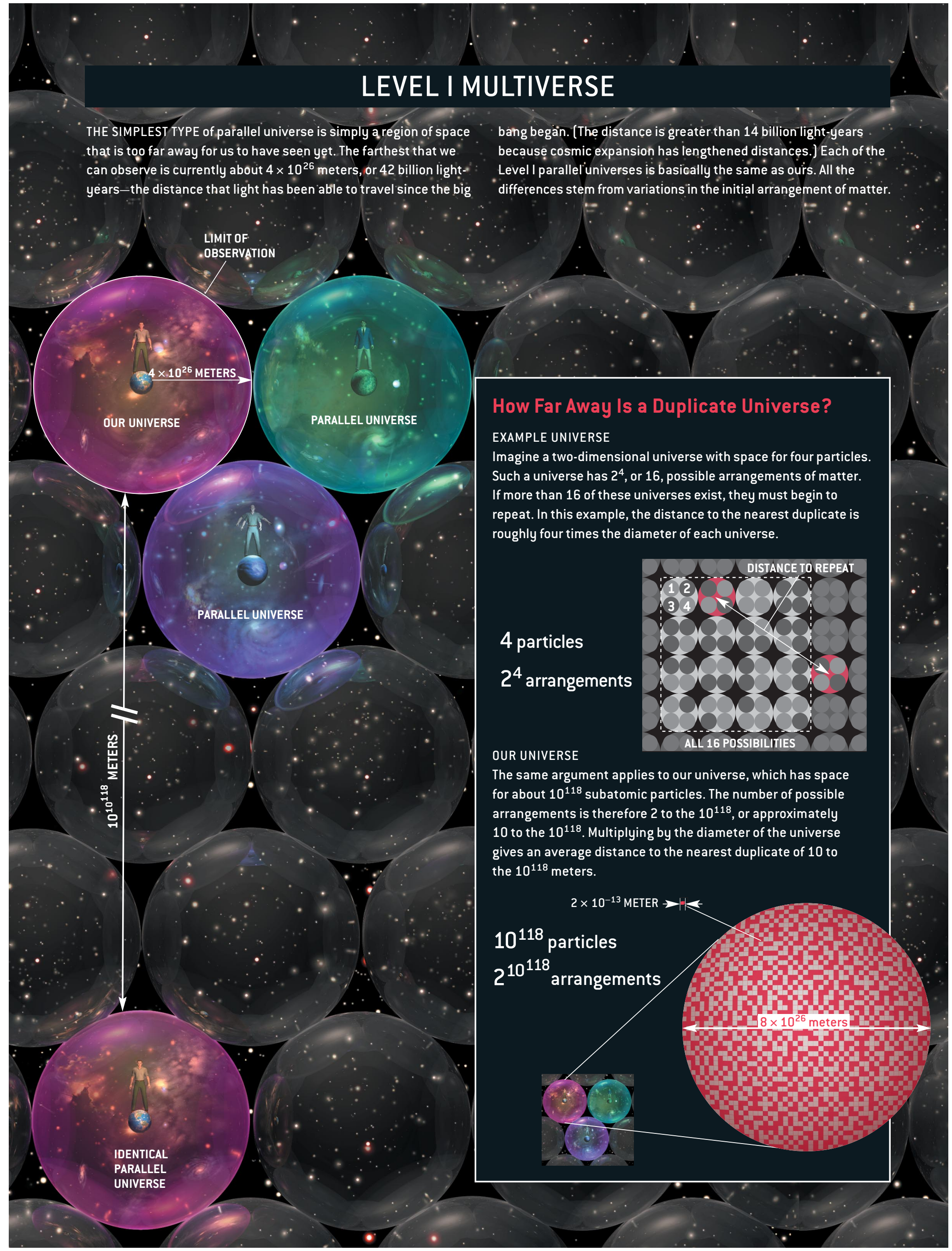
Your nearest doppelgänger is most likely to be much closer than these numbers suggest, given the processes of planet formation and biological evolution that tip the odds in your favor. Astronomers suspect that our Hubble volume has at least 10^{20} habitable planets; some might well look like Earth.

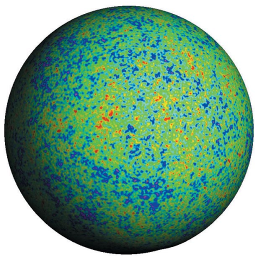
The Level I multiverse framework is used routinely to evaluate theories in modern cosmology, although this procedure is rarely spelled out explicitly. For instance, consider how cosmologists used the microwave background to rule out a finite spherical geometry. Hot and cold spots in microwave background maps have a characteristic size that depends on the curvature of space, and the observed spots appear too small to be consistent with a spherical shape. But it is important to be statistically rigorous. The average spot size varies randomly from one Hubble volume to another, so it is possible that our universe is fooling us—it could be spherical but happen to have abnormally small spots. When cosmologists say they have ruled out the spherical model with 99.9 percent confidence, they really mean that if this model were true, fewer than one in 1,000 Hubble volumes would show spots as small as those we observe.

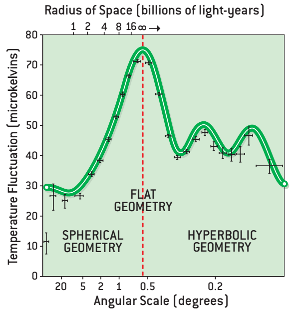
Overview/*Multiverses*

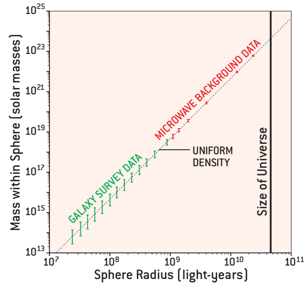
- One of the many implications of recent cosmological observations is that the concept of parallel universes is no mere metaphor. Space appears to be infinite in size. If so, then somewhere out there, everything that is possible becomes real, no matter how improbable it is. Beyond the range of our telescopes are other regions of space that are identical to ours. Those regions are a type of parallel universe. Scientists can even calculate how distant these universes are, on average.
- And that is fairly solid physics. When cosmologists consider theories that are less well established, they conclude that other universes can have entirely different properties and laws of physics. The presence of those universes would explain various strange aspects of our own. It could even answer fundamental questions about the nature of time and the comprehensibility of the physical world.

ALFRED T. KAMAJIAN (*background*); SARA CHEN (*inset*)









COSMOLOGICAL DATA support the idea that space continues beyond the confines of our observable universe. The WMAP satellite recently measured the fluctuations in the microwave background (*left*). The strongest fluctuations are just over half a degree across, which indicates—after applying the rules of geometry—that space is very large or infinite (*center*). (One caveat: some cosmologists speculate that the discrepant point on the left of the graph is evidence for a finite volume.) In addition, WMAP and the 2dF Galaxy Redshift Survey have found that space on large scales is filled with matter uniformly (*right*), meaning that other universes should look basically like ours.

MAX TEGMARK (*sphere*); SARA CHEN (*graphs*)

The lesson is that the multiverse theory can be tested and falsified even though we cannot see the other universes. The key is to predict what the ensemble of parallel universes is and to specify a probability distribution, or what mathematicians call a "measure," over that ensemble. Our universe should emerge as one of the most probable. If not—if, according to the multiverse theory, we live in an improbable universe—then the theory is in trouble. As I will discuss later, this measure problem can become quite challenging.

Level II: Other Postinflation Bubbles

IF THE LEVEL I MULTIVERSE was hard to stomach, try imagining an infinite set of distinct Level I multiverses, some perhaps with different spacetime dimensionality and different physical constants. Those other multiverses—which constitute a Level II multiverse—are predicted by the currently popular theory of chaotic eternal inflation.

Inflation is an extension of the big bang theory and ties up many of the loose ends of that theory, such as why the universe is so big, so uniform and so flat. A rapid stretching of space long ago can explain all these and other attributes in one fell swoop [see "The Inflationary Universe," by Alan H. Guth and Paul J. Steinhard; SCIENTIFIC AMERICAN, May 1984; and "The Self-Reproducing Inflationary Universe," by Andrei Linde, November 1994]. Such stretching is predicted by a wide class of theories of elementary particles, and all available evidence bears it out. The phrase "chaotic eternal" refers to what happens on the very largest scales. Space as a whole is stretching and will continue doing so forever, but some regions of space stop stretching and form distinct bubbles, like gas pockets in a loaf of rising bread. Infinitely many such bubbles emerge. Each is an embryonic Level I multiverse: infinite in size and filled with matter deposited by the energy field that drove inflation.

Those bubbles are more than infinitely far away from Earth, in the sense that you would never get there even if you traveled at the speed of light forever. The reason is that the space between our bubble and its neighbors is expanding faster than you could travel through it. Your descendants will never see their doppelgängers elsewhere in Level II. For the same reason, if cosmic expansion is accelerating, as observations now suggest, they might not see their alter egos even in Level I.

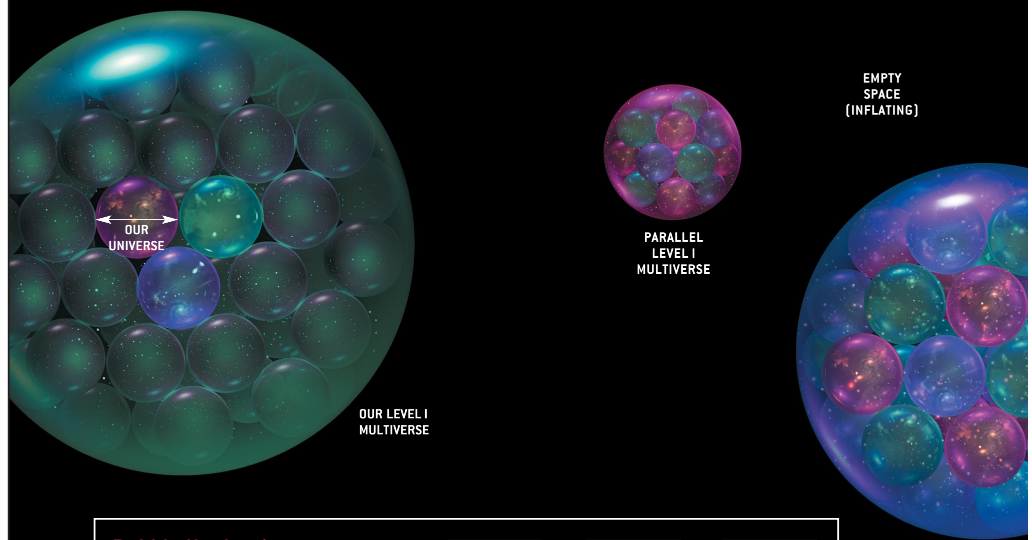
The Level II multiverse is far more diverse than the Level I multiverse. The bubbles vary not only in their initial conditions but also in seemingly immutable aspects of nature. The prevailing view in physics today is that the dimensionality of spacetime, the qualities of elementary particles and many of the so-called physical constants are not built into physical laws but are the outcome of processes known as symmetry breaking. For instance, theorists think that the space in our universe once had nine dimensions, all on an equal footing. Early in cosmic history, three of them partook in the cosmic expansion and became the three dimensions we now observe. The other six are now unobservable, either because they have stayed microscopic with a doughnutlike topology or because all matter is confined to a three-dimensional surface (a membrane, or simply "brane") in the nine-dimensional space.

Thus, the original symmetry among the dimensions broke. The quantum fluctuations that drive chaotic inflation could cause different symmetry breaking in different bubbles. Some might become four-dimensional, others could contain only two rather than three generations of quarks, and still others might have a stronger cosmological constant than our universe does.

Another way to produce a Level II multiverse might be through a cycle of birth and destruction of universes. In a scientific context, this idea was introduced by physicist Richard C. Tolman in the 1930s and recently elaborated on by Paul J. Steinhardt of Princeton University and Neil Turok of the University of Cambridge. The Steinhardt and Turok proposal and related models involve a second three-dimensional brane that is quite literally parallel to ours, merely offset in a higher dimension [see "Been There, Done That," by George Musser; News Scan, SCIENTIFIC AMERICAN, March 2002]. This parallel universe is not

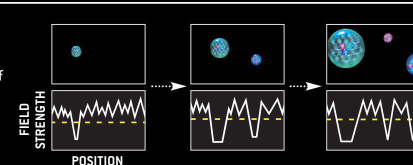
LEVEL II MULTIVERSE

A SOMEWHAT MORE ELABORATE type of parallel universe emerges from the theory of cosmological inflation. The idea is that our Level I multiverse—namely, our universe and contiguous regions of space—is a bubble embedded in an even vaster but mostly empty volume. Other bubbles exist out there, disconnected from ours. They nucleate like raindrops in a cloud. During nucleation, variations in quantum fields endow each bubble with properties that distinguish it from other bubbles.



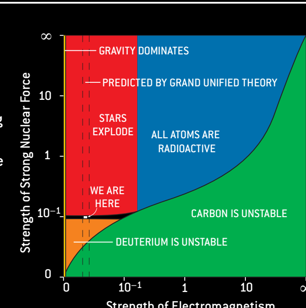
Bubble Nucleation

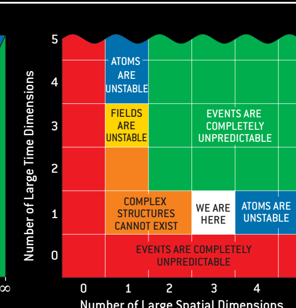
A QUANTUM FIELD known as the inflaton causes space to expand rapidly. In the bulk of space, random fluctuations prevent the field from decaying away. But in certain regions, the field loses its strength and the expansion slows down. Those regions become bubbles.



Evidence

COSMOLOGISTS INFER the presence of Level II parallel universes by scrutinizing the properties of our universe. These properties, including the strength of the forces of nature (*right*) and the number of observable space and time dimensions (*far right*), were established by random processes during the birth of our universe. Yet they have exactly the values that sustain life. That suggests the existence of other universes with other values.





ALFRED T. KAMAJIAN (*background*); CORNELIA BLIK (*top inset*); SARA CHEN (*bottom inset*) really a separate universe, because it interacts with ours. But the ensemble of universes—past, present and future—that these branes create would form a multiverse, arguably with a diversity similar to that produced by chaotic inflation. An idea proposed by physicist Lee Smolin of the Perimeter Institute in Waterloo, Ontario, involves yet another multiverse comparable in diversity to that of Level II but mutating and sprouting new universes through black holes rather than through brane physics.

Although we cannot interact with other Level II parallel universes, cosmologists can infer their presence indirectly, because their existence can account for unexplained coincidences in our universe. To give an analogy, suppose you check into a hotel, are assigned room 1967 and note that this is the year you were born. What a coincidence, you say. After a moment of reflection, however, you conclude that this is not so surprising after all. The hotel has hundreds of rooms, and you would not have been having these thoughts in the first place if you had been assigned one with a number that meant nothing to you. The lesson is that even if you knew nothing about hotels, you could infer the existence of other hotel rooms to explain the coincidence.

As a more pertinent example, consider the mass of the sun. The mass of a star determines its luminosity, and using basic physics, one can compute that life as we know it on Earth is possible only if the sun's mass falls into the narrow range between 1.6×10^{30} and 2.4×10^{30} kilograms. Otherwise Earth's climate would be colder than that of present-day Mars or hotter than that of present-day Venus. The measured solar mass is 2.0×10^{30} kilograms. At first glance, this apparent coincidence of the habitable and observed mass values appears to be a wild stroke of luck. Stellar masses run from 10^{29} to 10^{32} kilograms, so if the sun acquired its mass at random, it had only a small chance of falling into the habitable range. But just as in the hotel example, one can explain this apparent coincidence by postulating an ensemble (in this case, a number of planetary systems) and a selection effect (the fact that we must find ourselves living on a habitable planet). Such observer-related selection effects are referred to as "anthropic," and although the "A-word" is notorious for triggering controversy, physicists broadly agree that these selection effects cannot be neglected when testing fundamental theories.

What applies to hotel rooms and planetary systems applies to parallel universes. Most, if not all, of the attributes set by symmetry breaking appear to be fine-tuned. Changing their values by modest amounts would have resulted in a qualitatively different universe—one in which we probably would not exist. If protons were 0.2 percent heavier, they could decay into neutrons, destabilizing atoms. If the electromagnetic force were 4 percent weaker, there would be no hydrogen and no normal stars. If the weak interaction were much weaker, hydrogen would not exist; if it were much stronger, supernovae would fail to seed interstellar space with heavy elements. If the cosmological constant were much larger, the universe would have blown itself apart before galaxies could form.

Although the degree of fine-tuning is still debated, these examples suggest the existence of parallel universes with other values of the physical constants [see "Exploring Our Universe and Others," by Martin Rees; Scientific American, December 1999]. The Level II multiverse theory predicts that physicists will never be able to determine the values of these constants from first principles. They will merely compute probability distributions for what they should expect to find, taking selection effects into account. The result should be as generic as is consistent with our existence.

Level III: Quantum Many Worlds

THE LEVEL I AND LEVEL II multiverses involve parallel worlds that are far away, beyond the domain even of astronomers. But the next level of multiverse is right around you. It arises from the famous, and famously controversial, many-worlds interpretation of quantum mechanics—the idea that random quantum processes cause the universe to branch into multiple copies, one for each possible outcome.

In the early 20th century the theory of quantum mechanics revolutionized physics by explaining the atomic realm, which does not abide by the classical rules of Newtonian mechanics. Despite the obvious successes of the theory, a heated debate rages about what it really means. The theory specifies the state of the universe not in classical terms, such as the positions and velocities of all particles, but in terms of a mathematical object called a wave function. According to the Schrödinger equation, this state evolves over time in a fashion that mathematicians term "unitary," meaning that the wave function rotates in an abstract infinite-dimensional space called Hilbert space. Although quantum mechanics is often described as inherently random and uncertain, the wave function evolves in a deterministic way. There is nothing random or uncertain about it.

The sticky part is how to connect this wave function with what we observe. Many legitimate wave functions correspond to counterintuitive situations, such as a cat being dead and alive at the same time in a so-called superposition. In the 1920s physicists explained away this weirdness by postulating that the wave function "collapsed" into some definite classical outcome whenever someone made an observation. This add-on had the virtue of explaining observations, but it turned an elegant, unitary theory into a kludgy, nonunitary one. The intrinsic randomness commonly ascribed to quantum mechanics is the result of this postulate.

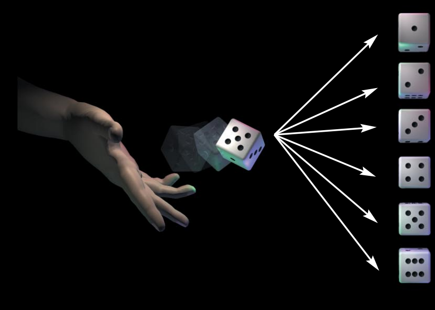
Over the years many physicists have abandoned this view in favor of one developed in 1957 by Princeton graduate student Hugh Everett III. He showed that the collapse postulate is unnecessary. Unadulterated quantum theory does not, in fact, pose any contradictions. Although it predicts that one classical reality gradually splits into superpositions of many such realities, observers subjectively experience this splitting merely as a slight randomness, with probabilities in exact agreement with those from the old collapse postulate. This superposition of classical worlds is the Level III multiverse.

Everett's many-worlds interpretation has been boggling minds inside and outside physics for more than four decades. But the theory becomes easier to grasp when one distinguishes

ALFRED T. KAMAJIAN

LEVEL III MULTIVERSE

QUANTUM MECHANICS PREDICTS a vast number of parallel universes by broadening the concept of "elsewhere." These universes are located elsewhere, not in ordinary space but in an abstract realm of all possible states. Every conceivable way that the world could be (within the scope of quantum mechanics) corresponds to a different universe. The parallel universes make their presence felt in laboratory experiments, such as wave interference and quantum computation.

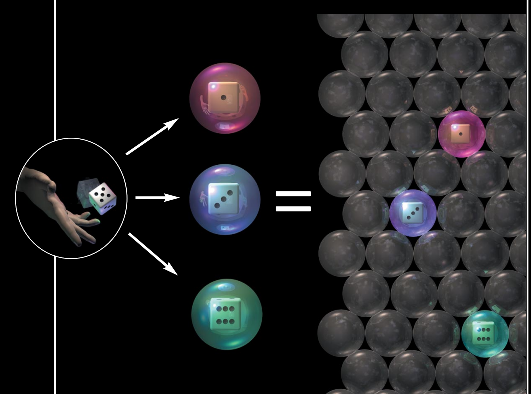


Quantum Dice

IMAGINE AN IDEAL DIE whose randomness is purely quantum. When you roll it, the die appears to land on a certain value at random. Quantum mechanics, however, predicts that it lands on all values at once. One way to reconcile these contradictory views is to conclude that the die lands on different values in different universes. In one sixth of the universes, it lands on 1; in one sixth, on 2, and so on. Trapped within one universe, we can perceive only a fraction of the full quantum reality.

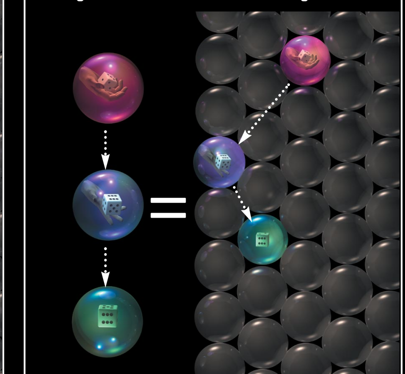
Ergodicity

ACCORDING TO THE PRINCIPLE of ergodicity, quantum parallel universes are equivalent to more prosaic types of parallel universes. A quantum universe splits over time into multiple universes (*left*). Yet those new universes are no different from parallel universes that already exist somewhere else in space—in, for example, other Level I universes (*right*). The key idea is that parallel universes, of whatever type, embody different ways that events could have unfolded.



The Nature of Time

MOST PEOPLE THINK of time as a way to describe change. At one moment, matter has a certain arrangement; a moment later, it has another (*left*). The concept of multiverses suggests an alternative view. If parallel universes contain all possible arrangements of matter (*right*), then time is simply a way to put those universes into a sequence. The universes themselves are static; change is an illusion, albeit an interesting one.



between two ways of viewing a physical theory: the outside view of a physicist studying its mathematical equations, like a bird surveying a landscape from high above it, and the inside view of an observer living in the world described by the equations, like a frog living in the landscape surveyed by the bird.

From the bird perspective, the Level III multiverse is simple. There is only one wave function. It evolves smoothly and deterministically over time without any kind of splitting or parallelism. The abstract quantum world described by this evolving wave function contains within it a vast number of parallel classical story lines, continuously splitting and merging, as well as a number of quantum phenomena that lack a classical description. From their frog perspective, observers perceive only a tiny fraction of this full reality. They can view their own Level I universe, but a process called decoherence—which mimics wave function collapse while preserving unitarity—prevents them from seeing Level III parallel copies of themselves.

Whenever observers are asked a question, make a snap decision and give an answer, quantum effects in their brains lead to a superposition of outcomes, such as "Continue reading the article" and "Put down the article." From the bird perspective, the act of making a decision causes a person to split into multiple copies: one who keeps on reading and one who doesn't. From their frog perspective, however, each of these alter egos is unaware of the others and notices the branching merely as a slight randomness: a certain probability of continuing to read or not.

As strange as this may sound, the exact same situation occurs even in the Level I multiverse. You have evidently decided to keep on reading the article, but one of your alter egos in a distant galaxy put down the magazine after the first paragraph. The only difference between Level I and Level III is where your doppelgängers reside. In Level I they live elsewhere in good old three-dimensional space. In Level III they live on another quantum branch in infinite-dimensional Hilbert space.

The existence of Level III depends on one crucial assumption: that the time evolution of the wave function is unitary. So far experimenters have encountered no departures from unitarity. In the past few decades they have confirmed unitarity for ever larger systems, including carbon 60 buckyball molecules and kilometer-long optical fibers. On the theoretical side, the case for unitarity has been bolstered by the discovery of decoherence [see "100 Years of Quantum Mysteries," by Max Tegmark and John Archibald Wheeler; Scientific American, February 2001]. Some theorists who work on quantum gravity have questioned unitarity; one concern is that evaporating black holes might destroy information, which would be a nonunitary process. But a recent breakthrough in string theory known as AdS/CFT correspondence suggests that even quantum gravity is unitary. If so, black holes do not destroy information but merely transmit it elsewhere. [*Editors' note: An upcoming article will discuss this correspondence in greater detail.*]

If physics is unitary, then the standard picture of how quantum fluctuations operated early in the big bang must change. These fluctuations did not generate initial conditions at random. Rather they generated a quantum superposition of all possible initial conditions, which coexisted simultaneously. Decoherence then caused these initial conditions to behave classically in separate quantum branches. Here is the crucial point: the distribution of outcomes on different quantum branches in a given Hubble volume (Level III) is identical to the distribution of outcomes in different Hubble volumes within a single quantum branch (Level I). This property of the quantum fluctuations is known in statistical mechanics as ergodicity.

The same reasoning applies to Level II. The process of symmetry breaking did not produce a unique outcome but rather a superposition of all outcomes, which rapidly went their separate ways. So if physical constants, spacetime dimensionality and so on can vary among parallel quantum branches at Level III, then they will also vary among parallel universes at Level II.

In other words, the Level III multiverse adds nothing new beyond Level I and Level II, just more indistinguishable copies of the same universes—the same old story lines playing out again and again in other quantum branches. The passionate debate about Everett's theory therefore seems to be ending in a grand anticlimax, with the discovery of less controversial multiverses (Levels I and II) that are equally large.

Needless to say, the implications are profound, and physicists are only beginning to explore them. For instance, consider the ramifications of the answer to a long-standing question: Does the number of universes exponentially increase over time? The surprising answer is no. From the bird perspective, there is of course only one quantum universe. From the frog perspective, what matters is the number of universes that are distinguishable at a given instant—that is, the number of noticeably different Hubble volumes. Imagine moving planets to random new locations, imagine having married someone else, and so on. At the quantum level, there are 10 to the 10^{118} universes with temperatures below 10^8 kelvins. That is a vast number, but a finite one.

From the frog perspective, the evolution of the wave function corresponds to a never-ending sliding from one of these 10 to the 10^{118} states to another. Now you are in universe A, the one in which you are reading this sentence. Now you are in universe B, the one in which you are reading this other sentence. Put differently, universe B has an observer identical to one in universe A, except with an extra instant of memories. All possible states exist at every instant, so the passage of time may be in the eye of the beholder—an idea explored in Greg Egan's 1994 science-fiction novel *Permutation City* and developed by physicist David Deutsch of the University of Oxford, independent physicist Julian Barbour, and others. The multiverse framework may thus prove essential to understanding the nature of time.

THE AUTHOR

MAX TEGMARK wrote a four-dimensional version of the computer game Tetris while in college. In another universe, he went on to become a highly paid software developer. In our universe, however, he wound up as professor of physics and astronomy at the University of Pennsylvania. Tegmark is an expert in analyzing the cosmic microwave background and galaxy clustering. Much of his work bears on the concept of parallel universes: evaluating evidence for infinite space and cosmological inflation; developing insights into quantum decoherence; and studying the possibility that the amplitude of microwave background fluctuations, the dimensionality of spacetime and the fundamental laws of physics can vary from place to place.

The Mystery of Probability: What Are the Odds?

AS MULTIVERSE THEORIES gain credence, the sticky issue of how to compute probabilities in physics is growing from a minor nuisance into a major embarrassment. If there are indeed many identical copies of you, the traditional notion of determinism evaporates. You could not compute your own future even if you had complete knowledge of the entire state of the multiverse, because there is no way for you to determine which of these copies is you (they all feel they are). All you can predict, therefore, are probabilities for what you would observe. If an outcome has a probability of, say, 50 percent, it means that half the observers observe that outcome.

Unfortunately, it is not an easy task to compute what fraction of the infinitely many observers perceive what. The answer depends on the order in which you count them. By analogy, the fraction of the integers that are even is 50 percent if you order them numerically (1, 2, 3, 4, . . .) but approaches 100 percent if you sort them digit by digit, the way your word processor would (1, 10, 100, 1,000, . . .). When observers reside in disconnected universes, there is no obviously natural way in which to order them. Instead one must sample from the different universes with some statistical weights referred to by mathematicians as a "measure."

This problem crops up in a mild and treatable manner at Level I, becomes severe at Level II, has caused much debate at Level III, and is horrendous at Level IV. At Level II, for instance, Alexander Vilenkin of Tufts University and others have published predictions for the probability distributions of various cosmological parameters. They have argued that different parallel universes that have inflated by different amounts should be given statistical weights proportional to their volume. On the other hand, any mathematician will tell you that $2 \times \infty = \infty$, so there is no objective sense in which an infinite universe that has expanded by a factor of two has gotten larger. Moreover, a finite universe with the topology of a torus is equivalent to a perfectly periodic universe with infinite volume, both from the mathematical bird perspective and from the frog perspective of an observer within it. So why should its infinitely smaller volume give it zero statistical weight? After all, even in the Level I multiverse, Hubble volumes start repeating (albeit in a random order, not periodically) after about 10 to the 10^{118} meters.

If you think that is bad, consider the problem of assigning statistical weights to different mathematical structures at Level IV. The fact that our universe seems relatively simple has led many people to suggest that the correct measure somehow involves complexity. *—M.T.*

SARA CHEN

Level IV: Other Mathematical Structures

THE INITIAL CONDITIONS and physical constants in the Level I, Level II and Level III multiverses can vary, but the fundamental laws that govern nature remain the same. Why stop there? Why not allow the laws themselves to vary? How about a universe that obeys the laws of classical physics, with no quantum effects? How about time that comes in discrete steps, as for computers, instead of being continuous? How about a universe that is simply an empty dodecahedron? In the Level IV multiverse, all these alternative realities actually exist.

A hint that such a multiverse might not be just some beer-fueled speculation is the tight correspondence between the worlds of abstract reasoning and of observed reality. Equations and, more generally, mathematical structures such as numbers, vectors and geometric objects describe the world with remarkable verisimilitude. In a famous 1959 lecture, physicist Eugene P. Wigner argued that "the enormous usefulness of mathematics in the natural sciences is something bordering on the mysterious." Conversely, mathematical structures have an eerily real feel to them. They satisfy a central criterion of objective existence: they are the same no matter who studies them. A theorem is true regardless of whether it is proved by a human, a computer or an intelligent dolphin. Contemplative alien civilizations would find the same mathematical structures as we have. Accordingly, mathematicians commonly say that they discover mathematical structures rather than create them.

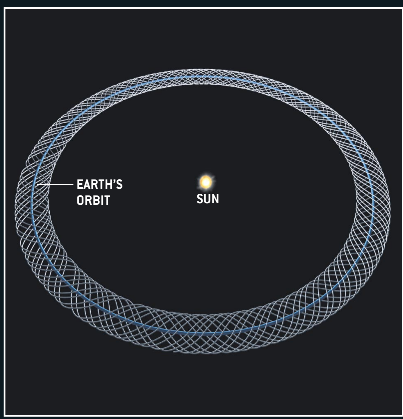
There are two tenable but diametrically opposed paradigms for understanding the correspondence between mathematics and physics, a dichotomy that arguably goes as far back as Plato and Aristotle. According to the Aristotelian paradigm, physical reality is fundamental and mathematical language is merely a useful approximation. According to the Platonic paradigm, the mathematical structure is the true reality and observers perceive it imperfectly. In other words, the two paradigms disagree on which is more basic, the frog perspective of the observer or the bird perspective of the physical laws. The Aristotelian paradigm prefers the frog perspective, whereas the Platonic paradigm prefers the bird perspective.

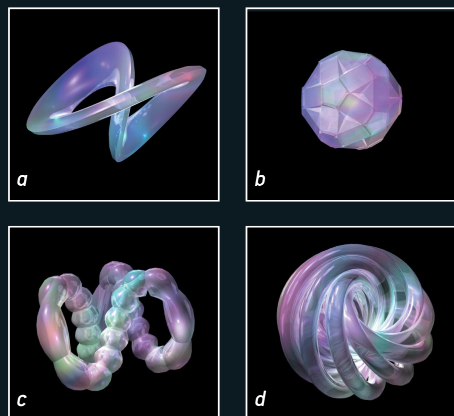
As children, long before we had even heard of mathematics, we were all indoctrinated with the Aristotelian paradigm. The Platonic view is an acquired taste. Modern theoretical physicists tend to be Platonists, suspecting that mathematics describes the universe so well because the universe is inherently mathematical. Then all of physics is ultimately a mathematics problem: a mathematician with unlimited intelligence and resources could in principle compute the frog perspective—that is, compute what self-aware observers the universe contains, what they perceive, and what languages they invent to describe their perceptions to one another.

A mathematical structure is an abstract, immutable entity existing outside of space and time. If history were a movie, the structure would correspond not to a single frame of it but to the entire videotape. Consider, for example, a world made up of pointlike particles moving around in three-dimensional space.

LEVEL IV MULTIVERSE

THE ULTIMATE TYPE of parallel universe opens up the full realm of possibility. Universes can differ not just in location, cosmological properties or quantum state but also in the laws of physics. Existing outside of space and time, they are almost impossible to visualize; the best one can do is to think of them abstractly, as static sculptures that represent the mathematical structure of the physical laws that govern them. For example, consider a simple universe: Earth, moon and sun, obeying Newton's laws. To an objective observer, this universe looks like a circular ring (Earth's orbit smeared out in time) wrapped in a braid (the moon's orbit around Earth). Other shapes embody other laws of physics (*a, b, c, d*). This paradigm solves various problems concerning the foundations of physics.





In four-dimensional spacetime—the bird perspective—these particle trajectories resemble a tangle of spaghetti. If the frog sees a particle moving with constant velocity, the bird sees a straight strand of uncooked spaghetti. If the frog sees a pair of orbiting particles, the bird sees two spaghetti strands intertwined like a double helix. To the frog, the world is described by Newton's laws of motion and gravitation. To the bird, it is described by the geometry of the pasta—a mathematical structure. The frog itself is merely a thick bundle of pasta, whose highly complex intertwining corresponds to a cluster of particles that store and process information. Our universe is far more complicated than this example, and scientists do not yet know to what, if any, mathematical structure it corresponds.

The Platonic paradigm raises the question of why the universe is the way it is. To an Aristotelian, this is a meaningless question: the universe just is. But a Platonist cannot help but wonder why it could not have been different. If the universe is inherently mathematical, then why was only one of the many mathematical structures singled out to describe a universe? A fundamental asymmetry appears to be built into the very heart of reality.

As a way out of this conundrum, I have suggested that complete mathematical symmetry holds: that all mathematical structures exist physically as well. Every mathematical structure corresponds to a parallel universe. The elements of this multiverse do not reside in the same space but exist outside of space and time. Most of them are probably devoid of observers. This hypothesis can be viewed as a form of radical Platonism, asserting that the mathematical structures in Plato's realm of ideas or the "mindscape" of mathematician Rudy Rucker of San Jose State University exist in a physical sense. It is akin to what cosmologist John D. Barrow of the University of Cambridge refers to as "π in the sky," what the late Harvard University philosopher Robert Nozick called the principle of fecundity and what the late Princeton philosopher David K. Lewis called modal realism. Level IV brings closure to the hierarchy of multiverses, because any self-consistent fundamental physical theory can be phrased as some kind of mathematical structure.

The Level IV multiverse hypothesis makes testable predictions. As with Level II, it involves an ensemble (in this case, the full range of mathematical structures) and selection effects. As mathematicians continue to categorize mathematical structures, they should find that the structure describing our world is the most generic one consistent with our observations. Similarly, our future observations should be the most generic ones that are consistent with our past observations, and our past observations should be the most generic ones that are consistent with our existence.

Quantifying what "generic" means is a severe problem, and this investigation is only now beginning. But one striking and encouraging feature of mathematical structures is that the symmetry and invariance properties that are responsible for the simplicity and orderliness of our universe tend to be generic, more the rule than the exception. Mathematical structures tend to have them by default, and complicated additional axioms must be added to make them go away.

BRYAN CHRISTIE DESIGN (*left*); ALFRED T. KAMAJIAN (*a–d*)

What Says Occam?

THE SCIENTIFIC THEORIES of parallel universes, therefore, form a four-level hierarchy, in which universes become progressively more different from ours. They might have different initial conditions (Level I); different physical constants and particles (Level II); or different physical laws (Level IV). It is ironic that Level III is the one that has drawn the most fire in the past decades, because it is the only one that adds no qualitatively new types of universes.

In the coming decade, dramatically improved cosmological measurements of the microwave background and the large-scale matter distribution will support or refute Level I by further pinning down the curvature and topology of space. These measurements will also probe Level II by testing the theory of chaotic eternal inflation. Progress in both astrophysics and high-energy physics should also clarify the extent to which physical constants are fine-tuned, thereby weakening or strengthening the case for Level II.

If current efforts to build quantum computers succeed, they will provide further evidence for Level III, as they would, in essence, be exploiting the parallelism of the Level III multiverse for parallel computation. Experimenters are also looking for evidence of unitarity violation, which would rule out Level III. Finally, success or failure in the grand challenge of modern physics—unifying general relativity and quantum field theory—will sway opinions on Level IV. Either we will find a mathematical structure that exactly matches our universe, or we will bump up against a limit to the unreasonable effectiveness of mathematics and have to abandon that level.

So should you believe in parallel universes? The principal arguments against them are that they are wasteful and that they are weird. The first argument is that multiverse theories are vulnerable to Occam's razor because they postulate the existence of other worlds that we can never observe. Why should nature be so wasteful and indulge in such opulence as an infinity of different worlds? Yet this argument can be turned around to argue *for* a multiverse. What precisely would nature be wasting? Certainly not space, mass or atoms—the uncontroversial Level I multiverse already contains an infinite amount of all three, so who cares if nature wastes some more? The real issue here is the apparent reduction in simplicity. A skeptic worries about all the information necessary to specify all those unseen worlds.

But an entire ensemble is often much simpler than one of its members. This principle can be stated more formally using the notion of algorithmic information content. The algorithmic information content in a number is, roughly speaking, the length of the shortest computer program that will produce that number as output. For example, consider the set of all integers. Which is simpler, the whole set or just one number? Naively, you might think that a single number is simpler, but the entire set can be generated by quite a trivial computer program, whereas a single number can be hugely long. Therefore, the whole set is actually simpler.

Similarly, the set of all solutions to Einstein's field equations is simpler than a specific solution. The former is described by a few equations, whereas the latter requires the specification of vast amounts of initial data on some hypersurface. The lesson is that complexity increases when we restrict our attention to one particular element in an ensemble, thereby losing the symmetry and simplicity that were inherent in the totality of all the elements taken together.

In this sense, the higher-level multiverses are simpler. Going from our universe to the Level I multiverse eliminates the need to specify initial conditions, upgrading to Level II eliminates the need to specify physical constants, and the Level IV multiverse eliminates the need to specify anything at all. The opulence of complexity is all in the subjective perceptions of observers—the frog perspective. From the bird perspective, the multiverse could hardly be any simpler.

The complaint about weirdness is aesthetic rather than scientific, and it really makes sense only in the Aristotelian worldview. Yet what did we expect? When we ask a profound question about the nature of reality, do we not expect an answer that sounds strange? Evolution provided us with intuition for the everyday physics that had survival value for our distant ancestors, so whenever we venture beyond the everyday world, we should expect it to seem bizarre.

A common feature of all four multiverse levels is that the simplest and arguably most elegant theory involves parallel universes by default. To deny the existence of those universes, one needs to complicate the theory by adding experimentally unsupported processes and ad hoc postulates: finite space, wave function collapse and ontological asymmetry. Our judgment therefore comes down to which we find more wasteful and inelegant: many worlds or many words. Perhaps we will gradually get used to the weird ways of our cosmos and find its strangeness to be part of its charm. SA

MORE TO EXPLORE

Why Is the CMB Fluctuation Level 10^{-5}? Max Tegmark and Martin Rees in *Astrophysical Journal*, Vol. 499, No. 2, pages 526–532; June 1, 1998. Available online at **arXiv.org/abs/astro-ph/9709058**

Is "The Theory of Everything" Merely the Ultimate Ensemble Theory? Max Tegmark in *Annals of Physics*, Vol. 270, No.1, pages 1–51; November 20, 1998. Available online at **arXiv.org/abs/gr-qc/9704009**

Many Worlds in One. Jaume Garriga and Alexander Vilenkin in *Physical Review*, Vol. D64, No. 043511; July 26, 2001. Available online at **arXiv.org/abs/gr-qc/0102010**

Our Cosmic Habitat. Martin Rees. Princeton University Press, 2001.

Inflation, Quantum Cosmology and the Anthropic Principle. Andrei Linde in *Science and Ultimate Reality: From Quantum to Cosmos*. Edited by J. D. Barrow, P.C.W. Davies and C. L. Harper. Cambridge University Press, 2003. Available online at **arXiv.org/abs/hep-th/0211048**

The author's Web site has more information at **www.hep.upenn.edu/~max/multiverse.html**