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HOW FAR OUT MUST WE GO TO GET INTO THE HUBBLE FLOW?

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ABSTRACT

The answer, we shall see, must be z equals 3—if the expansion is flat, matter-dominated, hence slow. I show that gravitational lenses are the best hope for definitely getting out into the Hubble flow and hence for measuring the true values of the cosmological parameters. The source of one known lens system, B1422+231, is definitely in the Hubble flow.

Subject headings: cosmology: theory — distance scale — gravitational lensing — large-scale structure of universe — quasars: individual (B1422+231)

1. INTRODUCTION

The growth of local perturbations since the big bang has not been known to obscure the global features of the universe. Structures of redshift size up to $z \sim 0.1$ have been observed (Strauss et al. 1992). The isotropy of the cosmic background radiation suggests that these enormous entities have been formed by the growth of local perturbations since the decoupling of matter and radiation. Many have hoped that the recent observations (Perlmutter et al. 1998; Garnavich et al. 1998) of Type I supernovae (SNs) at redshifts up to $z = 0.83$ have put us beyond the region of local perturbations. It is true that regions of the cosmic background radiation that are larger than about 2° in angular size have been out of causal contact since the initial singularity and hence reflect the global rather than the local structure. Thus, going out to a redshift of $z \sim 2000$ is *sufficient* to be in the Hubble flow.

However, there has been no calculation of the *minimum* redshift that one must go out in order to be beyond local effects. This minimum redshift could be as far out as a light ray outgoing from Earth's location at the beginning of time. The set of all such light rays forms the boundary of the future light cone $\partial J^+(\gamma)$ of Earth's world line γ, with future end point today (see Fig. 1). We see back into time along the boundary of the past light cone $\partial J^-(\gamma)$, so the minimum redshift is given by the location of $\partial J^+(\gamma) \cap \partial J^-(\gamma)$.

I shall show that if the universe is flat and if the expansion has been adiabatic since the initial singularity, then the location of $\partial J^+(\gamma) \cap \partial J^-(\gamma)$ is $z = 3$; going beyond $z = 3$ definitely puts us into the Hubble flow. Thus, if the universe is flat, then structures larger than $z = 3$ are primordial; they did not arise via the growth of local perturbations. It is well known that the emerging consensus that Hubble's constant H_0 is $\simeq 75$ km s^{-1} Mpc is inconsistent with the number $\simeq 30$ km s^{-1} Mpc preferred by theory (Tipler 1994; Bartlett et al. 1995). The consensus number comes from measurements on objects with $z \le 0.83$, so it is at least conceivable that we could be in a region that is expanding more rapidly—as a result, say, of a relativistic shock originating near the initial singularity—than the universe as a whole. If so, then measuring Hubble's constant at different redshifts out to $z = 3$ would necessarily yield different values between $z = 1$ and $z = 3$. I shall discuss the only known method of measuring the cosmological parameters at $z = 3$: gravitational lens systems.

2. FLAT FRIEDMANN UNIVERSE WITH MATTER AND RADIATION

The flat Friedmann universe has the metric

$$ds^2 = R^2(\tau)[-d\tau^2 + d\chi^2 + \chi^2(d\theta^2 + \sin^2\theta d\phi^2)] , \quad (1)$$

where $0 \le \theta \le \pi$, $0 \le \phi < 2\pi$, and $0 \le \chi < +\infty$. The time variable τ in equation (1) is conformal time, related to the proper time t by $dt = R(\tau)d\tau$. The proper time, flat Friedmann equation $(R'/R)^2 = A/R^3 + B/R^4$ for an adiabatic mixture of matter and radiation becomes in conformal time

$$(dR/d\tau)^2 = AR + B . \quad (2)$$

If the singularity is chosen as the zero of time $[R(0) = 0]$ and if τ_{eq} is defined to be the conformal time when the matter and radiation densities were equal [i.e., $A/R^3(\tau_{eq}) = B/R^4(\tau_{eq})$], then equation (2) can be integrated exactly to give

$$R(\tau) = \frac{A}{4}\tau[\tau + 2(1 + \sqrt{2})\tau_{eq}] . \quad (3)$$

In the small-angle approximation, two events on the surface of last scattering with angular separation $\Delta\theta$ will have been out of causal contact since the initial singularity if

$$\Delta\theta \ge \frac{\chi_{dec}}{\chi_M} = \frac{\tau_{dec}}{\tau_{now} - \tau_{dec}} = \frac{1}{(\tau_{now}/\tau_{dec}) - 1} , \quad (4)$$

where χ_{dec} is the radial coordinate of separation between the two events, and χ_M is the radial coordinate separation between the surface of last scattering and Earth (see Fig. 2). The last two equalities in equation (4) follow from the equation $\Delta\tau = \Delta\chi$ satisfied by null geodesics (set $ds^2 = 0$ in eq. [1]).

Letting z_{dec} and z_{eq} be, respectively, the redshifts of the last scattering surface (the decoupling redshift) and the time when the universe went from being radiation to matter dominated, we obtain from equation (3) and the redshift formulae

$$z_{dec} + 1 = \frac{R(\tau_{now})}{R(\tau_{dec})} = \frac{\tau_{now}[\tau_{now} + 2(1 + \sqrt{2})\tau_{eq}]}{\tau_{dec}(\tau_{dec} + 2(1 + \sqrt{2})\tau_{eq})} , \quad (5)$$

$$\frac{z_{dec} + 1}{z_{eq} + 1} = \frac{[R(\tau_{now})/R(\tau_{dec})]}{[R(\tau_{now})/R(\tau_{eq})]} = \frac{\tau_{eq}[\tau_{eq} + 2(1 + \sqrt{2})\tau_{eq}]}{\tau_{dec}[\tau_{dec} + 2(1 + \sqrt{2})\tau_{eq}]} . \quad (6)$$

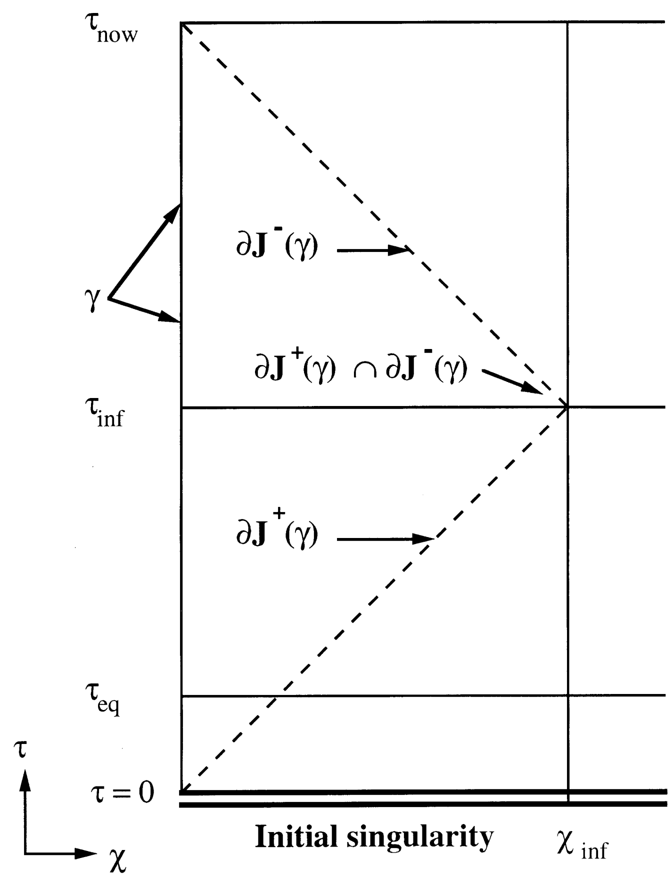

FIG. 1.—Events located at Earth's spatial position (world line γ—past endless, but future end point today) can have a causal influence only within the causal future $J^+(\gamma)$, with boundary $\partial J^+(\gamma)$. The intersection of $\partial J^+(\gamma)$ with the boundary of causal past $\partial J^-(\gamma)$ of Earth today (the region we can see) is at (χ_{inf}, τ_{inf}), where $\tau_{now} = 2\tau_{inf}$.

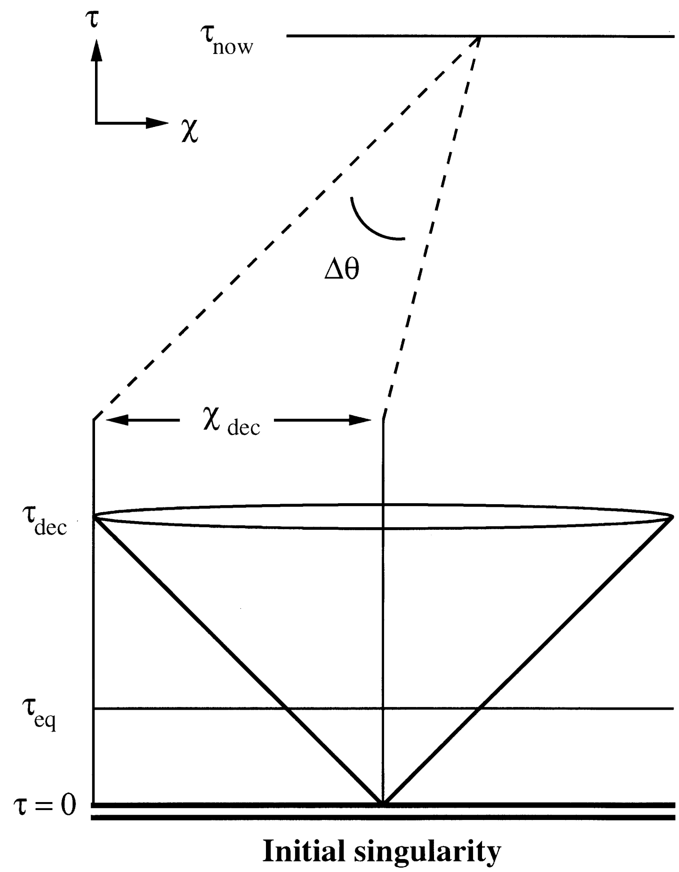

FIG. 2.—Angular separation $\Delta\theta$ of two events that first come into causal contact at the conformal time τ_{dec} when matter and radiation decouple (the surface of last scattering). The coordinate distance of separation between the two points is χ_{dec}. The time when the matter and radiation densities were equal is τ_{eq}, and τ_{now} is the time now. The initial singularity occurred at time $\tau = 0$.

These two equations can be solved for the ratio τ_{now}/τ_{dec} in terms of z_{dec} and z_{eq}, and the result substituted into equation (4). The result is

$$\Delta\theta \geq \{\sqrt{C^2(1+\sqrt{2})^2 + (z_{dec}+1)[1+2C(1+\sqrt{2})]} - C(1+\sqrt{2}) - 1\}^{-1}, \quad (7a)$$

where

$$C \equiv \left(\frac{z_{dec}+1}{z_{eq}+1}\right) \times \left\{\frac{(1+\sqrt{2}) + \sqrt{(1+\sqrt{2})^2 + [(z_{eq}+1)/(z_{dec}+1)](3+2\sqrt{2})}}{3+2\sqrt{2}}\right\}, \quad (7b)$$

and $\Delta\theta$ is in radians. Writing Hubble's constant as $H_0 = 100\ h$ km s^{-1} Mpc^{-1}, then the density of a matter-dominated flat universe is $\rho_{crit} = 3H_0^2/8\pi G = 1.87 \times 10^{-29}$ h^2 g cm^{-3}. If the radiation today consists of the cosmic background radiation (CBR) at $T = 2.735$ and three types of massless neutrinos, then $1 + z_{eq} = 2.37 \times 10^4\ h^2$ if the density parameter $\Omega_0 = 1$ (Peebles 1993, 167). Thus $h = 0.3$ gives $z_{eq} = 2100$. In the limit $z_{eq} \to \infty$ (no radiation), equation (7) becomes

$$\Delta\theta \geq \left(\frac{180}{\pi}\right)\frac{1}{\sqrt{z_{dec}+1} - 1}, \quad (8)$$

where $\Delta\theta$ is expressed in degrees. Since $\Omega_0 h = 0.013$ gives $z_{dec} = 1360$, and $\Omega_0 h^2 = 1$ gives $z_{dec} = 1520$ (Peebles 1993, 167), inequality (8) yields $\Delta\theta(z_{dec} = 1300) = 1^\circ\!.6$, and $\Delta\theta(z_{dec} = 1500) = 1^\circ\!.5$, which are the standard values one finds in the literature (e.g., Weinberg 1972, 525, who computes $2\Delta\theta$ under the no radiation assumption), although inequality (7b) does not appear to have been published. If $z_{dec} = z_{eq}$, inequality (7b) becomes

$$\Delta\theta \geq \left(\frac{180}{\pi}\right)\frac{1}{\sqrt{(2+z_{dec})(3+2\sqrt{2})} - (2+\sqrt{2})}.$$

Let χ_{inf} be the coordinate distance of the location of $\partial J^+(\gamma) \cap \partial J^-(\gamma)$. [See Tipler 1994, 404, and Hawking et al. 1973 for precise definitions of $J^{\pm}(\gamma)$.] This distance is defined by all light rays from the origin of coordinates at the initial singularity going to χ_{inf} and returning to the origin of coordinates at τ_{now} (see Fig. 1). Any event at a larger distance cannot have been affected by anything in the causal future $J^+(\gamma)$ of the coordinate origin before time τ_{inf}, the time the light from χ_{inf} left it in order to reach us now. We have $\tau_{now} = 2\tau_{inf}$, and

$$z_{inf} + 1 = \frac{R(\tau_{now})}{R(\tau_{inf})} = \frac{\tau_{now}[\tau_{now} + 2(1+\sqrt{2})\tau_{eq}]}{\tau_{inf}[\tau_{inf} + 2(1+\sqrt{2})\tau_{eq}]} = 4\left[\frac{(\tau_{inf}/\tau_{eq}) + (1+\sqrt{2})}{(\tau_{inf}/\tau_{eq}) + 2(1+\sqrt{2})}\right], \quad (9)$$

$$z_{\rm eq}+1=\frac{R(\tau_{\rm now})}{R(\tau_{\rm inf})}=\frac{2\tau_{\rm inf}[2\tau_{\rm inf}+2(1+\sqrt{2})\tau_{\rm eq}]}{\tau_{\rm eq}[\tau_{\rm eq}+2(1+\sqrt{2})\tau_{\rm eq}]}$$
$$=4\left(\frac{\tau_{\rm inf}}{\tau_{\rm eq}}\right)\left(\frac{(\tau_{\rm inf}/\tau_{\rm eq})+(1+\sqrt{2})}{3+2\sqrt{2}}\right). \quad (10)$$

Solving equations (9) and (10) for $z_{\rm inf}$ in terms of $z_{\rm eq}$ gives

$$z_{\rm inf}=4\left[\frac{\sqrt{(3+2\sqrt{2})(2+z_{\rm eq})}+(1+\sqrt{2})}{\sqrt{(3+2\sqrt{2})(2+z_{\rm eq})}+3(1+\sqrt{2})}\right]-1\approx 3\,. \quad (11)$$

So to reach definitely the Hubble flow in a flat universe with an adiabatic mixture of matter and radiation, we have to go out to a redshift of $z_{\rm inf}=3$ ($z_{\rm inf}=2.83$ if $h=0.3$ and $1+z_{\rm eq}=2.37\times 10^4\,h^2$, which comes from the observed amounts of matter and radiation; $z_{\rm inf}=3$ exactly if there is no radiation, only matter). In a matter-only flat universe, an object at $z=3$ is today at a proper distance of exactly the Hubble distance, and its proper age at $z=3$ was exactly 1/12 the Hubble time.

3. GENERAL FRIEDMANN UNIVERSE WITH MATTER ONLY

Recall that in the general (not necessarily flat) Friedmann universe, the metric (1) is replaced by

$$ds^2=R^2(\tau)[-d\tau^2+d\chi^2+\Sigma^2(\chi)(d\theta^2+\sin^2\theta\,d\phi^2)]\,, \quad (12)$$

where

$$\Sigma(\chi)=\begin{cases}\sin\chi\,, & \text{if } k=+1\,,\\ \chi\,, & \text{if } k=0\,,\\ \sinh\chi\,, & \text{if } k=-1\,.\end{cases} \quad (13)$$

The closed, flat, and open Friedmann universes are defined, respectively, by $k=+1$, $k=0$, and $k=-1$. We have $0\le\chi\le\pi$ in the closed case, and $0\le\chi<+\infty$ for the flat and open cases. As in the flat case, $0\le\theta\le\pi$, $0\le\phi<2\pi$. The time variable τ in equation (12) is as before conformal time. If the pressure is zero (no radiation), the Friedmann equation is

$$\frac{1}{R^2}\frac{dR}{dt}\equiv H^2=\frac{1}{R^4}\frac{dR}{d\tau}=\frac{8\pi G\rho_{m0}R_0^3}{3R^3}+\frac{k}{R^2}\,, \quad (14)$$

where $8\pi G\rho_{m0}R_0^3/3$ is a constant. Equation (14) evaluated at $\tau_{\rm now}$ can be written as

$$\Omega_0=1+\frac{k}{H_0^2R_0^2}\,, \quad (15)$$

where $\Omega_0\equiv\rho_0/\rho_{\rm crit}$ is the density parameter. It is well known that the solutions to equation (14) are

$$R(\tau)=\begin{cases}\dfrac{R_{\rm max}}{2}(1-\cos\tau)\,, & \text{if } k=+1\,;\\ R_1\tau^2\,, & \text{if } k=0\,;\\ \dfrac{R^2}{2}(\cosh\tau-1)\,, & \text{if } k=-1\,.\end{cases} \quad (16)$$

where the R_i are constants, and, as in the flat case, the zero of conformal time is set at the initial singularity ($R(0)=0$). Substituting equation (16) into equation (15) gives

$$\frac{2}{\Omega_0}-1=\begin{cases}\cos\tau_{\rm now}\,, & \text{if } k=+1\,;\\ 1\,, & \text{if } k=0\,;\\ \cosh\tau_{\rm now}\,, & \text{if } k=-1\,.\end{cases} \quad (17)$$

Since equation (12) is conformally flat, we have $\tau_{\rm now}=2\tau_{\rm inf}$, as in the flat case. Thus the redshift of the intersection of Earth's event horizon $\partial J^-(\gamma)$ with its particle horizon $\partial J^+(\gamma)$ is given by

$$z_{\rm inf}+1=\frac{R(\tau_{\rm now})}{R(\tau_{\rm inf})}=\frac{(R_{\rm max}/2)[1-\cos\tau_{\rm now}]}{(R_{\rm max}/2)[1-\cos(\tau_{\rm now}/2)]}$$
$$=\frac{1-\cos\tau_{\rm now}}{1-\sqrt{(1+\cos\tau_{\rm now})/2}} \quad (18)$$

in the $k=+1$ case, where it has been assumed that we are currently in the expanding phase of the closed universe. Repeating the calculation for the other two cases and putting equation (17) into the results gives

$$z_{\rm inf}=1+\frac{2}{\sqrt{\Omega_0}} \quad (19)$$

in all three cases of pressureless Friedmann universe models, closed ($\Omega_0>1$), flat ($\Omega_0=1$), and open ($\Omega_0<1$).

The calculations in § 2 demonstrate that the presence of radiation introduces only a small correction in the value of $z_{\rm inf}$. I have argued elsewhere (Tipler 1994) that the universe must be closed, but with $\Omega_0\approx 1$ on the largest scales, in which case $z_{\rm inf}=3$ would again put us into the Hubble flow. (If we happen to be in a low-density region with $\Omega_0<1$, then (19) shows that $z_{\rm inf}$ could be quite larger than 3 in such a region, for instance $z_{\rm inf}=5.5$ if we use the central value $\Omega_0=0.2$ recently obtained (Perlmutter 1998) using high-redshift supernovae. But such a large low-density region would have to be quite rare cosmologically—it would have to be essentially unique in the visible universe—and we would have to be located near the center of such a region, in order for such a region to be consistent (Shi et al. 1996) with the observed isotropy of the CBR.)

4. GRAVITATIONAL LENSES POSSIBLY IN HUBBLE FLOW

Gravitational lens systems can, at least in principle, definitely put us out into the Hubble flow. A determination of Hubble's constant using a gravitational lens requires measuring both the source redshift z_S and the lens redshift z_L. Twenty-three definite lens systems have been found, but we have z_S and z_L for only six (Kochanek 1996). I shall discuss the three most promising of these six.

A preliminary measurement of Hubble's constant H_0 using time delay of images in the gravitational lens 0957+561 has been made (Hewitt 1993), with $0.3<h<0.9$, but the lensing cluster has $z_L=0.355$ (Garrett et al. 1992) and the source quasar has $z_S=1.41$ (Press et al. 1992), both of which are short of the magic number $z=3$. The 12 day variability of lens B0218+357 has led to the hope that it can be used to measure H_0 (Patnaik et al. 1995), but once again $z_S=0.96$ (Grundahl et al. 1995), too close to be definitely in the Hubble flow. (If an isothermal sphere is used as the model for the lens, then $0.52<h<0.83$ (Grundahl et al. 1995), but the lens potential of B0218+357 is known to be nonspherical.)

The best candidate lens system discovered to date for measuring H_0 is the system in CL 1358+62 that has $z_L=$

0.33 and $z_S = 4.92$ (Franx et al. 1997). The second best candidate lens system is B1422+231, which has $z_L \approx 1$ (Lawrence et al. 1992) and $z_S = 3.62$ (Patnaik et al. 1992). Thus the source quasar is definitely in the Hubble flow, but the lens may not be. It is not known if B1422+231 has the required variability. The third best candidate is 4C 39.24, the lens of which is the most distant "giant" radio galaxy known, with $z_L = 1.883$ (Law-Green et al. 1995).

I propose that a search be made for gravitational lenses for which both components have $z > 3$ and, preferably, for which $z_L > 3$ and $z_S > 3z_L > 9$. (The constraint of z_S comes from requiring that the source was out of causal contact with the lens until the instant that the source light was lensed by the lens.) Such objects may exist; there are more than 50 quasars with $z > 4$ (Shaver 1995), and there has been a recent observation of a galaxy with $z = 4.4$. Active galactic nuclei—potential lens sources—are thought to have first appeared at $z \sim 10$ or earlier (Peebles 1993, 611).

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