Braaaaaaaains! The Undead Humbug Production Function: Now With Human Capital

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Abstract

This paper demonstrates that the human-capital augmented Cobb-Douglas function is identically equal to the rules of aggregate accounting with any factor indices and an arbitrary 'human capital' variable thrown in. It is demonstrated empirically that the term for 'total factor productivity' does not show total factor productivity at all, but rather a factor share weighted geometric mean of the profit rate and the quotient of the wage rate and human capital. It is demonstrated empirically with randomly generated data that both the calculation of this term as well as tests of its explanatory power in development economics are the result of using an arbitrary variable correlative with wages. It is also a story about zombies.

Keywords: capital, development accounting, human capital, methodology JEL Classifications: A13, B4, C43, E13, E24

> "Zombie no go think unless you tell am to think."

> > - Fela Anikulapo Kuti, "Zombie"

I Patient Zero

There is a virus among macroeconomists. It saps them of their reasoning skills, and makes them crave brains. The New Growth/New Classical/New Keynesian/New Whatever You Like school of economics has developed an obsession with human capital.¹ For mainstream economic theory, human capital provides quite a postmodern buffer against critique in the mainstream's refusal to define exactly what they mean by it. In a Stevensonian fit of cynicism, these economists see no problem defining human capital for the sake of empirics, but insist that those exact definitions are not what is meant by human capital.

As Branko Milanovic (2014), among others, has pointed out, the term human capital is as misleading as it is insidious. Heuristically, it is described as if it were indeed a machine that one invests in. Mathematically, it is treated as an augmenter of labor power. Empirically, it

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¹For the sake of brevity, I will be using the term "New Consensus" to describe the whole of these cosmetically distinct schools of thought.

is treated as a mark-up on wages. Practically, it is treated as a carrot on a stick to batter economists who refuse to follow its marginalist lead.

The whole affair is based on the Cobb-Douglas function.² In their original paper, Cobb & Douglas (1928) sought to explain an empirical regularity in terms of natural laws of economic production. Since then, the functional form Income = Shift Variable $\times \prod_i \operatorname{Factor}_i^{\gamma_i} | \sum_i \gamma_i = 1$ has been misinterpreted as a function describing production. With such an interpretation, this function has been wielded as an analytical shotgun, blasting the heads off of every criticism that income is unfairly distributed.

Many economists of this stripe will sing odes to marginal productivity and efficiency to hide naked venom of their assurances: the poor deserve to be poor. Some economists attempt to patch the callousness of this elitist nihilism: the poor don't deserve to be poor, but they are naturally poor; let's give them something. A relatively recent current of this milieu suggests: the poor should be given something so they can stop deserving to be poor.

This 'something' is human capital – a metaphysical jumble of social welfare components treated as a bionic limb one grafts into one's torso. So equipped, these otherwise malproductivity-afflicted derelicts of capitalist justice gain +1 health or +1 speed (or +1 anything really) in order to acquire +1 coin and perhaps a level up. Some play this version of *Left 4 Dead* as an MMORPG,³ allowing an individual's human capital inventory to be shared with the rest of the party.

No matter the attempts made by those questioning the theoretical and mathematical foundations of production functions in general⁴ and constant returns production functions in particular,⁵ this analytical framework continues to respawn. Over and over again, the marginal productivity framework of income distribution is called into question as internally inconsistent, terminologically imprecise, and algebraically circular. Over and over again, the zombies of mainstream economics droningly plod forth, cannibalizing the brains of each new generation of economists.⁶

II Opening Credits

Since its publication in the *Handbook of Economic Growth*, Francesco Caselli's 2005 article "Accounting for Cross-Country Income Differences" has become a primary point of reference for human capital treatments of income growth and distribution. None of its 68 pages are wasted. Caselli presents a careful, detailed, and systematic review of the literature. In spite of the mathematical oversight described below, Caselli's article is and will remain to be indispensable to literature on growth with human capital so long as literature on growth with human capital remains indispensable.

Caselli identifies Mankiw, Romer, and Weil (1992) as the beginning of the *ad-hoc* mutation of the Cobb Douglas in the New Consensus thirst for brains. Whereas that trio uses a functional form that specifies the three "factors" – labor, capital, and brains – as follows:

$$Y = K^{\alpha} H^{\beta} (AL)^{1-\alpha-\beta} \tag{1}$$

Caselli opts to follow the labor-augmenting functional form of Hall and Jones (1999). He uses data from the Penn World Tables of Heston, Summers, and Aten (2002) and on educational attainment from Barro and Lee (2001). To calibrate the model of educational attainment, Caselli uses a variation of a success measure developed by Klenow and Rodriguez-Clare (1997).

 $^{^{2}}$ Velupillai (1973) among others notes that the 'Cobb-Douglas' function itself had made its debut in economics at least as early as Knut Wicksell. We could perhaps refer to it as the Wicksell function, but perhaps Wicksell already has enough things named after him that no one in the mainstream of economics seems to want to seriously analyze.

 $^{^{3}}$ MMORPG, for the uninitiated, is an acronym for Massive Multiplayer Online Role Playing Game which allows players to play the game cooperatively or competitively, e.g., *World of Warcraft, Call of Duty* and of course *Left 4 Dead*.

⁴See for example Robinson (1953), Robinson (1961), Garegnani (1970), and Pasinetti (1966).

⁵For critique of the Cobb-Douglas, see Shaikh (1974), Carter (2011), and Simon (1979). For a critique of the CES function, see Felipe & McCombie (2001).

⁶ It would hardly be appropriate to write a zombie-themed paper in economics without referencing the wonderful book by John Quiggin (2010). While Quiggin successfully blows the heads off of many zombies trudging through the streets, he overlooks the zombie of the production function lurking in the backseat of the car. Hopefully, this can serve as an addendum to his already thorough New Consensus zombie apocalypse survival guide.

Since its publication, Caselli (2005) has been featured in a wide range of papers within New Consensus scholarship insofar as New Consensus scholarship can be said to be wide ranging. With 1118 citations, the paper features in Weil (2015), Jones and Romer (2010), Lucas (2009), Hayashi and Prescott (2006), and Temple (2005).

The empirical conventions swarm around a nebulous notion of "Total Factor Productivity," represented by A in both the Mankiw, Romer Weil and Hall and Jones specifications. In the conventions from Cobb and Douglas through its application to macroeconomic growth in Solow (1957) to the dawn of New Consensus human capital modeling, this term had been used to represent some nebulous factor transforming factor inputs into revenues generated from outputs.

In the New Consensus approach, this term's nebulousness is intolerable, but only to the degree that it is not explicitly accounted for by other variables. This is particularly troubling when the income differences across countries are largely explained by this nebulous "Total Factor Productivity" variable. Thus, New Consensus approaches seek to "explain" these difference with various ad hockeries⁷ including statistics on educational attainment, health, and overall quality of life.

Despite the apparent success of this approach, a number of red flags ought to arise in its application as a "production function." First, one ought to be suspicious of the wide range of institutionally and structurally diverse economic systems the approach is applied to including capitalist representative democracies, kleptocratic dictatorships, and feudal kingdoms. Second, one ought to be suspicious of the variety of successful measurement specifications to which the approach is subject. Third, one might be concerned about the lack of a unit measurement for "Total Factor Productivity" altogether and wonder why none of the amendments would serve to convert the diverse factor indices into the monetary value of salable goods.

Given the breadth of applicability to such a variety of social systems and measurement conventions, it's a wonder why New Consensus economists don't question whether what they are measuring is a relationship of production at all. This is the major question that this paper seeks to answer in the negative.

III Epidemiology

One of the hallmarks of New Consensus economics is a bizarre obsession with implicit form equations. The appeal of these functions is that they afford the ability to define behavioral and structural features of a model without having to assume a particular functional form.⁸ As such, Caselli (2005) boils production down to:

$$Income = F(factors, efficiency)$$
(2)

For Caselli, this rendition of the production function serves the purpose of development economics. As far as Caselli is concerned, income differentials across countries must be a result of production facts. The general functional form he chooses leaves two options for cross-country inequality. Either the differences are attributable to factors or efficiency.

If the explanation is factors, then development economists must focus on differential accumulation rates. Presumably this might also mean that development economists should focus on the cross-country distribution of factors as well, but as this contravenes the rules of zombie movies,⁹ it doesn't even register on Caselli's radar.

If the explanation for income inequality is efficiency, then additional research must be done to determine what impacts efficiency. For Caselli, this means the tireless search for additional factors that can be added to the production function.

However, before we can even begin to diagnose the composition of income differentials, we are presented with two problematics. First is that of the functional form. Since New Consensus economists are scared to death of structural equations, they find themselves in a constant search

⁷ To borrow an abbreviated form of Richard Day's *ad hoc shockeries* coined in Day (1992).

⁸ For an application of implicit form functions to derive the Cobb-Douglas and CES production functions, see Ferguson (1969). ⁹Always check in the backseat before starting a car. Shoot every zombie twice to make sure they're dead.

Let the free market decide.

for surviving functional forms to join their party. Like all zombie movies, these survivors are more often tropes than well-developed characters. This preoccupation largely revolves around the impact a given functional form will have on the measurement of efficiency as a residual of factor effects. The second problematic is that of measurement. The measurement of factors, particularly heterogeneous factors, will necessarily have an impact on the results of the analysis.

In order to buttress his analysis against criticisms of *ad hockery*, Caselli begins by discussing some robustness checks. He begins by presuming a slightly less general form equation,

$$J = A y_{KH} \tag{3}$$

where y is per capita income, y_{KH} is a factors-only function,¹⁰ and A is a residual that Caselli, afflicted with the New Classical virus, interprets as "total factor productivity." Using this rather general-form equation, Caselli uses variance decomposition to derive a statistical measure of success in minimizing the variation in the "total factor productivity" residual across countries. The resulting equation

$$\operatorname{var}[\log(y)] = \operatorname{var}[\log(y_{KH})] + \operatorname{var}[\log(A)] + 2\operatorname{cov}[\log(A), \log(y_{KH})]$$
(4)

allows us to construct a measure of success in explaining income differences in terms of factors. If a factors-only explanation were the case, the last two terms of equation 4 would be equal to zero. In such a scenario, dividing through by var[log(y)] would be equal to one. From here we derive our success measure

$$success = \frac{\operatorname{var}[\log(y_{KH})]}{\operatorname{var}[\log(y)]}$$
(5)

As we can see, the more that variations in y_{KH} explains variations in y, the closer success will be to one. Conversely, the less related these two quantities are, the closer success will be to zero. Given that this measure of success uses variance which is notorious for its sensitivity to outliers, Caselli buttresses this measure of success with a second based on 90/10 ratios rather than log variances.¹¹ Regardless, he finds that in his analysis there isn't much difference between the two approaches, and uses the latter mostly as a robustness check against the former.

The exact functional form that Caselli investigates is a standard Cobb-Douglas with labor augmented by human capital.

$$Y = AK^{\alpha}(hL)^{1-\alpha} \tag{6}$$

Given the notorious nebulousness of the term "human capital," Caselli goes to great lengths to demonstrate just how convoluted he can be.

$$h = A_h e^{\phi(s)} \tag{7}$$

The underlying features of this measure makes human capital strictly positive. A_h is an index of education quality, whereas $\phi(s)$ is a function of average years of schooling. This latter function is piecewise linear of the form

$$\phi(s) = \begin{cases} 0.134 \cdot s & s \leq 4\\ 0.134 \cdot 4 & + & 0.101 \cdot (s-4) & 4 < s \leq 8\\ 0.134 \cdot 4 & + & 0.101 \cdot 4 & + & 0.068 \cdot (s-8) & s > 8 \end{cases}$$
(8)

where s is the number of years of schooling such that s = 4 means that the working population on average has a high school diploma. The coefficients themselves are derived from an imperialist assumption based on education-earnings profiles. The first is the percentage earnings increase from an additional year of schooling on average in Sub-Saharan African countries, the second is the same for the world overall, and the last is the same for OECD countries.

¹⁰ Also known as the "underlying" production function.

¹¹The implied takeaway is that the countries that are so poor as to be in the bottom ten percent or so wealthy as to be in the top 10 percent are outside of the purview of development economics. This might be disappointing for Mali, Kiribati, Rwanda, Burkina Faso, The Gambia, Ethiopia, Comoros, Togo, Madagascar, Guinea-Bissau, Guinea, Eritrea, Mozambique, Niger, Burundi, Liberia, Malawi, The Democratic Republic of Congo, and the Central African Republic.

The second component A_h of the human capital index takes the form

$$A_h = p^{\phi_p} m^{\phi_m} k_h^{\phi_h} h_t^{\phi_h} \tag{9}$$

Here, the indices are all variables, and the exponents are all strictly positive parameters. From left to right, the variables are the teacher-pupil ratio, the teaching material per student, the structures per student, and the human capital of teachers. This last element implies a degree of recursion, which Caselli resolves by assuming a steady state such that $h = h_t$.

He then begins his calibration, beginning with h_t and moving on to p, m, and k_h . He does this by first setting the parameters on all other variables to zero, and adjusting ϕ_h until his measures of success reach 1. He then retains this value for the equation for A_h and repeats the procedure with the next parameter.

For all of his work, his procedure proves unnecessarily convoluted, not in a failure to obtain meaningful results, but in a failure to obtain meaningful understanding of what he is doing mathematically. Fundamentally, Caselli's "production function" is not a production function at all. Rather, as Shaikh (1974) pointed out about the original Cobb-Douglas function, it is merely a restatement of a basic accounting identity.

IV The Antidote

The fundamental failure of mainstream economics is a refusal to utilize economic facts which must be true while using behavioral and logistical assumptions that are never true. To demonstrate the thoroughgoing brain-deadedness of this human capital approach, I begin with the standard aggregate accounting identity which must hold true across all economies all the time.

$$Y(t) \equiv W(t) + \Pi(t) \tag{10}$$

Here we have income divided between workers in the form of wages and capitalists in the form of profits.¹² Since all income must be distributed (lest it not be income), this identity must hold true for all economies at all times. Using two arbitrary indices L and K we can derive factor price values ω and r, respectively.

$$\omega(t) \equiv \frac{W(t)}{L(t)} \tag{11a} \qquad r(t) \equiv \frac{\Pi(t)}{K(t)} \tag{11b}$$

This of course yields

$$Y(t) \equiv \omega(t)L(t) + r(t)K(t)$$

At this point, we can choose yet another arbitrary index h to further subdivide the wage component ω into a "human capital" component h and a wage residual w.

$$\omega(t) \equiv w(t)h(t) \tag{12}$$

It will be important to note that this implies that, for any given wage ω , the wage residual w will covary inversely with h. This procedure gives us

$$Y(t) \equiv w(t)h(t)L(t) + r(t)K(t)$$

If we assign y and k as the amount of Y and K per unit of L,

$$y(t) \equiv \frac{Y(t)}{L(t)}$$
(13a) $k(t) \equiv \frac{K(t)}{L(t)}$ (13b)

 $^{^{12}}$ What sorts of income count as wages and what sorts count as profits (or who count as workers and who count as capitalists) is an important question – particularly for Classical Political Economy – but largely irrelevant to the present discussion.

we arrive at

$$y(t) \equiv w(t)h(t) + r(t)k(t)$$

From here, we can take time derivatives to yield

$$\dot{y} \equiv \dot{w}h + w\dot{h} + \dot{r}k + r\dot{k}$$

that, with a little algebraic manipulation gives us

$$\dot{y} \equiv wh\frac{\dot{w}}{w} + wh\frac{\dot{h}}{h} + rk\frac{\dot{r}}{r} + rk\frac{\dot{k}}{k}$$

Dividing through by y gives us

$$\frac{\dot{y}}{y} \equiv \frac{wh}{y}\frac{\dot{w}}{w} + \frac{wh}{y}\frac{\dot{h}}{h} + \frac{rk}{y}\frac{\dot{r}}{r} + \frac{rk}{y}\frac{\dot{k}}{k}$$

Since according to Kaldor (1957) factor shares are roughly constant across countries (and since Caselli presumes this so in his analysis), we can set these shares equal to parameter values as follows

$$\frac{wh}{q} \equiv 1 - \alpha$$
 (14a) $\frac{rk}{q} \equiv \alpha$ (14b)

Implicit in this is that, from the initial accounting identity, the quantities α and $1 - \alpha$ must sum to one. Substituting these values and allowing \hat{x} to represent the rate of change in a given variable x, we can write

$$\hat{y} \equiv (1-\alpha)\hat{w} + (1-\alpha)\hat{h} + \alpha\hat{r} + \alpha\hat{k}$$

If we gather the terms $(1 - \alpha)\hat{w}$ and $\alpha\hat{r}$ into a common variable \hat{A} as below

$$\hat{A} \equiv \left[(1 - \alpha)\hat{w} + \alpha \hat{r} \right] \tag{15}$$

we arrive at

$$\hat{y} \equiv \hat{A} + (1 - \alpha)\hat{h} + \alpha\hat{k}$$

Taking the integral with respect to time and taking anti-logs yields

$$y \equiv Ak^{\alpha}h^{1-\alpha}$$

which multiplying by our labor index L gives Caselli's "production function"

$$Y \equiv AK^{\alpha}(hL)^{1-\alpha} \tag{6}$$

Thus, just as Shaikh (1974) showed about the original Cobb-Douglas function, the measure of "total factor productivity" is not a measure of productivity at all, but of factor prices. More precisely, $A = w^{1-\alpha}r^{\alpha}c_0$ where c_0 is a strictly positive constant of integration.¹³

The point of this exercise is not to show that Caselli's function does not work, but rather quite the opposite: it must work by definition. To test this proposition, I follow Shaikh (1974) in investigating how closely the percentage change in the residual calculated from $\frac{y}{k^{\alpha}h^{1-\alpha}}$ resembles the percentage change of $w^{1-\alpha}r^{\alpha}$.¹⁴

$$\frac{w_t^{1-\alpha}r_t^{\alpha}c_0 - w_{t-1}^{1-\alpha}r_{t-1}^{\alpha}c_0}{w_{t-1}^{1-\alpha}r_{t-1}^{\alpha}c_0} = \frac{w_t^{1-\alpha}r_t^{\alpha} - w_{t-1}^{1-\alpha}r_{t-1}^{\alpha}}{w_{t-1}^{1-\alpha}r_{t-1}^{\alpha}}\frac{c_0}{c_0} = \frac{w_t^{1-\alpha}r_t^{\alpha} - w_{t-1}^{1-\alpha}r_{t-1}^{\alpha}}{w_{t-1}^{1-\alpha}r_{t-1}^{\alpha}}$$

 $^{^{13}}$ The constant is strictly positive because it is technically the result of being an exponential function with the initial constant of integration as the exponent.

¹⁴The constant cancels out using a percentage change since

V The Zombie Apocalypse¹⁵

Since my contention is that this can be done with any arbitrary data that conforms to rules of accounting, I begin by using the same data used by Shaikh (1974) in figure 1.



Figure 1

As in Shaikh (1974), plotted on the horizontal axis is an arbitrary arrangement of units of capital per capita (k from above) and on the vertical axis a similarly arbitrary arrangement of output per capita (y from above). Plotted, they spell "HUMBUG."

Since according to the labor theory of value capital is dead labor and in keeping with the theme of this paper, we must conclude that human capital, plotted on the horizontal axis, must be "UNDEAD" as in figure 2.



Figure 2

 $^{^{15}\,\}mathrm{`Apocalypse'}$ here should be understood by its etymological meaning as a revelation.

Using these data in conjunction with randomly generated capital share data between 34-36%, I calculate A using both Caselli's and Shaikh's method. The results are very telling. Figure 3 is actually two lines. There is the red line representing the Shaikhian calculation, and there is a blue line representing Caselli's. The reason that the blue line does not appear to be on the chart is because it is identically equal to the red line.



Figure 3

A two-sample independent t-test yields a t-score of 0 with a corresponding p-value of 1. In other words, there is a 100% chance that the null hypothesis that these two vectors are equivalent is true. Similarly, the correlation between these two vectors is also 1. However, this alone doesn't necessarily undermine Caselli's methodology. Since his project is not to explain the *value* of the Solow residual A, but rather to minimize its variation within his sample, I have tested the validity of this conclusion in light of Caselli's first success measure. Since, as equation 5 stated

$$success = \frac{\operatorname{var}[\log(y_{KH})]}{\operatorname{var}[\log(y)]}$$
(5)

it must necessarily also be true from equation 4 that the success measure can be alternatively calculated

$$success = 1 - \frac{\operatorname{var}[\log(A)] + 2\operatorname{cov}[\log(A), \log(y_{KH})]}{\operatorname{var}[\log(y)]}$$
(16)

Since additive constants drop out of variance calculations (since they define only the position of the mean), we should be able to plug in values for A based on the factor prices and shares alone to arrive at the same value for the success measure. As such, I created and ran a simulation to generate random aggregate accounting samples and compute the difference between these two success measures. The results, in figure 4 tell a harrowing tale about the undead humbug production function.

As before, the two-sample independent t-test between these two success measures yields a t-score of 0 and a p-value of one with a correlation coefficient between them of 1. Thus, what Caselli is actually doing with these hacks on the Cobb-Douglas is in fact attempting to minimize the effect of factor prices in the deviations of incomes across countries.

Thus, what we have shown is that Caselli and all those others who use his methodology do not succeed in "explaining" the variation in "total factor productivity" with the use of "human capital." Rather they succeed in reducing the variation of price effects with the use of a divisor. A successful measure of "human capital" in actuality can be any variable correlated with either or both of the factor prices. If such a scenario obtains (as it does between the developed and developing world) we would expect that we wouldn't have to do much calibration to get a "successful" model.

Difference in Success Scores



Figure 4

VI The Aftermath

In the final analysis, the human-capital augmented Cobb-Douglas function fairs no better than its brain-dead predecessor under the scrutiny of basic calculus and algebra. The droning unresponsiveness of the mainstream to ignore the tautological nature of their supposed marginal approach¹⁶ continues not only at the peril of mainstream economists, but also the millions of people left in the lurch by the policy implications of their claims.

After these zombies are thoroughly eradicated, it becomes clear that factor prices, far from reflecting factor productivity, merely reflect the rate at which factor shares will remain constant for a given quantity of labor, capital and income. In essence, what the Cobb-Douglas hides is that, far from being a fact of production, the level of income is a result of summing up distribution.

The question arises, however, as to why human capital would improve the factors-only explanation of cross country income differences. It can be safely said that, in general wealthier countries will have higher rates of human capital as well as higher wages. Given that from equation 12 we can say that

$$\hat{\omega} \equiv \hat{w} + \hat{h} \tag{17}$$

any deviations across countries in wages will be captured by the correlative deviations in human capital. Whereas we could say that high incomes allow acquisition of high amounts of human capital were the contents of the term A made explicit, leaving it implicit allows the zombies of mainstream theory to explain in terms of efficiency what are clearly structural facts on the one hand¹⁷ and decisions of capitalists on the other.¹⁸ Thus, in more ways than one the New Consensus assurance is: the poor are too stupid not to be poor.

¹⁶I say 'supposed' because when the factor shares are substituted into the first order conditions of the Cobb-Douglas, the result is the tautology that the wage rate equals the wage rate and the profit rate equals the profit rate. In other words, the results are exactly what you can expect from taking the first derivative of a first-degree linear equation. Given this analysis, the New Consensus "explanation" of distribution reduces to "it is what it is" which is not so much an explanation as a dismissal of the question.

¹⁷Since we are in fact talking about rules of accounting.

¹⁸Since in the real world it is capitalists who set the wage rate, not "the market."

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