Infinite Sets: A Costume Party

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October 14, 2015

1 Introduction

Note: The author section was configured in this way because the authorship of this work[1] is an infinite chain dependent on every contribution made to it. The names listed serve as examples of the discrete benchmarks in this chain

Attempting to demonstrate the bijection between the set of natural and real numbers has been a wonderful journey. However, it has finally come to an end. We make use of a new tool to describe numbers: costumes. You may be wondering how numbers could possibly be related to costumes. Before this, a brief account of costumes. Consider for a moment you and your friends are getting ready for halloween. Say your friend Jane has chosen to dress up as a ghost, Mark is going as a vampire, and Phil is ready to be the greatest zombie to ever not live. When they are not wearing their costumes they are Jane, Mark and Phil. As soon as they put them on they become a ghost, vampire and zombie respectively. Nonetheless, if you see them at the party, you will know they are Jane, Mark and Phil because you either know their costume or you know their face. In a similar fashion, numbers can have costumes too.This is not to say that the number three, for instance, is a ghost, or any other similar spooky concept for that matter.

2 What should I dress up as?

The number asks itself. Upon looking at its friends it notices they have all dressed up for the rational themed party, so it puts on its rational costume

[1] As well as any work

and goes to the party, where all the other numbers have a rational costume as well. Turning away from this costume idea for a moment, we shall examine the natural numbers and the integers. We know a bijection may be found between these two sets. The method for doing this was explained as such:

1. Group the negative and positive counterparts of a number together
2. Identify this grouping as one element
3. Since every natural number has a negative counterpart in the integers, we found a bijection

Z= $\{-1, 1, -2, 2, -3, 3, -4, 4, -5, 5, ...\}$

N = $\{1, 2, 3, 4, 5, ...\}$

This demonstrates a bijection between two infinite sets. Since it exists, we call the infinities of the naturals and the integers one and the same. Specifically we identify them both as *countably* infinite. Before proceding, we must eliminate this rather contradictory manner of identifying infinity from our minds. If it is countable, it cannot possibly be infinite.

3 The Integer Costume

Say instead of going to the rational party, our number wants to go to the integer party. It goes into the closet, finds its negative and puts it on. At the party all the numbers have done the same. All of our number's friends know who it is, regardless of its costume. The theme for this party defines the integers.

4 Rational Numbers

Can we define the set of rational numbers as a party where all the numbers have put on their rational costumes? Before even thinking about an answer we need to define the rational numbers. The following can be said of any rational number.

1. It is infinitely long.[2]
2. It is unique.[3]

Following from these two definitions we employ the language of permutations. A rational number is a string length infinity whose characters are defined by the number base it is represented in. We are particularly familiar with base ten, and so if we employ this system to express our numbers, the alphabet for the characters is the set of elements one through ten. We could therefore say there are 10 to the infinity possible ways to arrange any one rational number, but this means nothing given that there are also 9 to the infinity possible ways, 8 to the infinity possible ways and even infinity to the infinity possible ways, depending on the size of our alphabet. The useful aspect of defining them as strings is that every string is distinct. Whether its length is infinite or not, it is only one string. Now we can apply our costume model to the rational numbers.

5 The Rational Costume

As stated in the introduction, the number saw that its friends had put on their rational costumes and so it went into its closet to get its costume ready for the rational themed party. How does it put on such a costume? It sees the one costume, defined as the unique string previously discussed, and puts it on.

6 Who is our number?

If we wish to show a bijection between one infinite set and another, say for instance N and R we say that the first set corresponds to a building where all the elements of that set live. Moreover we know that they are all friends[4]. Next we say that the second set is a party with a theme to which all the numbers are invited (and also go). The numbers each only have one disguise,

[2]This is true for natural numbers in the rationals since 3 is just 3.00000000... .

[3]Here I refer to the bijection between shapes and numbers. Pi is a unique number because a circle is a unique shape. There is no other pi.

[4]The use of friends in this context is to communicate the fact that the numbers all have relationships to each other. This is the basis for number sort

since their friends can always recognize them or their costume, because they have all seen each other's costume. Borrowing from current mathematical notation, we may define this costume as a function.

7 A Functional Definition

The function is identified in the following manner:

1. The function takes in a 2-tuple parameter, corresponding to the two sets we are examining[5] and a number from the set defined in the first number of the tuple.

2. Our function returns the value of the input number in terms of the second set in the tuple.

We define several examples of this function in use:

1. F(13, (N, R)) = 13.0000...

2. F(11, (N, Q)) = -11, 11

3. F(pi, (R, N)) = 31,415,926,535,897,932,384,626,433,832,795,028,841,971,693,993,751...

8 Things that are Different are Inherently Related

It was necessary to incorporate a system of definition seemingly unrelated to numbers. This allowed for a clear insight into numbers. Following from the premise that there is a bijection between any infinte set and any other infinite set, we needed to determine what it might mean for a number to be unique, since this is a key feature in defining a bijection. A number may only have one link coming out of it, corresponding to another number, also with one link coming out of it. The notions of numbers as shapes allows for a visual guide in understanding uniqueness. If we further examine this linking

[5] The order of these elements corresponds to the direction of the bijection we wish to show. If it is that N maps to R then the tuple is N,R

definition we can picture a bijection as a disconnected graph made entirely of two-vertex-one-edge graphs. In the case of a bijection between infinite sets we know each side has infinite vertices. Thus the elements of the first set, perhaps located on the left side of a paper or a brain image[6], only have one edge coming out of them, which corresponds to the same edge coming out of the right side of your visual representation of this graph.

9 Questions

The continuum hypothesis was determined undecidable. Interestingly, this undecidability was described as being the result of a limitation inherent in the natural numbers. This undecidability must therefore be eliminated by revisiting the natural numbers.

Once all the obstacles have been clearly defined there is no longer a problem.

What are our problems today?

1. Cancer has not been cured. (Un-posed questions: What defines cancer as being cured?[7])
2. World hunger. (Un-posed question: What determines one person's hunger and another's satiation?[8]
3. Problem (Un-posed question: Question defined by problem, Question defined by problem...)
4. Sudoku
5. ...
6. ...

Whenever we solve problems we must therefore ask the sufficient questions which will allow us to understand the problem, and solve it.

[6]Think of a square. What do you see and where do you see it?

[7]There is no more division of cells? There is a slower division? How slow?...

[8]How much food is in their stomach? How much food is required? More than none? How much more?...)

10 End Note: Rational Costume Revisited

Upon examination of example number three in section seven we notice that the only difference between a natural number and a rational number is a decimal point.