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Black Holes and Everyday Physics[1]

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Abstract

Black holes have piqued much curiosity. But thus far they have been important only in "remote" subjects like astrophysics and quantum gravity. We show that the situation can be improved. By a judicious application of black hole physics, one can obtain new results in "everyday physics." For example, black holes yield a quantum universal upper bound on the entropy-to-energy ratio for ordinary thermodynamical systems which was unknown earlier. It can be checked, albeit with much labor, by ordinary statistical methods. Black holes set a limitation on the number of species of elementary particles–quarks, leptons, neutrinos–which may exist. And black holes lead to a fundamental limitation on the rate at which information can be transferred for given message energy by any communication system.

The astrophysical roles of black holes are by now widely appreciated. Their importance for understanding quantum gravity is also accepted. Still, most would regard the black hole as a curiosity devoid of relevance for everyday physics. After all, astrophysics is far from the laboratory, and quantum gravity may never be testable experimentally! This view is unduly pessimistic. As I shall show, a judicious application of black hole theory can give new insights into everyday physics–ordinary thermodynamics, particle physics, communication theory.

Black holes have suggested a new principle in statistical thermodynamics: "the entropy-to-energy ratio S/E of *any* system which can be enclosed in a sphere of radius R cannot exceed $2\pi R/\hbar c$" [1] (we take $k = 1$). To see why this is so, at least for systems with negligible self-gravity, consider (for simplicity) a spherical system of radius R, *rest* energy E, entropy S, which we allow to be

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absorbed by a large black hole, so large that Hawking's radiance contributes negligibly to the entropy bookkeeping. In the process the entropy of the exterior world, S_{ext}, decreases by S; the black hole entropy S_{bh} [2, 3] increases by $\frac{1}{4}\Delta A/\hbar$ (units with $G = c = 1$), where ΔA is the growth in event horizon area. Two distinct arguments set the lower bound $\Delta A \geqslant 8\pi ER$ if the system's self-gravity is negligible. One argument [2] extends Christodoulou's reasoning [4] about reversible processes to the case when the injected particle has finite radius R. It shows that the quoted lower bound is attainable for "reversible" injection orbits. The second argument [5] obtains the same lower bound by integrating the Newman–Penrose equations for the growth in horizon area in response to the passage of stress-energy. Thus $S_{bh} \geqslant 2\pi ER/\hbar c$ (we restore c).

The generalized second law of thermodynamics [2, 3] (whose success in processes where Hawking's radiance plays a role [3] leaves little doubt about its general validity) now requires that $S \leqslant 2\pi ER/\hbar c$, for otherwise $S_{ext} + S_{bh}$ could be made to decrease by injecting the system in a "reversible" orbit. Thus, arguments based on black hole thermodynamics (which were already implicit in the early papers on the subject [2, 5]) lead to the bound $S/E \leqslant 2\pi R/\hbar c$ for *any* ordinary spherical system with negligible self-gravity. This result in statistical thermodynamics is apparently new. That the bound on S/E is trivially satisfied by systems of nonrelativistic particles was known early [5]. The applicability of the bound to systems of massless quanta, i.e., radiation, is not so trivial to demonstrate directly, but this has been done recently.

The most transparent demonstration is that of Gibbons [6]. He considered a quantum scalar field of definite energy E confined to a cavity of some shape. The entropy of the system he computed from Boltzmann's formula $S = \ln W$ in terms of the number W of many-particle states with the given energy. Gibbons worked out S/E for several cavity shapes, and showed that in every case $S/E < 2\pi R/\hbar c$ if R is taken to be the radius of the sphere which circumscribes the cavity. A more general, though more involved, approach is to compute S and the mean energy $\overline{E}$ of a quantum field in a cavity from the density matrix. One includes in $\overline{E}$ the vacuum energy in order that $\overline{E}$ be the gravitating energy—this relates it directly to the E in our gedanken experiment featuring the black hole. It has been possible to show [1] that, for arbitrary container shape, and for an arbitrary admixture of scalar, electromagnetic, and neutrino fields, $S/\overline{E} < 2\pi R/\hbar c$, where R, again, is the radius of the circumscribing sphere. In a rather different approach Wald, Sorkin, and Jiu [7] have shown that a self-gravitating sphere of radiation, which may be as close to its Schwarzschild radius as allowed by the wave character of its contents, also obeys $S/E < 2\pi R/\hbar c$.

The role played by black holes in bringing to light this new principle in thermodynamics has been primary. The argument from black holes suggested a well-defined bound for S/E. The role of the statistical arguments just described has been to confirm, to clarify the meaning of R, and to show that the bound is also valid for self-gravitating systems.

The versatility of the bound is especially highlighted when it is applied to situations not ordinarily regarded as within the province of thermodynamics. For example, one can use it to gain some information about the number of elementary building blocks of matter—quarks and leptons. The present picture of particle physics is that mesons and baryons are two or three quark composites, respectively. Quarks and leptons are arranged in "generations." The first generation contains the electron, the e-neutrino, two quarks with charges $+\frac{2}{3}$ and $-\frac{1}{3}$, respectively (two quark flavors), and all their antiparticles. Three such generations are known today, all exactly analogous in structure. There could be more; in fact, there could be twenty, or fifty, and particle physics would be saddled with hundreds of "building blocks." This depressing possibility may be ruled out by appealing again to the bound $S/E \leqslant 2\pi R/\hbar c$. An early argument of this type dealing with leptons is due to Mariwalla [8]. Because it is hadrons, not leptons, which are composite, the bound is more credible for hadrons, and it is here applied to them.

The idea is to think of, say, a baryon as a cavity or "bag" containing three quarks. One is interested in the ground states of such systems, namely, in all unexcited baryons. As a first approximation one could neglect mass differences between the various quark species, and regard all unexcited baryons as having the same radius R. The generic baryon would thus be endowed with a definite value for $ER/\hbar c$. Since it is composite, the baryon should be larger than its Compton length. Hence $ER/\hbar c$ should be somewhat larger than unity. One is at liberty to dump the generic baryon into a black hole, and recover for it the bound $S \leqslant 2\pi ER/\hbar c$. But what is S here? It is not thermal entropy—the baryon is unexcited. So S must be exclusively "composition entropy"—the measure of the information required to single out one baryon species (one set of the relevant quantum numbers) from all permitted ones. S subsumes missing information about baryon number, strangeness, hypercharge . . . of the baryon. It does not subsume charge and spin projection, because without even examining the baryon, one knows its charge and spin projection from measuring the charge and angular momentum of the hole before and after its assimilation of the baryon. Hence, S refers to the entropy associated with a generic baryon of definite charge q and spin projection S_z.

Evidently $S(q, S_z) = \ln W(q, S_z)$, where W is the number of permitted three-quark (antiquark) combinations corresponding to a baryon or antibaryon with the given q and S_z. As an example take $q = +1, S_z = +\frac{1}{2}$. $W(+1, +\frac{1}{2})$ can be computed by recalling that the three quarks or antiquarks must have different color quantum numbers. One assigns spins to the colors in all ways compatible with $S_z = +\frac{1}{2}$. One then assigns flavor quantum number to each color while respecting the constraint $q = +1$. If the quarks can be selected out of g analogous generations, there are $W = 3(g^3 + 2g)$ distinct permitted combinations. Now we demand that $\ln W \leqslant 2\pi ER/\hbar c$. For $g = 3, 4, 5, 10, 20, 50$ the left-hand side takes on values 4.60, 5.38, 6.00, 8.03, 10.1, 12.8. The bound in the right-hand side should be somewhat larger than 2π. Thus a few generations look all right. But many more than ten strain our credulity. Hence, the second law, via black holes, constrains the number of quark, lepton, and neutrino species that may exist.

Communications theory is also enriched by like considerations. One of its central problems is to establish the maximum rate at which information may be transferred given certain constraints. With an eye on radio and telecommunications, the constraint usually assumed is that the bandwidth is limited. There results the famous Shannon bound [9] on the rate of information transfer $\dot{I}$. But with a more physical motivation one might like to set as constraint that the energy E of the "message" is fixed. One may then argue as follows. The message—a material system or a packet of radiation—must be subject to thermodynamics. Thus, if one can assign a radius to it, its maximum conceivable entropy S_m should obey $S_m < 2\pi ER/\hbar c$. But S_m also gives the maximum information that can be coded into the message by employing every one of its (quantum) states as a symbol [10]. Thus, the information I in bits borne by the message is $I < 2\pi ER/(\hbar c \ln 2)$.

Now think of the message as spherical. It cannot travel faster than light; hence, it sweeps by a given point in time $T \geqslant 2R/c$. Thus an appropriate receiver cannot acquire from it information at a rate higher than $\pi E/\hbar \ln 2$ bits/sec. A more elaborate analysis allowing for general message geometry and for the Lorentz contraction supports this conclusion [11]. The rate of information acquisition $\dot{I}$ from any message of energy E cannot exceed $\pi E/\hbar \ln 2$ bits/sec (all quantities measured in frame of receiver). Thus, by combining a thermodynamic result suggested by black hole physics with causality restrictions, one gets an important rule in communication theory. This is particularly fitting since it was information considerations which very early suggested the concept of black-hole entropy [2].

The rule about $\dot{I}$ was foreshadowed in ideas of Bremermann [12], though his interpretation of it was quite different. The rule is fully consistent with Shannon's bound, but of vastly wider applicability [11]. For example, combined with a minimum of information about atomic structure, it leads to the conclusion that an *ideal* digital computer cannot perform more than 10^{15} elementary arithmetical operations per second.

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