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BIOGRAPHICAL SKETCH

The author, J. D. Bekenstein, went to undergraduate school at the Polytechnic Institute of Brooklyn from which he received both the B. S. and the M. S. degrees in physics. Since 1969 he has been at Princeton University as a graduate student. His thesis research, done under the supervision of Professor J. A. Wheeler, is concerned mostly with black-hole physics. He expects to receive his Ph. D. by June, 1972.

BLACK HOLES AND THE SECOND LAW

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ABSTRACT

Entropy can disappear down a black hole, thus leading to an apparent transcendence of the second law of thermodynamics. Yet, black holes are known to exhibit a tendency to irreversibly increase their areas, a phenomenon reminiscent of the second law. We propose a generalized form of the second law, "common entropy plus black-hole entropy never decreases", which is not transcended when entropy goes down a black hole, and which shows the tendency of the area to increase to be a consequence of the second law. We also arrive a the expression for the entropy of a black hole.

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The history of physics records many suggestions as to how the second law of thermodynamics might be transcended. Undoubtedly, the most famous of these is the one commonly called "Maxwell's demon". No satisfactory resolution of the paradox posed by Maxwell's demon was available until Brillouin[1] pointed out that information must be reckoned as negative entropy. Only then did it become clear that Maxwell's demon cannot transcend the second law at any stage of its job. The importance of Brillouin's contribution lies in his having recognized the necessity of generalizing the concept of entropy to include information in order to forestall a violation of the second law.

Black hole physics apparently offers another way for transcending the second law. Drop some entropy into a black hole, thus decreasing the entropy of the observable universe. One knows that a black hole in equilibrium has only three degrees of freedom: Mass, charge, and angular momentum.[2] Thus an exterior observer cannot possibly determine the entropy <u>inside</u> the black hole, and thus cannot verify that the increase of entropy inside the black hole compensates for the decrease of entropy in the exterior world. It thus appears that the second law is transcended in the same sense that the law of conservation of baryon number is transcended in black hole physics.[3]

Yet, it would be a mistake to consider the matter as closed. After all, the second law is essentially a statement that "natural processes are generally irreversible." And nowhere is this statement better exemplified than in black hole physics. For example, the capture of a particle by a black hole is an irreversible process. Also, a theorem of Hawking,[4] a special case of which was first established by Christodoulou,[5] states that an increase in the area of a black hole is always irreversible. Thus, changes of a black hole are generally irreversible! We have here a striking parallelism between black hole physics and thermodynamics. What does it signify? It is clear that the Hawking-Christodoulou theorem plays, at least formally, the role of the second law for black holes. Formally, the area of a black hole is its entropy; it can never decrease. Yet, if one believes in the unity of physics, one cannot accept two second laws: One for thermodynamics, and one for black hole physics. One would like to see the familiar second law and the Hawking-Christodoulou theorem emerge as two aspects of a single generalized second law. However, the apparent transcendence of the second law which we mentioned earlier warns us that the generalized second law cannot be stated in terms of common entropy alone, but must be stated in terms of a generalized entropy. Thus, inspired by Brillouin's example, we shall seek a generalization of the concept of entropy with the double purpose of saving the second law from being transcended in black hole physics, and at the same time exhibiting the Hawking-Christodoulou theorem as a consequence of the (generalized) second law.

What is the generalized entropy? Our earlier discussion suggests that the area of a black hole behaves somewhat as entropy. Thus we expect the generalized entropy to be a sum of the common entropy in the black hole exterior plus some monotonically increasing function of the black hole's area (the black-hole entropy). The generalized second law will then read: Generalized entropy never decreases. We shall now subject this proposed second law to a critical test by reconsidering the problem of the disappearance of entropy down a black hole.

Suppose that a container carrying some common entropy is sent into a Kerr black hole of mass M and angular momentum $\vec{L}$. The entropy will disappear from the observable world. At the same time M and $\vec{L}$ will increase by amounts equal to the energy and angular momentum of the container, respectively. Is it possible to arrange things in such a way that the changes in the black hole parameters correspond to no increase in the area of the black hole? If this were possible, then clearly the process here envisaged would transcend not only the familiar second law (common entropy never decreases), but also the generalized second law (common entropy plus a function of the black hole's area never decreases)! Is the above process alowed? We owe to Christodoulou[5] the analysis of the fall of a particle into a Kerr black hole under the assumption that the particle's orbit is a geodesic. He finds that in general the black hole's area increases in the process. Only when $E = \Omega p_\phi$ where Ω is the rotational frequency of the black hole, and E and p_ϕ are the energy and the component of angular momentum along the symmetry axis of the particle, respectively, does the infall of the particle cause no change in the area (reversible process). Christodoulou's reversible process appears to supply us with a counterexample to the generalized second law, for it permits the disappearance of the entropy contained in the particle without implying a compensating irreversible change of the black hole! However, recent results indicate that the assumption of geodesic motion of the particle is not appropriate in the last stages of the infall, so that Christodoulou's conclusion must be reevaluated.

Davis, Ruffini, and Tiomno[6] have found that a particle of rest mass m falling freely and radially into a Schwarzschild black hole emits <u>into</u> the black hole gravitational waves whose energy is proportional to m , and which can be distinguished from the particle itself. The fact that the energy radiated is proportional to m indicates that the change in the particle's energy E due to radiation damping is non-negligible in the later stages of infall even if m is very small. There is no reason to suppose that a similar phenomenon will not also occur for a particle falling into a Kerr black hole along a general orbit.[7] If we accept the proposition that radiation damping causes the energy of the infalling particle to vary by an amount of O(m), then a simple argument[7] shows that the particle can be captured by the black hole only if its <u>initial</u> energy and angular momentum satisfy $E = \Omega p_\phi + |O(m)|$. By Christodoulou's methods one can then show[7] that the minimum possible increase in the area of the black hole A is

$$dA = \tau_1 A^{\frac{1}{2}} m + O(m^2) \quad . \qquad (1)$$

Here τ_1 is a dimensionless quantity which can depend only on L/M^2, the only dimensionless parameter of the black hole. The τ_1 has a lower bound of order unity.

One may also consider a process in which the particle is <u>lowered</u> into the black hole by a string. An analysis for the Schwarzschild case[7] shows that the minimum possible increase in the area of the black hole is again given by (1) with a somewhat different τ_1 . What emerges from this discussion is that the black hole compensates for the infall of the particle (which could carry entropy) by irreversibly increasing its area by a non-negligible amount. This is just what we would expect from the generalized second law!

What is the expression for the entropy of a black hole? Consider a container of mass m being sent into a black hole of area A . What is the maximum entropy that the container can carry? In nature, black-body radiation is the thing with the largest entropy-to-mass ratio for given temperature T . This ratio is[8] $S/m = \frac{4}{3} T^{-1}$. Imagine the container to be filled with black-body radiation of temperature T and to have the least massive walls which will support the radiation pressure. Its overall entropy-to-mass ratio is[7]

$$S/m = \tfrac{4}{3}\tau_2 \, T^{-1} \qquad (2)$$

where τ_2 is a dimensionless constant which corrects for the fact that the entropy-to-mass ratio of the walls is smaller than that of the radiation ($\tau_2 < 1$).

The container must go down the black hole; thus, in the optimum case, it can be no larger than $\tau_3 A^{\frac{1}{2}}$ where τ_3 is a dimensionless quantity which again can depend only on L/M^2 ($\tau_3 < 1$). It follows that the majority of the wavelengths of the radiation in the container must be smaller than $\tau_3 A^{\frac{1}{2}}$. Otherwise, the radiation would not be in equilibrium and could not have a well defined temperature! Recalling that the characteristic wavelength associated with temperature T is $\lambda \approx \hbar/kT$ (k is Boltzmann's constant), we see that the temperature of the radiation cannot be smaller than $\approx \hbar/k\tau_3 A^{\frac{1}{2}}$. Thus the maximum entropy that the container can carry is (see (2))

$$S_{max} \approx \tfrac{4}{3}\tau_2 \, \tau_3 \, k A^{\frac{1}{2}} m/\hbar \, . \qquad (3)$$

If we identify the corresponding increase in the black-hole entropy dS_{bh} with S_{max}, we insure that the generalized second law will hold for any process in which a package of entropy is sent into the black hole. With tha aid of (1) we then have

$$dS_{bh} = \tfrac{4}{3}(\tau_2 \tau_3/\tau_1)\ k\ dA/\hbar \ . \qquad (3)$$

In order that S_{bh} depend only on A, and not on L and M separately, we must assume that $\eta \equiv \tfrac{4}{3}(\tau_2 \tau_3/\tau_1)$ turns out to be independent of L/M^2. Then the entropy of a black hole takes the form (conventional units restored)

$$S_{bh} = (\eta\, k\, L_P^{-2})\ A \ = \ 1.05 \times 10^{48}\ \mathrm{erg/K\text{-}cm^2}\ \eta\ A\ , \qquad (4)$$

where $L_P = (\hbar G/c^3)^{\frac{1}{2}}$ is the Planck length 1.6×10^{-33} cm. Our argument suggests that the constant η is considerably smaller than unity. The large value of the entropy S_{bh} makes manifest the highly irreversible character of gravitational collapse.

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