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Experimental violation and reformulation of the Heisenberg's error-disturbance uncertainty relation

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The uncertainty principle formulated by Heisenberg in 1927 describes a trade-off between the error of a measurement of one observable and the disturbance caused on another complementary observable such that their product should be no less than the limit set by Planck's constant. However, Ozawa in 1988 showed a model of position measurement that breaks Heisenberg's relation and in 2003 revealed an alternative relation for error and disturbance to be proven universally valid. Here, we report an experimental test of Ozawa's relation for a single-photon polarization qubit, exploiting a more general class of quantum measurements than the class of projective measurements. The test is carried out by linear optical devices and realizes an indirect measurement model that breaks Heisenberg's relation throughout the range of our experimental parameter and yet validates Ozawa's relation.

The uncertainty principle formulated by Heisenberg in 1927¹ can be stated as: Any measurement of the position Q of a particle with the error $\epsilon(Q)$ causes the disturbance $\eta(P)$ on its momentum P satisfying

$$\epsilon(Q)\eta(P) \geq \frac{\hbar}{2}. \quad (1)$$

It should be emphasized that Heisenberg¹ not only derived this relation from the famous γ -ray microscope thought experiment, but he also gave a mathematical justification¹, in which he used the relation

$$\sigma(Q)\sigma(P) \geq \frac{\hbar}{2} \quad (2)$$

for the standard deviations $\sigma(Q)$, $\sigma(P)$ of the position Q and the momentum P , defined, for instance, by $\sigma(Q)^2 = \langle Q^2 \rangle - \langle Q \rangle^2$, where $\langle \dots \rangle$ stands for the mean value in a given state.

Heisenberg¹ indeed proved Eq. (2) for Gaussian wave functions, and subsequently Kennard² proved it for general wave functions. Later, Eq. (2) has often been explained as the formal expression of Heisenberg's relation (1)³⁻⁶. However, Eq. (2) does not conclude the limitation of measurement as stated by Eq. (1), since Heisenberg's argument to derive Eq. (1) from Eq. (2) uses additional assumptions, which the prevailing view has ignored.

Heisenberg's argument has been reconstructed in the modern language as follows^{7,8}. Heisenberg assumes that (H1) the measurement with the error $\epsilon(Q)$ collapses the wave function so that the post-measurement standard deviation $\sigma(Q)$ is no more than $\epsilon(Q)$, i.e., $\epsilon(Q) \geq \sigma(Q)$, and that (H2) the error $\epsilon(Q)$ and the disturbance $\eta(P)$ do not depend on the pre-measurement state. Under assumption (H2), we can assume without loss of generality that the pre-measurement momentum is so small that all the post-measurement momentum is caused by the measurement, i.e., $\eta(P) = \langle P^2 \rangle^{1/2} \geq \sigma(P)$, where $\langle \dots \rangle$ stands for the post-measurement mean value. Then, he uses Eq. (2) to conclude Eq. (1).

In 1929, Robertson⁹ generalized Eq. (2) to an arbitrary pair of observables A , B in the form

$$\sigma(A)\sigma(B) \geq \frac{1}{2} |\langle [A, B] \rangle|, \quad (3)$$

where $[A, B] = AB - BA$. Accordingly, the generalized form of Heisenberg's relation

$$\epsilon(A)\eta(B) \geq \frac{1}{2} |\langle [A, B] \rangle| \quad (4)$$



has been accepted to hold for the error $\epsilon(A)$ of any A -measurement and the disturbance $\eta(B)$ caused by that measurement on an observable B , whereas the relation has been proven only in limited circumstances^{10–13}. Note that Eq. (4) is derived from Eq. (3) under certain assumptions^{10–13}, as Heisenberg assumed (H1) and (H2) to derive Eq. (1) from Eq. (2). Thus, Eq. (4) would not be universally valid, under conditions out of the assumptions. Nevertheless, Eq. (4) is often regarded as the generalized form of Heisenberg's original claim, Eq. (1), and we hereafter refer to Eq. (4) as Heisenberg's uncertainty relation.

In 1980, Braginsky, Vorontsov, and Thorne¹⁴ claimed that the uncertainty principle (1) leads to a sensitivity limit, called the standard quantum limit, for gravitational wave detectors using the monitoring of free mass position such as interferometer type detectors. However, following Yuen's¹⁵ proposal of exploiting "contractive states," Ozawa^{16–19} in 1988 showed a solvable model of an error-free position measurement that breaks both the standard quantum limit and the uncertainty principle (1). In a double-slit experiment, it has also been claimed that one can perform a which-way measurement of a particle without disturbing its momentum^{20,21}. Nowadays, the so-called Heisenberg limit, (1) or (4), is taken in theory to be rather a breakable limit²², but then we should ask: What is the unbreakable limit, which Heisenberg might have originally intended? Also, is the Heisenberg limit really breakable in experiment?

In 2003, Ozawa¹³ proposed an alternative relation for error and disturbance that he theoretically proved to be universally valid: Any measurement of an observable A in a state $|\psi\rangle$ with the error $\epsilon(A)$ causes the disturbance $\eta(B)$ on another observable B satisfying

$$\epsilon(A)\eta(B) + \epsilon(A)\sigma(B) + \sigma(A)\eta(B) \geq \frac{1}{2}|\langle[A, B]\rangle|, \quad (5)$$

where $\sigma(A)$ and $\sigma(B)$ stand for the standard deviations in the state $|\psi\rangle$. Ozawa's relation has two additional correlation terms, the presence of which allows the error-disturbance product $\epsilon(A)\eta(B)$ to be much below the lower bound of Eq. (4). An experimental demonstration of Ozawa's relation has been proposed by Lund and Wiseman²³, exploiting the "weak-measurement technique" used for measuring momentum transfer in Ref. 21. Recently, Erhart *et al.* have experimentally demonstrated Ozawa's relation in neutron spin measurements²⁴, using the "three-state method" for measuring error and disturbance proposed in Ref. 25.

In this paper, we report an experimental test of Ozawa's relation using the "three-state method" for a single-photon polarization qubit carried out by linear optical devices. Our test realizes an indirect measurement model, a standard model of measuring process, that validates Ozawa's relation and breaks Heisenberg's relation throughout the range of our experimental parameter, the "measurement strength" defined below. In the previous attempt²⁴, the projective measurement of a spin component is implemented by a pair of projective operations, each of which is carried out in an independent experimental set-up by a spin-analyser, which passes the measured object for only one fixed outcome (+1 or -1) of measurement. This is unlike any indirect measurement model, in which the apparatus always passes the measured object for two possible outcomes (+1 and -1) in a single experimental set-up. Moreover, our measurements are of a more general class of quantum measurements than the class of projective measurements, which were tested previously²⁴.

Results

Indirect measurement model. For the general indirect measurement model depicted in Fig. 1, the error $\epsilon(A)$ and the disturbance $\eta(B)$ are defined by¹⁹

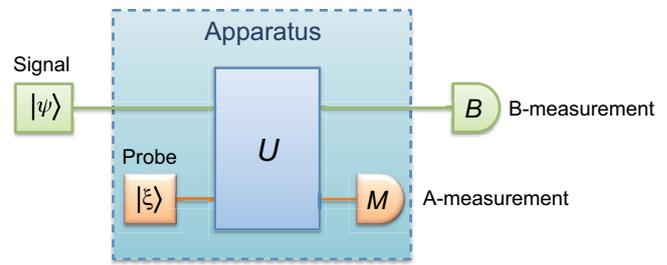


Figure 1 | Indirect measurement model for A -measurement with detection of B -disturbance. The probe initialized in the state $|\xi\rangle$ is introduced in the apparatus to make an indirect measurement of an observable A in the state $|\psi\rangle$ by interacting with the system through a unitary U . The meter observable M in the probe is precisely measured after the interaction to obtain the measurement outcome, which is used for estimating the error $\epsilon(A)$. Another observable B is also precisely measured after the interaction to obtain the data for estimating the disturbance $\eta(B)$.

$$\begin{aligned} \epsilon(A) &= \left\langle \left(U^\dagger (I \otimes M) U - A \otimes I \right)^2 \right\rangle^{1/2}, \\ \eta(B) &= \left\langle \left(U^\dagger (B \otimes I) U - B \otimes I \right)^2 \right\rangle^{1/2}, \end{aligned} \quad (6)$$

where the average is taken over the system-probe composite state on input (See Supplementary Information). The error $\epsilon(A)$ is the root-mean-square of the difference between the meter observable M after the interaction and the observable A before the interaction. The disturbance $\eta(B)$ is the root-mean-square of the change in the observable B during the measuring interaction. Note that these definitions of error and disturbance are generalizations of their classical definitions. Indeed, when $U^\dagger(I \otimes M)U$ (or $U^\dagger(B \otimes I)U$) commutes with $A \otimes I$ ($B \otimes I$), the definitions of $\epsilon(A)$ ($\eta(B)$) becomes identical to the classical root-mean-square error (disturbance).

In our experiment, both the system and the probe are qubits, called the signal qubit and the probe qubit. Let X , Y , and Z be the Pauli matrices; $|0\rangle$ and $|1\rangle$ denote the eigenstates of Z with eigenvalues $+1$ and -1 , respectively. The measurement is carried out by an interaction U between the signal qubit and the probe qubit initialized in the state $|\xi\rangle = |0'\rangle$; we use the prime symbol for probe observables and probe states, when a distinction is necessary. We take the meter observable M in the probe as $M = Z'$. The measurement operators $M_m = \langle m' | U | 0'\rangle$ with $m = 0, 1$ describe the measurement as

$$U(|\psi\rangle \otimes |0'\rangle) = M_0|\psi\rangle \otimes |0'\rangle + M_1|\psi\rangle \otimes |1'\rangle. \quad (7)$$

In this paper, we employ a general form of measurement given as

$$\begin{aligned} M_0 &= \cos\theta|0\rangle\langle 0| + \sin\theta|1\rangle\langle 1|, \\ M_1 &= \sin\theta|0\rangle\langle 0| + \cos\theta|1\rangle\langle 1|, \end{aligned} \quad (8)$$

where $0 \leq \theta \leq \pi/4$ (See Supplementary Information). The measurement strength s of this measurement is quantified by $s = \cos 2\theta = \cos^2\theta - \sin^2\theta$, varying from unity at the full-strength measurement ($\theta = 0$) to zero at the weakest measurement ($\theta = \pi/4$). The positive operator valued measure (POVM) elements corresponding to the outcomes $\lambda_0 = 1$, $\lambda_1 = -1$ are

$$\begin{aligned} \Pi_0 &= M_0^\dagger M_0 = \frac{I}{2} + \frac{s}{2}Z, \\ \Pi_1 &= M_1^\dagger M_1 = \frac{I}{2} - \frac{s}{2}Z. \end{aligned} \quad (9)$$

A theoretically simple procedure (quantum circuit) to realize the generalized measurement given in (7) and (8) is shown in Fig. 2 (a). In our experiment, as described later, to realize this measurement we employ a different procedure shown in Fig. 2 (b) that is optically

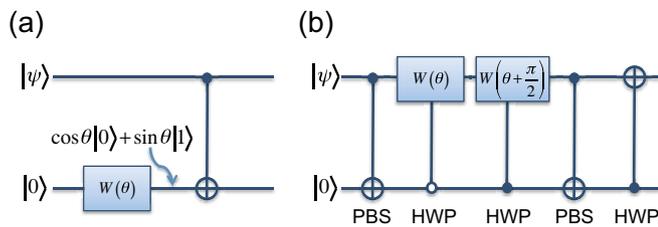


Figure 2 | Quantum circuit realization of the indirect measurement model to be tested. A theoretically simple circuit (a) and the circuit used in our experiment (b), where $W(\theta) \equiv \begin{pmatrix} \cos \theta & \sin \theta \\ \sin \theta & -\cos \theta \end{pmatrix}$. PBS and HWP in (b) stand for polarization beamsplitters and half-wave plates in Fig. 3, respectively.

implemented as in Fig. 3. Note that both circuits provide the same measurement operators defined in (7) and (8) for the probe input state $|0\rangle$, although the explicit interactions are different (See Supplementary Information). This optical implementation was previously introduced by Baek, Cheong, and Kim²⁶. The same measurement was proposed by Lund and Wiseman²³ for testing Ozawa’s relation using the “weak-measurement technique.”

Error and disturbance of the experimental model. We take the signal observable to be measured as $A = Z$ and consider the disturbance in the signal observable $B = X$. From Eq. (6), the measurement error and the disturbance for this model are calculated as

$$\epsilon(Z) = 2 \sin \theta, \quad \eta(X) = 2 \sin\left(\frac{\pi}{4} - \theta\right). \quad (10)$$

An equivalent result was given in Ref. 23. For this particular measuring apparatus, both the error and the disturbance are independent of the input state $|\psi\rangle$.

To compare Ozawa’s relation with Heisenberg’s relation, we choose the input signal state as an eigenstate of Y since it gives the maximum value of $C(Z, X) \equiv |\langle \psi | [Z, X] | \psi \rangle| / 2 = |\langle \psi | Y | \psi \rangle| = 1$, and is thus the most stringent test for these relations. For this input state the standard deviations are $\sigma(X) = 1$ and $\sigma(Z) = 1$. Heisenberg’s error-disturbance product and Ozawa’s quantity are

$$H(\theta) \equiv \epsilon(Z)\eta(X) = 4 \sin \theta \sin\left(\frac{\pi}{4} - \theta\right), \quad (11)$$

$$O(\theta) \equiv \epsilon(Z)\eta(X) + \epsilon(Z)\sigma(X) + \sigma(Z)\eta(X) = (2 \sin \theta + 1) \left(2 \sin\left(\frac{\pi}{4} - \theta\right) + 1 \right) - 1. \quad (12)$$

Then, we have $0 \leq H(\theta) \leq 2 - \sqrt{2} \leq 1$ and $\sqrt{2} \leq O(\theta)$ for $0 \leq \theta \leq \pi/4$. Thus, Ozawa’s relation (5) always holds, while Heisenberg’s relation (4) fails for all measurement strengths. Detailed materials are available as Supplementary Information.

The violation of Heisenberg’s relation in this model has been in part anticipated from a previous analysis²⁷, in which we found that the projective measurement, which corresponds to the case where $s = 1$, or $\theta = 0$, of a spin component violates Heisenberg’s relation, since the error should be zero but the disturbance should be at most two. Thus, if the measurement strength s varies continuously from $s = 1$, it can be expected from the continuity of the error and the disturbance that Heisenberg’s relation would be violated in some interval of the measurement strength including $s = 1$. In the present experiment, it also happens that the product would be zero at the opposite end, where $s = 0$, since the disturbance should be zero but the error should be at most two. The product is eventually below Heisenberg’s limit for all values of s .

Experimental test of error-disturbance relation. We study the error-disturbance relation for the measurement of a photon polarization qubit; horizontal and vertical polarizations are chosen as the eigenstates $|0\rangle$ and $|1\rangle$ of Z , respectively, and $\pm 45^\circ$ polarizations correspond to the eigenstates $|0_X\rangle$ and $|1_X\rangle$ of X . Our experimental scheme is shown in Fig. 3, which realizes the quantum circuit in Fig. 2 (b) (See Methods).

The experimentally measured error and disturbance quantities are shown in Fig. 4. Solid circles denote the measurement error (a) and disturbance (b) as functions of measurement strength. We clearly see that as the measurement strength increases, the measurement error of the observable Z decreases, while the disturbance of the observable X increases. The dashed lines show the theoretically calculated error and disturbance for the ideal generalized measurements, which are

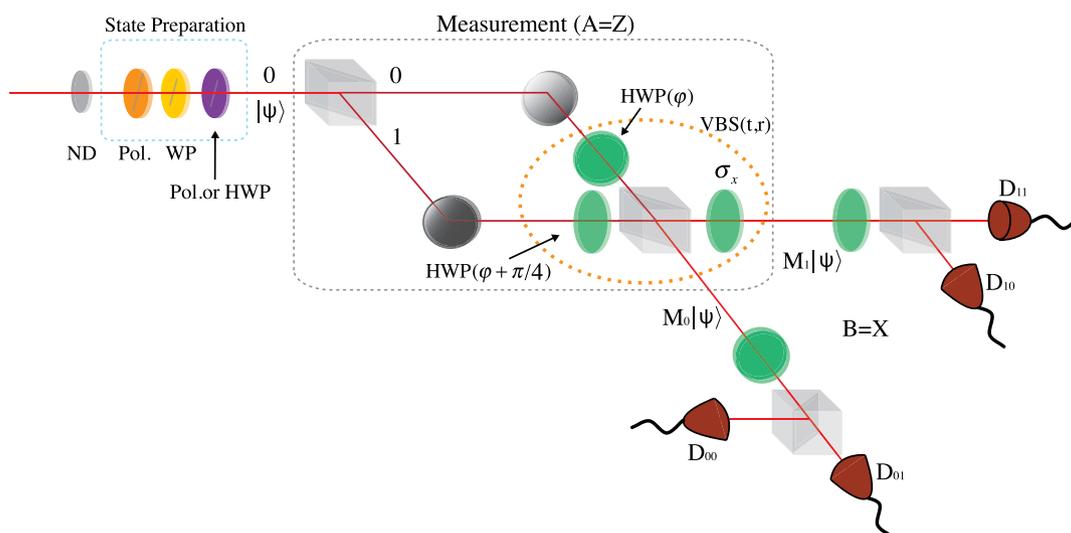


Figure 3 | Experimental setup to test the error-disturbance relation. ND (neutral density filter), Pol. (vertical polarizer), and WP (wave plates) prepare the initial polarization qubit $|\psi\rangle$. Pol. or HWP is inserted to prepare $Z|\psi\rangle$, $(Z + I)|\psi\rangle$, $X|\psi\rangle$, and $(X + I)|\psi\rangle$. VBS(t,r) is realized by using a pair of HWPs and a PBS. A HWP at an angle of 22.5° and a PBS are used to carry out the projective measurement of X .

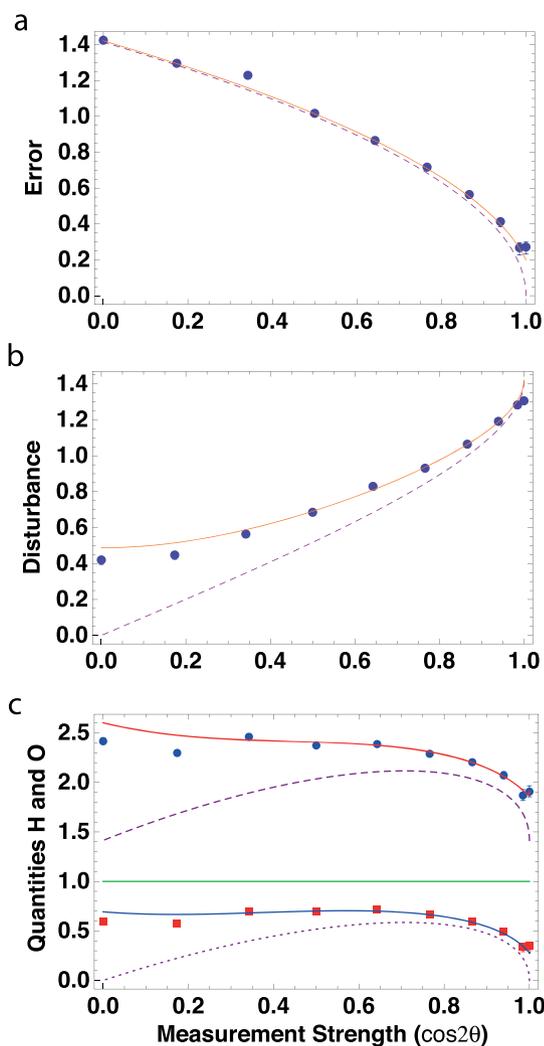


Figure 4 | Measurement error (a) and disturbance (b) as functions of measurement strength $s = \cos 2\theta$. Solid lines show the theoretical error and the disturbance after the non-ideal extinction ratio of a PBS is taken into account. Dashed lines show theoretical curves for an ideal PBS, which has perfect extinction ratio. Experimentally measured quantities O (solid circles) and H (solid squares) appearing in Ozawa's quantity (12) and Heisenberg's quantity (11), respectively (c). Upper and lower solid lines are corresponding theoretical plots as functions of measurement strength after the non-ideal PBS extinction ratio is taken into account. Dashed and dotted lines are theoretical plots for an ideal PBS. From Eq. (4) and Eq. (5), both uncertainty relations have the same lower bound $C(Z, X) = 1$ (middle solid line). The data clearly demonstrate that Ozawa's relation is always valid, whereas Heisenberg's relation is false for all measurement strengths.

defined in Eqs. (7) and (8). It is found that the error and the disturbance quantities are affected by the imperfect extinction ratio, i.e., the polarization contrast, of the polarization beamsplitters (PBSs) used in the measurement (see Supplementary Information). The experimentally measured error and disturbance closely follow the theoretically calculated error and disturbance after the PBS extinction ratio is taken into account (solid lines).

From the experimentally measured error and disturbance, we evaluate Ozawa's quantity (solid circles) and Heisenberg's quantity (solid squares) in Fig. 4(c). The upper and lower solid lines are the corresponding theoretical plots after the non-ideal PBS extinction ratio is taken into account. The dashed and dotted lines are theoretical plots for an ideal PBS. The same curves were given in Fig. 3 in Ref. 23. As shown in Eq. (4) and Eq. (5), both relations have the same

lower bound $C(Z, X) = 1$ (middle solid line). The data clearly demonstrate that Ozawa's relation always holds, whereas Heisenberg's relation fails for all measurement strengths.

Discussion

In this paper, we have proposed and demonstrated a method for experimentally testing Ozawa's universally valid reformulation of Heisenberg's trade-off relation of the error and the disturbance in photon polarization measurements. Based on generalized quantum measurements of the single-photon polarization qubit, we demonstrated an interesting case where Ozawa's relation always holds but Heisenberg's relation always fails.

Following to Ozawa's works^{8,16–19}, other approaches to the trade-off relations between the error and the disturbance have been stimulated^{28–33}. Some of those proposed the use of state-independent measures of error and disturbance^{28–31}. Although such state-independent measures would be useful in some cases, state-dependent measures such in Eq. (6) are still valuable, as Kennard and Robertson's formulae, Eq. (2) and Eq. (3), are indeed state-dependent. We also note the case of "unbiased measurements", i.e.,

$$\begin{aligned} \langle U^\dagger(I \otimes M)U \rangle - \langle A \otimes I \rangle &= 0, \\ \langle U^\dagger(B \otimes I)U \rangle - \langle B \otimes I \rangle &= 0, \end{aligned} \quad (13)$$

which are sometimes assumed when the trade-off relations between the error and the disturbance are dealt with. Actually, this condition is included in the assumptions necessary to derive Eq. (4) from Eq. (3)^{10–13}. Thus, if one assumes the unbiased measurements, it is quite natural that Heisenberg's relation holds. On the contrary, Ozawa's approach does not require such assumptions and thus Ozawa's relation is universally valid even in circumstances out of such assumptions. In our experiments, which are indeed out of such assumptions because the measurements used are not unbiased, Heisenberg's relation is violated yet Ozawa's relation holds.

A correct understanding and experimental confirmation of a fundamental limitation of measurements will not only foster insight into foundational problems but also advance the precision measurement technology in quantum information processing. We have confirmed that the "three-state-method" successfully determines the error and the disturbance of the photonic measuring apparatus. This opens a way to a new technology for treating the error and the disturbance as measurable quantities in prospect for applications to secure quantum communication. The new universal limit of measurements that we have confirmed will give an ultimate limit for quantum metrology, in which it is now more important to know the unbreakable limit than to break the old one.

Note added. While completing this manuscript, we became aware that Rozema *et al.*³⁴ have experimentally examined the error-disturbance (or measurement-disturbance) relationship by the weak-measurement technique using polarization-entangled photons.

Methods

In our experimental setup (Fig. 3), we use a strongly attenuated diode laser, i.e., a weak coherent light, as the photon source. A polarizer and wave plates prepare the initial polarization qubit $|\psi\rangle = (|0\rangle + i|1\rangle)/\sqrt{2} = |0_Y\rangle$, one of the eigenstates of Y . A half-wave plate (HWP) or a polarizer is used to prepare the states $Z|\psi\rangle$, $X|\psi\rangle$, $(Z + I)|\psi\rangle = 2|0\rangle\langle 0|\psi\rangle$, and $(X + I)|\psi\rangle = 2|0_X\rangle\langle 0_X|\psi\rangle$, which are required for the three-state method for the Z and X measurements (see Supplementary Information). The probe path qubit, which is initialized at $|0\rangle$, is introduced to the same single-photon state by directing the photon at one (labelled 0) of the two input modes of the polarization beamsplitter (PBS).

For the prepared initial two-qubit state $|\psi\rangle \otimes |0\rangle$, the CNOT operation is implemented with the PBS. The conditional Hadamard-like operations are implemented with the HWPs placed in the path modes 0 and 1. The HWP with angle φ corresponds to the Hadamard-like operation $W(2\varphi)$ in Fig. 2 (b). The second CNOT operation is again implemented with the second PBS, and the third CNOT is carried out by the HWP ($\varphi = \pi/4$) placed in mode 1 after the second PBS. In this way, the Z measurement with arbitrary measurement strength is implemented and the outcome is held in the probe, i.e., the path qubit.



The signal qubit is now subjected to the X measurement, i.e., about $\pm 45^\circ$ polarizations, with a HWP and a PBS followed by two (for a total of four) detectors, D_{ij} . The subscripts i and j denote the Z and X measurement outcomes, respectively. We record the photon counts $N_{ij}(\xi)$ of the four detectors D_{ij} for the input signal state $|\xi\rangle$. From the results for the input states $|\xi\rangle = |\psi\rangle, Z|\psi\rangle, X|\psi\rangle, (Z + I)|\psi\rangle$, and $(X + I)|\psi\rangle$, we evaluate the error (disturbance) in the Z (X) measurement. Detailed materials are available as Supplementary Information.

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Author contributions

This work was based on the theoretical study by M.O. K.E. conceived and supervised the experimental research. S.-Y.B. and K.E. designed the experimental implementation. S.-Y.B. and F.K. carried out the experiment. S.-Y.B., F.K. and K.E. analysed the data. S.-Y.B., M.O. and K.E. co-wrote the manuscript.

Additional information

Supplementary information accompanies this paper at <http://www.nature.com/scientificreports>

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